

LECTURE NOTES ON

FLUID MECHANICS & FLUID POWER(Th.4)



PREPARED BY

Er. Soma Dalbehera, LECTURER STAGE-II

DEPARTMENT OF MECHANICAL ENGINEERING
GOVT. POLYTECHNIC, NAYAGARH-752080

Chapter-1

Properties of Fluid

Fluid

Definition:

A fluid is a substance which is capable of flowing or a substance which deforms continuously when subjected to external shearing force.

Characteristics:

- It has no definite shape of its own but will take the shape of the container in which it is stored.
- A small amount of shear force will cause a deformation.

Classification:

A fluid can be classified as follows:

- Liquid
- Gas

Liquid:

It is a fluid which possesses a definite volume and assumed as incompressible

GAS:

It possesses no definite volume and is compressible.

Fluids are broadly classified into two types.

- Ideal fluids
- Real fluids

Ideal fluid:

An ideal fluid is one which has no viscosity and surface tension and is incompressible actually no ideal fluid exists.

Real fluids:

A real fluid is one which has viscosity, surface tension and compressibility in addition to the density.

PROPERTIES OF FLUIDS:**1. density or mass density : (S)**

Density of a fluid is defined as the ratio of the mass of a fluid to its vacuum. It is denoted by δ The density of liquids are considered as constant while that of gases changes with pressure & temperature variations.

Mathematically

$$\rho = \frac{\text{mass}}{\text{volume}}$$

$$\text{Unit} = \frac{\text{kg}}{\text{m}^3}$$

$$\rho_{\text{water}} = 1000 \frac{\text{kg}}{\text{m}^3}$$

$$\text{or } \frac{\text{gm}}{\text{cm}^3}$$

2. **Specific weight or weight density((W):**

Specific weight of a fluid is defined as the ratio between the weights of a fluid to its volume. It is denoted by W.

$$\text{Mathematically } W = \frac{\text{weight of fluid}}{\text{volume of fluid}}$$

$$= \text{mg/v}$$

$$W = \rho g$$

$$\text{Unit} - \frac{N}{\text{m}^3}$$

3. **Specific volume:**

Specific volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called specific volume.

Mathematically

$$\text{Specific volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\frac{\text{Mass of fluid}}{\text{Volume}}} = \frac{1}{\rho}$$

$$\text{Unit: } \frac{\text{m}^3}{\text{kg}}$$

4. **Specific gravity:**

Specific gravity is defined as the ratio of the weight density of a fluid to the density or when density standard fluid.

For liquids the standard fluid is water.

For gases the standard fluid is air.

It is denoted by the symbol S

$$\text{Mathematically, } S(\text{for liquids}) = \frac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$$

$$S(\text{for gases}) = \frac{\text{Weight density (density) of gas}}{\text{Weight density (density) of air}}$$

$$\begin{aligned}\text{Thus weight density of a liquid} &= S \times \text{Weight density of water} \\ &= S \times 1000 \times 9.81 \text{ N/m}^3\end{aligned}$$

$$\begin{aligned}\text{The density of a liquid} &= S \times \text{Density of water} \\ &= S \times 1000 \text{ kg/m}^3.\end{aligned}$$

Simple Problems:

Problem: - 1

Calculate the specific weight, density and specific gravity of one litre of a liquid which weighs 7N.

Solution. Given :

$$\text{Volume} = 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \quad \left(\because 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \text{ or } 1 \text{ litre} = 1000 \text{ cm}^3 \right)$$

$$\text{Weight} = 7 \text{ N}$$

$$(i) \text{ Specific weight } (w) = \frac{\text{Weight}}{\text{Volume}} = \frac{7 \text{ N}}{\left(\frac{1}{1000} \right) \text{ m}^3} = 7000 \text{ N/m}^3. \text{ Ans.}$$

$$(ii) \text{ Density } (\rho) = \frac{w}{g} = \frac{7000}{9.81} \text{ kg/m}^3 = 713.5 \text{ kg/m}^3. \text{ Ans.}$$

$$(iii) \text{ Specific gravity} = \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000} \quad \{ \because \text{Density of water} = 1000 \text{ kg/m}^3 \}$$
$$= 0.7135. \text{ Ans.}$$

Problem: - 2

Calculate the density, specific weight and specific gravity of one litre of petrol of specific gravity = 0.7

Solution. Given : $\text{Volume} = 1 \text{ litre} = 1 \times 1000 \text{ cm}^3 = \frac{1000}{10^6} \text{ m}^3 = 0.001 \text{ m}^3$

$$\text{Sp. gravity} \quad S = 0.7$$

(i) Density (ρ)

Using equation (1.1.A),

$$\text{Density } (\rho) = S \times 1000 \text{ kg/m}^3 = 0.7 \times 1000 = 700 \text{ kg/m}^3. \text{ Ans.}$$

(ii) Specific weight (w)

$$\text{Using equation (1.1),} \quad w = \rho \times g = 700 \times 9.81 \text{ N/m}^3 = 6867 \text{ N/m}^3. \text{ Ans.}$$

(iii) Weight (W)

$$\text{We know that specific weight} = \frac{\text{Weight}}{\text{Volume}}$$

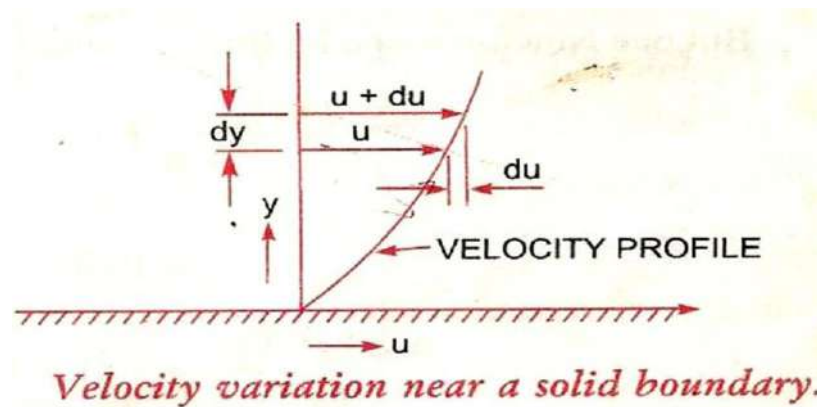
$$w = \frac{W}{0.001} \text{ or } 6867 = \frac{W}{0.001}$$

$$\therefore W = 6867 \times 0.001 = 6.867 \text{ N. Ans.}$$

Viscosity:

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

Let two layers of a fluid at a distance dy apart, move one over the other at different velocities u and $u + du$.



The viscosity together with the relative velocity between the two layers while causes a shear stress acting between the fluid layers, the top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer.

This shear stress is proportional to the rate of change of velocity with respect to y . It is denoted by r .

Mathematically

$$r \propto \frac{du}{dy}$$
$$r = \mu \frac{du}{dy}$$

Where μ = co-efficient of dynamic viscosity or constant of proportionality or viscosity

$\frac{du}{dy}$ = rate of shear strain or velocity gradient

$$\mu = \frac{c}{\frac{du}{dy}}$$

If $\frac{du}{dy} = 1$,

then $\mu = c$

Viscosity is defined as the shear stress required to produce unit rate of shear strain.

Unit of viscosity in S.I system - $\frac{Ns}{m^2}$

in C.G.S - $\frac{Dyne \ s}{cm^2}$

in M.K.S. - $\frac{kgfs}{m^2}$

$\frac{Dyne \ s}{cm^2} = 1 \text{ Poise}$

$1 \frac{Ns}{m^2} = 10 \text{ poise}$

$1 \text{ Centipoise} = \frac{1}{100} \text{ poise}$

Kinematic Viscosity:

It is defined as the ratio between the dynamic viscosity and density of fluid.

It is denoted by ν .

Mathematically

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho} \quad \dots(1.4)$$

The units of kinematic viscosity is obtained as

$$\begin{aligned} \nu &= \frac{\text{Units of } \mu}{\text{Units of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \frac{\text{Mass}}{(\text{Length})^3}} = \frac{\text{Force} \times \text{Time}}{\frac{\text{Mass}}{\text{Length}}} \\ &= \frac{\text{Mass} \times \frac{\text{Length}}{(\text{Time})^2} \times \text{Time}}{\left(\frac{\text{Mass}}{\text{Length}} \right)} \quad \left\{ \begin{array}{l} \because \text{Force} = \text{Mass} \times \text{Acc.} \\ = \text{Mass} \times \frac{\text{Length}}{\text{Time}^2} \end{array} \right\} \\ &= \frac{(\text{Length})^2}{\text{Time}} \end{aligned}$$

In MKS and SI, the unit of kinematic viscosity is metre²/sec or m²/sec while in CGS units it is written as cm²/s. In CGS units, kinematic viscosity is also known stoke.

$$\text{Thus, one stoke} = \text{cm}^2/\text{s} = \left(\frac{1}{100} \right)^2 \text{ m}^2/\text{s} = 10^{-4} \text{ m}^2/\text{s}$$

$$\text{Centistoke means} = \frac{1}{100} \text{ stoke.}$$

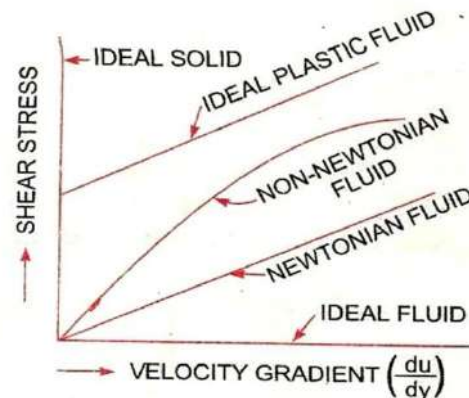
Newton's law of viscosity:

It states that the shear stress on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity.

Mathematically

$$\tau = \mu \frac{du}{dy}$$

Fluids which obey the above equation or law are known as Newtonian fluids & the fluids which do not obey the law are called Non-Newtonian fluids.



Surface tension:

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a stretched membrane under tension. The magnitude of this force per unit length of the free will has the same value as the surface energy per unit area.

It is denoted by σ

Mathematically
$$\sigma = \frac{F}{L}$$

Unit in si system is N/m

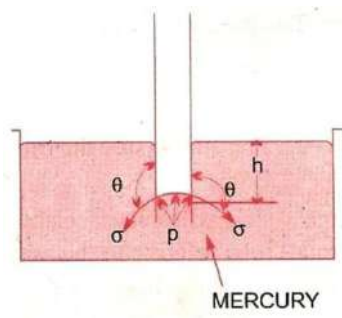
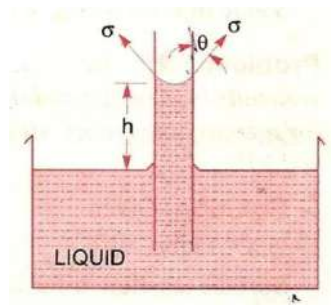
CGS system is Dyne/cm

MKS system is kgf/m

Capillarity:

Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression.

It is expressed in terms of cm or mm of liquid



Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

Chapter-2

Fluid Pressure And It's Measurements

Syllabus:

- 2.1 Definitions and units of fluid pressure, pressure intensity and pressure head
- 2.2 Concept of atmospheric pressure, gauge pressure, vacuum pressure and absolute pressure
- 2.3 Pressure measuring instruments Manometers: Simple and differential Bourdon tube pressure gauge (Simple Numerical)

Pressure of a Fluid:

When a fluid is contained in a vessel, it exerts force at all points on the sides & bottoms of the container. The force exerted per unit area is called pressure.

If P = Pressure at any point

F = Total force uniformly distributed over an area

A = unit area

$$P = F/A$$

Unit of pressure - $\frac{kgf}{m^2}$ in M.K.S.

- $\frac{N}{m^2}$ in S.I.

- $\frac{Dyne}{cm^2}$

$$1 \text{ pascal} = 1 \text{ N/m}^2$$

$$1 \text{ kpa} = 1000 \text{ N/m}^2$$

Pressure head of a liquid:

A liquid is subjected to pressure due to its own weight, this pressure increases as the depth of the liquid increases.

Let a bottomless cylinder stand in the liquid

Let w = specific weight of the liquid.

H = height of the liquid in the cylinder.

A = Area of the cylinder.

$$P = \frac{F}{A} = \frac{\text{weight of the liquid in the cylinder}}{\text{Area of the cylinder}}$$

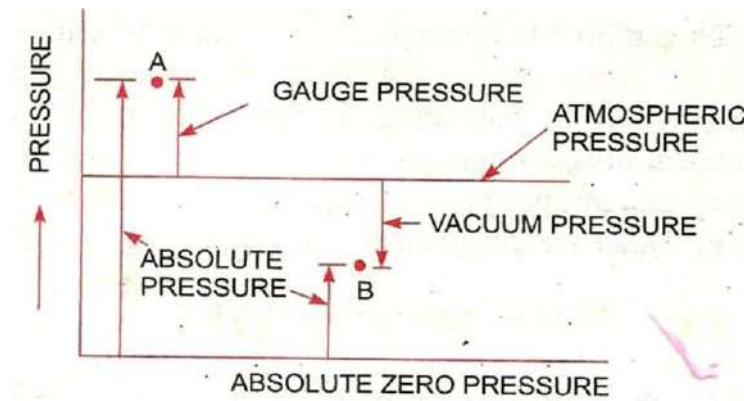
$$= \frac{W \times A h}{A}$$

$$= Wh$$

$$= \rho gh$$

So intensity of pressure at any point in a liquid is proportional to its depth.

ABSOLUTE, GAGUE, ATOMOSPHERIC, AND VACCUME PRESSURES:



Atmospheric Pressure:

The atmospheric air exerts a normal pressure upon all surfaces with which it is in contact & known as atmospheric pressure.

Absolute pressure:

It is defined as the pressure which is measured with reference to absolute vacuum pressure or absolute zero pressure.

Gauge pressure:

It is defined as the pressure which is measured with the help of a pressure measuring instrument in which the atmospheric pressure is taken as datum. The atmospheric pressure on the scale is marked as zero.

Vacuum pressure:

It is defined as the pressure below the atmospheric pressure.

Mathematically:

$$\text{Absolute pressure} = \text{Atmospheric pressure} + \text{gauge pressure}$$

$$\text{Or } P_{\text{abs}} = P_{\text{atm}} + P_{\text{gauge}}$$

$$\text{Vacuum pressure} = \text{Atmospheric pressure} - \text{Absolute pressure}$$

$$P_{\text{vacuum}} = P_{\text{atm}} - P_{\text{abs}}$$

Pressure Measuring Instruments:

The pressure of a fluid is measured by the following devices :

1. **Manometers**
2. **Mechanical Gauges.**

Manometers:

Manometers are defined as the device used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same another column of the fluid. They are classified as:

- (a) **Simple manometers.**
- (b) **Differential Manometers.**

Mechanical Gauges:

Mechanical gauges are defined as the device used for measuring the pressure by balancing the fluid column by the spring or dead weight. Commonly used mechanical pressure gauges are :

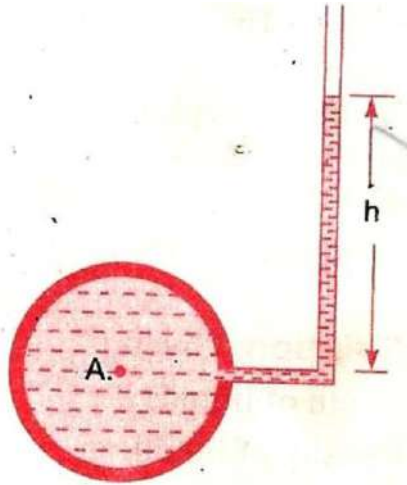
- **Diaphragm pressure gauge**
- **Bourdon tube pressure gauge**
- **Dead –weight pressure gauge**
- **Bellow pressure gauge**

Simple Manometres:

A simple manometer of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :

- **Piezometer**
- **U- tube Manometer**
- **Single Column Manometer**

Piezometer:

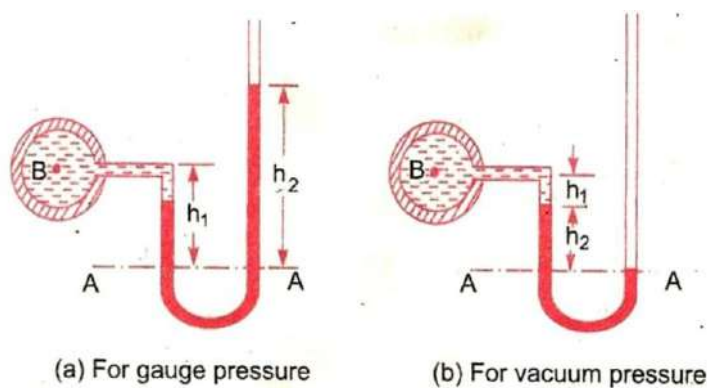


It is the simple form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Figure. The rise of liquid gives the pressure head at that point A. Then pressure at A

$$P_A = pgh$$

U – tube Manometer:

It consist of glass tube bent in U- shape , one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere as shown in figure. The tube generally contains mercury.



(a) For Gauge Pressure:

Let p be the point which is to be measured, whose value is p . The datum line is A-A.

Let h_1 = Height of light liquid above the datum line

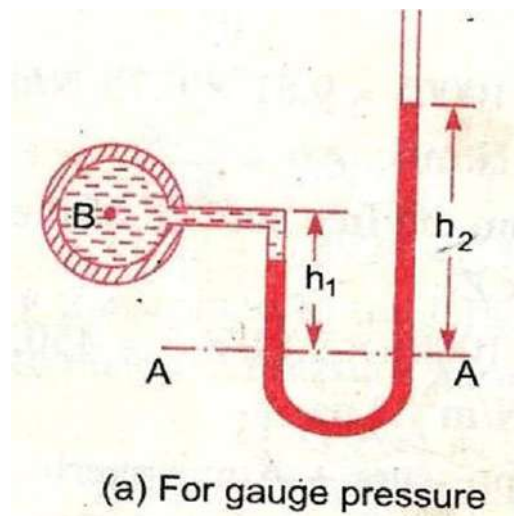
h_2 = Height of heavy liquid above the datum line

S_1 = Sp. gr. of light liquid

ρ_1 = Density of light liquid = $1000 \times S_1$

S_2 = Sp. Gr. Of heavy weight

ρ_2 = density of heavy weight = $1000 \times S_2$



Pressure is same in a horizontal surface. Hence pressure above the horizontal datum surface line A-A in the left column and in the right column of U-tube manometer should be same pressure above A-A in the left column

$$= p_A + \rho_1 \times g \times h_1$$

Pressure above A-A in the right column

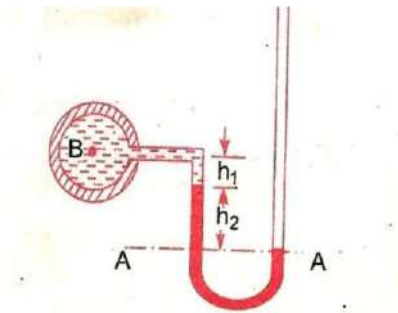
$$= \rho_2 \times g \times h_2$$

Hence equating the two pressures

$$p_A + \rho_1 g h_1 = \rho_2 g h_2$$

$$p_A = (\rho_2 g h_2 - \rho_1 g h_1).$$

(b) For Vacuum Pressure:



For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in figure. Then Pressure above A-A in the left column

$$= \rho_2 g h_2 + \rho_1 g h_1 + p_A$$

Pressure head in the right column above A - A = 0

$$\rho_2 g h_2 + \rho_1 g h_1 + p_A = 0$$

$$p_A = - (\rho_2 g h_2 + \rho_1 g h_1)$$

Single Column Manometer:

Single column Manometer is modified form of a U- tube manometer in which a reservoir, having a large cross- sectional area (about 100 times as compared to the area of the tube) is connected to one of the limbs (say left limb) of the manometer as shown in figure. Due to large cross- sectional area of the reservoir, for any variation in pressure, the change in the liquid level in the reservoir will be very small which may be neglected and hence the pressure is given by the height of liquid in the other limb. The other limb may be vertical or inclined. Thus there are two types of single column manometer as:

- **Vertical Single Column Manometer**
- **Inclined Single Column Manometer**

1. Vertical Single Column Manometer:

Let X-X be the datum line in the reservoir and in the right limb of the manometer, when it is not connected to the pipe. When the manometer is connected to the pipe, due to high pressure at A, the heavy liquid in the reservoir will be pushed downward and will rise in the right limb.

Let Δh = Fall of heavy liquid in reservoir

H_2 = rise of heavy liquid in right limb

H_1 = height of center of pipe above X-X

P_A = Pressure at A, which is to be measured

A = Cross – sectional area of the reservoir

a = Cross sectional area of the right limb

S_1 = Sp.gr.of liquid in pipe

S_2 = Sp.gr. of heavy weight liquid in reservoir and right limb

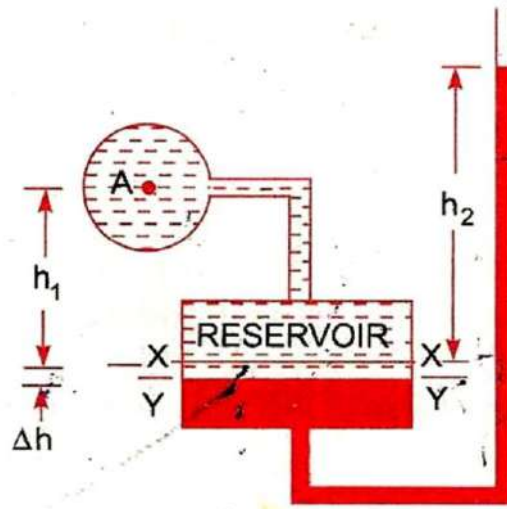
P_1 = Density in liquid in pipe

P_2 = Density of liquid in the reservoir

Fall of heavy liquid in the reservoir will cause a rise of heavy liquid level in the right limb.

$$\therefore A \times \Delta h = a \times h_2$$

$$\therefore \Delta h = \frac{a \times h}{A} \dots\dots\dots (i)$$



Now consider the datum line Y-Y as shown in Fig 2.15. Then pressure in the right limb above Y-Y.

$$= \rho_2 \times g \times (\Delta h + h_2)$$

Pressure in left limb above Y-Y $= \rho_1 \times g \times (\Delta h + h_1) + p_A$

Equating the pressure, we have

$$\rho_2 \times g \times (\Delta h + h_2) = \rho_1 \times g \times (\Delta h + h_1) + P_A$$

$$P_A = \rho_2 g (\Delta h + h_2) - \rho_1 g (\Delta h + h_1)$$

$$= \Delta h [\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g$$

But from equation (i), $\Delta h = \frac{a \times h}{A}$

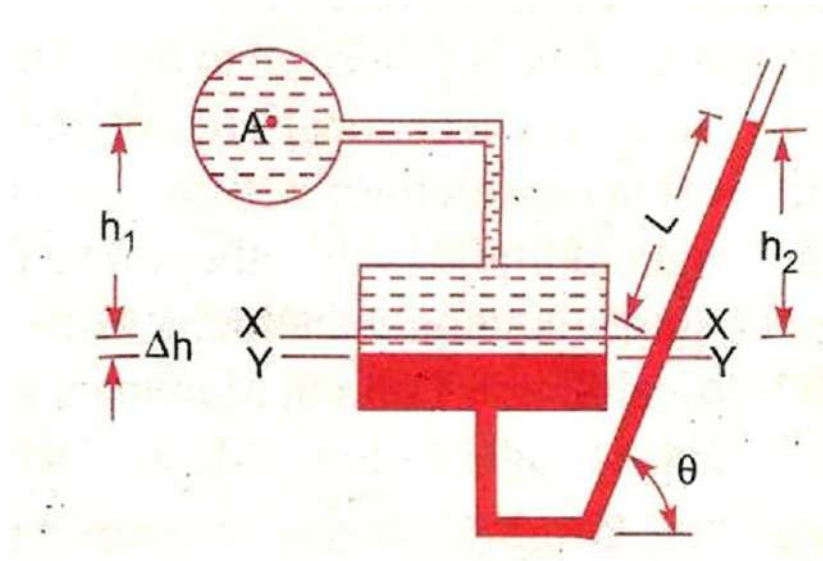
$$\text{So, } P_A = \frac{a \times h}{A} [\rho_2 g - \rho_1 g] + h_2 \rho_2 g - h_1 \rho_1 g$$

As the area A is very large as compared to a, hence ratio $\frac{a}{A}$ becomes very small and can be neglected.

$$\text{Then } P_A = h_2 \rho_2 g - h_1 \rho_1 g$$

2. Inclined Single Column Manometer:

The given figure shows the inclined single column manometer which is more sensitive. Due to inclination the distance moved by the heavy liquid in the right limb will be more.



Let L = length of heavy liquid moved in right limb from X-X

θ = Inclination of right limb with horizontal

h_2 = Vertical rise of heavy liquid in right limb from X-X

$$= L \times \sin\theta$$

From the above equation for the pressure in the single column manometer the pressure at A is

$$P_A = h_2 \rho_2 g - h_1 \rho_1 g .$$

Substituting the value of h_2 , we get

$$P_A = \sin\theta \rho_2 g L - h_1 \rho_1 g .$$

DIFFERENTIAL MANOMETERS:

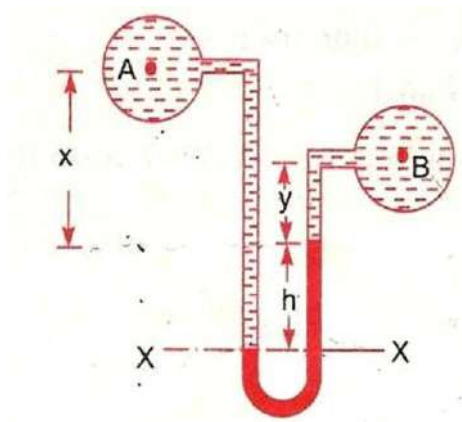
Differential manometers are the device use for measuring the difference of pressures between two points in a pipe or in two different pipes. A differential manometer consists of a U- tube, containing a heavy liquid, whose two ends are connected to the points, whose difference of pressure is to be measured. Most commonly used differential manometers are :

1. **U-tube differential manometer**
2. **Inverted U-tube differential manometer**

U-tube differential manometer:

Two points A and B are at different level

The given figure shows the differential manometers of U-tube type.



Let the two points A and B are at different level also contains liquids of different sp.gr. These points are connected to the U-tube differential manometer. Let the pressure at A and B are P_A and P_B .

Let h = Difference of mercury level in the U- tube.

y = Distance of the center of B, from the mercury level in the right limb.

ρ_1 = Density of liquid at A.

ρ_2 = Density of liquid at B.

ρ_g = Density of heavy liquid or mercury.

Taking datum line at X-X .

Pressure above X-X in the limb

$$= \rho_1 g(h + x) + P_A$$

Where pressure P_A = Pressure at A.

Pressure above X-X in the right limb

$$= \rho_g \times g \times h + \rho_2 \times g \times y + p_B$$

Where pressure p_B = pressure at B.

Equating the two pressure, we have

$$P_1 g(h + x) + P_A = \rho_g \times g \times h + \rho_2 g y + p_B$$

$$\begin{aligned} \therefore P_A - p_B &= \rho_g \times g \times h + \rho_2 g y - \rho_1 g(h + x) \\ &= h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x \end{aligned}$$

$\therefore$ Different of pressure at A and B

$$= h \times g(\rho_g - \rho_1) + \rho_2 g y - \rho_1 g x$$

Two points A and B are at same level

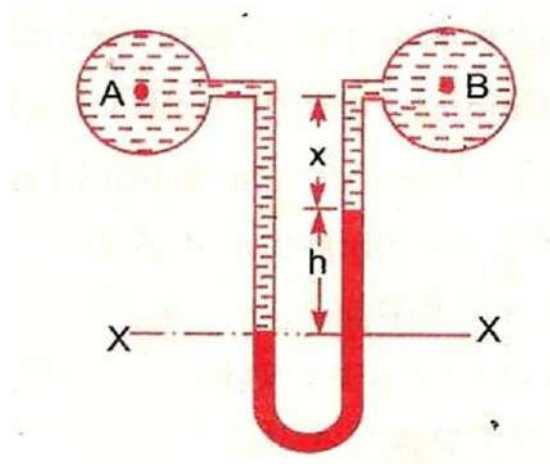
In the given figure A and B are the same level and contains the same liquid of density ρ_1 , then

Pressure above X-X in right limb

$$= \rho_g \times g \times h + \rho_1 \times g \times X + p_B$$

Pressure above X-X in left limb

$$= P_1 \times g \times (h + x) + P_A$$



Equating the two pressure

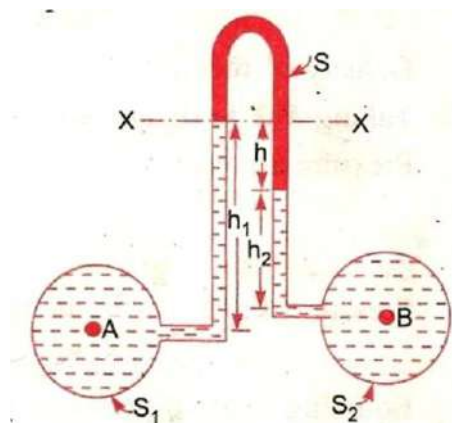
$$p_g \times g \times h + P_1 \times g \times X + p_B = P_1 \times g \times (h + x) + P_A$$

$$\therefore P_A - p_B = P_g \times g \times h + P_1 g x - P_1 g \times (h + x)$$

$$= g \times h (P_g - P_1)$$

Inverted U-tube Differential Manometer:

It consists of an inverted U-tube, containing a light liquid. The two ends of the U-tube are connected to the points whose difference of pressure is to be measured. It is used for measuring difference of low pressures. Fig 2.21 shows an inverted U-tube differential manometer connected to the points A and B. Let the pressure at A is more than the pressure at B.



Let h_1 = Height of liquid in the left limb below the datum line X-X

h_2 = Height of liquid in the right limb

h = Difference of light liquid

ρ_1 = Density of liquid at A

ρ_2 = Density of liquid at B

ρ_s = Density of light liquid

p_A = Pressure at A

p_B = Pressure at B.

Taking X-X datum line.

Then pressure in the left limb below X-X

$$= P_A - \rho_1 \times g \times h_1.$$

Pressures in the right limb below X-X

$$= P_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

Equating the two pressure

$$P_A - \rho_1 \times g \times h_1 = P_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

$$P_A - P_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

Bourdon's Tube Pressure Gauge:

- The pressure above or below the atmospheric pressure may be easily measured with the help of Bourdon tube pressure gauge.
- It consists of an elliptical tube ABC bent into an arc of a circle. This bent up tube is called Bourdon tube.
- When the gauge tube is connected to the C, the fluid under pressure flows into the tube the bourdon tube as a result of the increased pressure tends to straighten itself.
- Since the tube is encased in a circular cover therefore it tends to become circular instead of straight.
- The elastic deformation of the bourdon rotates the pointer.
- The pointer moves over a calibrated scale which directly gives the pressure.

Numerical problems:

Q.1 The right limb of a simple U-tube manometer containing mercury is open to the atmosphere while the left limb is connected to a pipe in which a fluid of sp. gravity 0.9 is flowing. The centre of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe if the difference of mercury level in the two limbs is 20 cm.

Q.2 A single column manometer is connected to a pipe containing a liquid of sp. Gravity 0.9. Find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube for manometer reading. The sp. Gravity of mercury is 13.6.

Q.3 A differential manometer is connected at the two points A and B of two pipes. The pipe A contains a liquid of sp. Gravity = 1.5 while pipe B contains a liquid of sp. Gravity 0.9. The pressure at A and B are 1 kg/cm^2 and 1.80 kg/cm^2 respectively. Find the difference in mercury level in the differential manometer.

Q.4 Water is flowing through two difference pipes to which an inverted differential manometer having an oil of sp. Gravity 0.8 is connected. The pressure head in the pipe A is 2 m of water, find the pressure in the pipe B for the manometer readings.

Chapter-3

Hydrostatics

Syllabus:

- 3.1 Definition of hydrostatic pressure**
- 3.2 Total pressure and centre of pressure on immersed bodies (Simple Numericals)**
- 3.3 Archimedis' principle, concept of buoyancy, metacentre and metacentric height**
- 3.4 Concept of floatation**

Hydrostatics:

Hydrostatics means the study of pressure exerted by the liquid at rest & the direction of such a pressure is always right angle to the surface on which it acts.

Total pressure and center of pressure:

Total pressure

Total pressure is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with surfaces. This force always acts normal to the surface.

Center of pressure:

Center of pressure is defined as the point of application of the total pressure on the surface.

There are four cases of submerged surfaces on which the total pressure force and center of pressure is to be determined. The submerged surfaces may be:

1. **Vertical plane surface**
2. **Horizontal plane surface**
3. **Inclined plane surface**
4. **Curved surface.**

Vertical plane surface submerged in liquid

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown in figure

Let A = total area of the surface

H = distanced of C.G. of the area from free surface of liquid

G = center of gravity of plane surface

P = center of pressure

h^* = distance of center of pressure from free surface of liquid.

Total pressure(F):

The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on surface is then calculated by integrating the force on small strip.

Consider a strip of thickness dh & width b at a depth of h from free surface of liquid.

Pressure intensity on the strip

$$p = \rho gh$$

Area of the strip, $dA = b \times dh$

Total pressure force on strip, $dF = qdA$

$$= qgh \times b \times dh$$

Total pressure force on the whole surface

$$F = \int dF = \int qgh \times b \times dh$$

$$= qg \int h \times b \times dh$$

$\int h \times dA$ = moment of surface area about the free surface of liquid

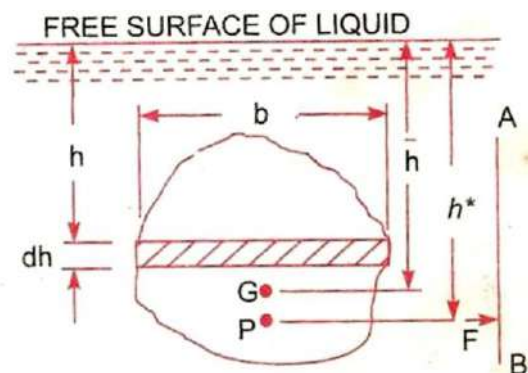
= Area of surface $\times$ distance of C.G. from the free surface

$$= A \times \bar{h}$$

So, $F = qgA\bar{h}$

Centre of the pressure: (h^*)

Centre of pressure is calculated by using the principle of moments which states that the moment of resultant force about an axis is equal to the sum of moments of the components about the same axis.



The resultant force F is acting at P , at a distance h^* from the free surface of liquid.

Hence moment of force F about free surface of liquid = $F \times h^*$

But moment force dF acting on a strip about the free surface of liquid = $dF \times h$

Sum of moments of all such forces about free surface of liquid

$$= \int qgh \times b \times dh \times h$$

$$= qg \int h \times b \times dh \times h$$

$$= qg \int bh^2 dh$$

$$= qg \int h^2 dA$$

$\int h^2 dA$ = moment of inertia of the surface area about the free surface of liquid = I_o

Sum of the moments about free surface

$$= qg I_o$$

$$F \times h^* = qg I_o$$

$$qg A \bar{h} \times h^* = qg I_o$$

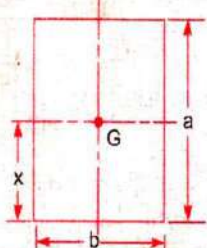
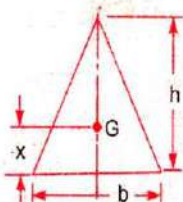
$$h^* = \frac{qg I_o}{qg A \bar{h}}$$

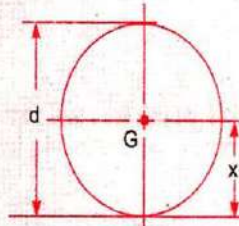
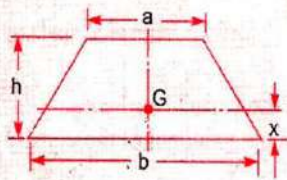
$$= \frac{I_o}{A \bar{h}}$$

By the parallel axis theorem, we have

$$I_o = I_G + A \times (\bar{h})^2$$

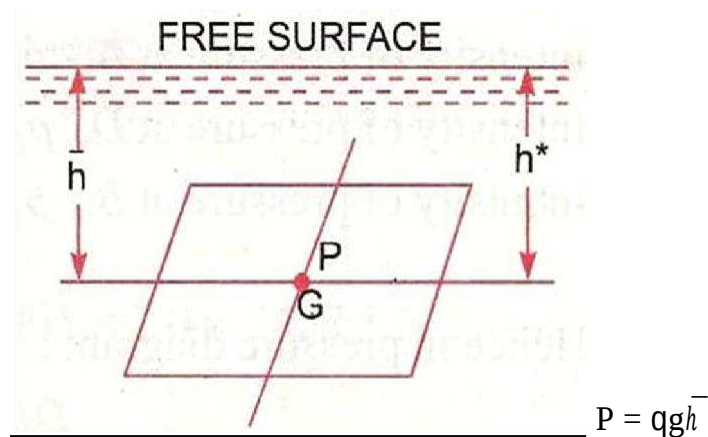
$$h^* = \frac{I_G + A \bar{h}^2}{A \bar{h}} = \frac{I_G}{A \bar{h}} + \bar{h}$$

Plane surface	C.G. from the base	Area	Moment of inertia about an axis passing through C.G. and parallel to base (I_G)	Moment of inertia about base (I_0)
1. Rectangle 	$x = \frac{d}{2}$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
2. Triangle 	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$

Plane surface	C.G. from the base	Area	Moment of inertia about an axis passing through C.G. and parallel to base (I_G)	Moment of inertia about base (I_0)
3. Circle 	$x = \frac{d}{2}$	$\frac{\pi d^2}{4}$	$\frac{\pi d^4}{64}$	—
4. Trapezium 	$x = \left(\frac{2a+b}{a+b} \right) \frac{h}{3}$	$\frac{(a+b)}{2} \times h$	$\left(\frac{a^2 + 4ab + b^2}{36(a+b)} \right) \times h^3$	—

Horizontal plane surface submerged in liquid:

Consider a plane horizontal surface immersed in a static fluid as every point of the surface is at the same depth from the free surface of the liquid, the pressure intensity will be equal on the entire surface.

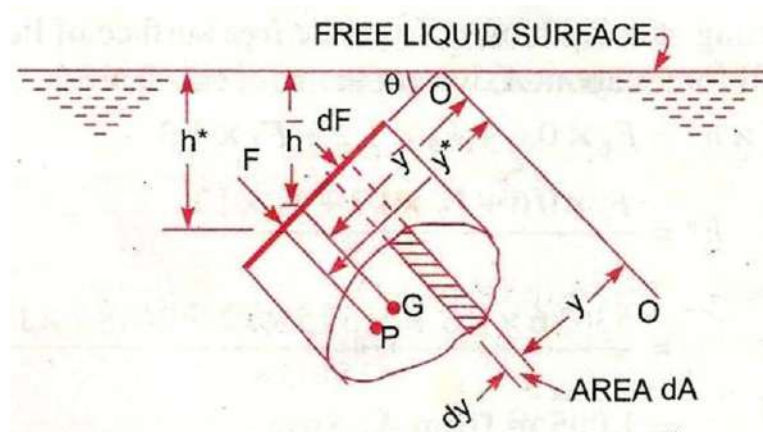


A = total area

$$F = P \times A$$

$$= \rho g A h$$

Inclined plane surface submerged in liquid:



Let A = total area of the inclined surface

H = depth of C.G. of inclined area from free surface.

h^* = distance of center of pressure from free surface of liquid.

θ = angle made by the plane of surface with free liquid surface.

Let the plane of the surface if produced meet the free liquid surface at O . Then $O-O$ is the axis parallel to the plane of the surface

$\bar{y}$ = distance of C.G of the inclined surface from $O-O$.

y^* = distance of the centre of pressure from $O-O$.

Consider a small strip of area dA at a depth ' h ' from free surface & at a distance y from axis $O-O$.

$$P = \rho gh$$

$$dF = p dA$$

$$= \rho gh dA$$

Total pressure force

$$F = \int dF = \int \rho gh dA$$

$$h = y \sin \theta$$

$$F = \int \rho g y \sin \theta dA$$

$$= \rho g \sin \theta \int y dA$$

$$= \rho g \sin \theta I_o$$

$$= \rho g \sin \theta A \bar{y}$$

$$= \rho g A \bar{y} \sin \theta$$

$$= \rho g A h^*$$

Centre of pressure:

Pressure force on the strip $dF = qgh \, dA$

$$= qg y \sin \theta \, dA$$

Moment of the force dF about $O - O$

$$= dF \times y = qgy^2 \sin \theta \, dA$$

Sum of moments of all such forces about $O - O$

$$= qg \sin \theta \int y^2 dA$$

$$\int y^2 dA = \text{moment of inertia of the surface about } O - O = I_O$$

$$= qg \sin \theta I_O$$

Moment of total force about $O - O$

$$= F y^*$$

$$F y^* = qg \sin \theta I_O$$

$$qg A \bar{h} \times \frac{h^*}{\sin \theta} = qg \sin \theta I_O$$

$$h^* = \frac{\sin^2 \theta}{A \bar{h}} I_O$$

$$= \frac{\sin^2 \theta}{A \bar{h}} [I_G + A \times (\bar{y})^2]$$

$$\text{Here } \frac{\bar{h}}{\bar{y}} = \sin \theta$$

$$\bar{y} = \frac{\bar{h}}{\sin \theta}$$

$$h^* = \frac{\sin^2 \theta}{A \bar{h}} [I_G + A \times (\frac{\bar{h}}{\sin \theta})^2]$$

$$h^* = \frac{I_G \sin^2 \theta}{A \bar{h}} + \bar{h}$$

Archimedes principle:

When a body is immersed in a fluid either wholly or partially, it is buoyed or lifted up by a force, which is equal to the weight of fluid displaced by the body.

Buoyancy:

Whenever a body is immersed wholly or partially in a fluid it is subjected to an upward force which tends to lift it up. This tendency for an immersed body to be lifted up in the fluid due to an upward force opposite to action of gravity is known as buoyancy this upward force is known as force of buoyancy.

Centre of Buoyancy:

It is defined as the point through which the force of buoyancy is supposed to act. The force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body.

Center of buoyancy will be the centre of gravity of the fluid displaced.

Problem-1:

Find the volume of the water displaced & position of centre of buoyancy for a wooden block of width 2.5m & of depth 1.5m when it floats horizontally in water. The density of wooden block is 650 kg/m^3 & its length 6.0m.

Solution:

Width = 2.5 m

Density of wooden block = 650 kg/m^3

Depth = 1.5m

Length = 6m

Volume of the block

$$= 2.5 \times 1.5 \times 6$$

$$= 22.50 \text{ m}^3$$

Volume of the block = Wt of water displaced

$$= W \times V$$

$$= \rho g \times V$$

$$= 650 \times 9.81 \times 6$$

$$= 143471 \text{ N}$$

Volume of water displaced

$$= \frac{\text{weight}}{\rho_w \times g}$$

$$= \frac{143471}{1000 \times 9.81}$$

$$= 14.625 \text{ m}^3$$

Position of centre of buoyancy

Volume of wooden block in water = volume of water displaced

$$2.5 \times 6 \times h = 14.625$$

$$\Rightarrow h = \frac{14.625}{2.5 \times 6}$$

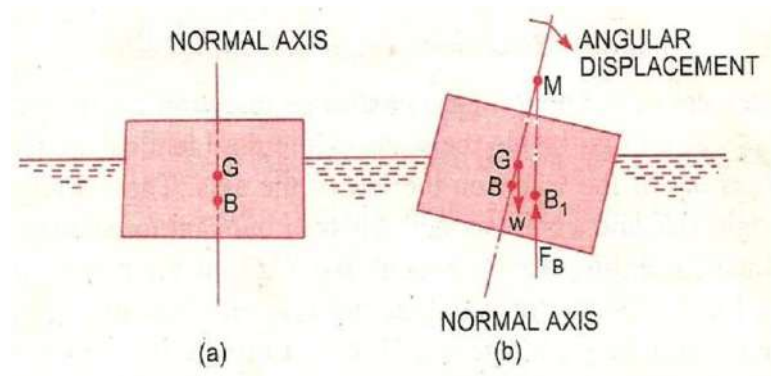
$$= 0.975 \text{ m}$$

$$\text{Centre of buoyancy} = \frac{0.975}{2}$$

$$= 0.4875 \text{ m from base.}$$

Meta-centre:

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The meta centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis. Of the body when the body is given a small angular displacement.



Meta centre height:

The distance between the meta centre of a floating body and the centre of gravity of the body is called meta-centric height i.e the distance MG.

Concept of flotation:

Flotation:

When a body is immersed in any fluid, it experiences two forces. First one is the weight of body W acting vertically downwards, second is the buoyancy force F_b acting vertically upwards in case W is greater than F_b , the weight will cause the body to sink in the fluid. In case $W = F_b$ the body will remain in equilibrium at any level. In case W is small than F_b the body will move upwards in fluid. The body moving up will come to rest or top moving up in fluid when the fluid displaced by it's submerged part is equal to its weight W , the body in this situation is said to be floating and this phenomenon is known as flotation.

Principle of flotation:

The principle of flotation states that the weight of the floating body is equal to the weight of the fluid displaced by the body.

Consider a body floating at the free surface of the liquid. The shaded part of the body is inside the fluid and it has volume V_1 . The other part of the body is in air and it has volume V_2 . Now the body can be considered to be in two fluids viz. air and liquid. Hence buoyant force

$$F_p = \rho_{\text{liquid}} V_1 g + \rho_{\text{air}} V_2 g = W$$

Since $\rho_{\text{air}} \ll \rho_{\text{liquid}}$

$$F_p = \rho_{\text{liquid}} V_1 g = W$$

Buoyancy force is equal to weight of the liquid displaced

The ways to make the body float:

The body can be made to float:

1. Decreasing the weight of the body while keeping the volume same. For example, making body hollow.
2. Increasing the volume of the body while keeping the body same. For example, attaching life jacket to a person fixed the person floating.

Chapter-4

KINEMATICS

Syllabus:

Types of fluid flow

Continuity equation (Statement and proof for one dimensional flow)

Bernoulli's theorem (Statement and proof)

**Applications and limitations of Bernoulli's theorem
(Venturimeter, pitot tube)
(Simple Numerical)**

Introduction:-

This chapter includes the study of forces causing fluid flow. The dynamics of fluid flow is the study of fluid motion with the forces causing flow. The dynamic behaviour of the fluid flow is analysed by the Newton second law of motion, which relates the acceleration with the forces. The fluid is assumed to be incompressible and non-viscous.

TYPES OF FLOW:-

The fluid flow is classified as follows:

- **STEADY AND UNSTEADY FLOW**
- **UNIFORM AND NON- UNIFORM FLOWS**
- **LAMINAR AND TURBULANT FLOWS**
- **COMPRESSIBLE AND INCOMPRESSIBLE FLOWS**
- **ROTATIONAL AND IRROTATIONAL FLOWS**
- **ONE, TWO, THREE DIMENSIONAL FLOW**

➤ **STEADY AND UNSTEADY FLOW:-**

1. Steady flow:-

Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density at a point do not change with time.

Thus, mathematically

$$\left(\frac{\partial v}{\partial t}\right)_{0,y_0,z_0} = 0$$

$$\left(\frac{\partial p}{\partial t}\right)_{0,y_0,z_0} = 0$$

$$\left(\frac{\partial \rho}{\partial t}\right)_{0,y_0,z_0} = 0$$

Where x_0, y_0, z_0 is a point in fluid flow .

2. Unsteady flow:-

Unsteady flow is defined as that type of flow in which the velocity, pressure, and density at a point changes w.r.t time.

Thus, mathematically

$$\left(\frac{\partial v}{\partial t}\right)_{0,y_0,z_0} \neq 0,$$

$$\left(\frac{\partial p}{\partial t}\right)_{0,y_0,z_0} \neq 0,$$

$$\left(\frac{\partial \rho}{\partial t}\right)_{0,y_0,z_0} \neq 0$$

➤ **UNIFORM AND NON- UNIFORM FLOWS:-**

1. **Uniform flow:-**

It is defined as the flow in which velocity of flow at any given time does not change w.r.t length of flow or space.

Mathematically,

$$\left(\frac{dv}{ds}\right)_{=constant} = 0$$

where ∂v = velocity of flow ,

∂s = length of flow ,

T = time

2. **Non- uniform flows:-**

It is defined as the flow in which velocity of flow at any given time changes w.r.t length of flow.

Mathematically,

$$\left(\frac{dv}{ds}\right)_{=constant} \neq 0$$

➤ **LAMINAR AND TURBULANT FLOWS:-**

1. **Laminar flow:-**

Laminar flow is that type of flow in which the fluid particles are moved in a well defined path called streamlines. The paths are parallel and straight to each other.

2. **Turbulent flow:-**

Turbulent flow is that type of flow in which the fluid particles are moved in a zig-zag manner.

For a pipe flow the type of flow is determined by Reynolds number (R_e)

Mathematically

$$R_e = \frac{VD}{\nu}$$

Where V = mean velocity of flow

D = diameter of pipe

ν = kinematic viscosity

If $R_e < 2000$, then flow is laminar flow.

If $R_e > 4000$, then flow is turbulent flow.

If R_e lies in between 2000 and 4000, the flow may be laminar or turbulent.

➤ **COMPRESSIBLE AND INCOMPRESSIBLE FLOWS :-**

1. **Compressible flow:-**

Compressible flow is that type of flow in which the density of fluid changes from point to point.

So, $\rho \neq \text{constant}$.

2. **Incompressible flow:-**

Incompressible flow is that type of flow in which the density is constant for the fluid flow.

So, $\rho = \text{constant}$

➤ **ROTATIONAL AND IRROTATIONAL FLOWS:-**

1. **Rotational flow:-**

Rotational flow is that of flow in which the fluid particles while flowing along stream lines also rotate about their own axis.

2. **Ir-rotational flow:-**

Irrotational flow is that type of flow in which the fluid particles while flowing along streamlines do not rotate about their own axis.

➤ **ONE, TWO, THREE DIMENSIONAL FLOW:-**

1. **One dimensional flow:-**

One dimension flow is defined as that type of flow in which velocity is a function of time and one space co-ordinate only.

For a steady one dimensional flow, the velocity is a function of one space co-ordinate only.

$$\text{So, } U = f(x),$$

$$V = 0,$$

$$W = 0$$

U, V, W are velocity components in x, y, z direction respectively.

2. **Two-dimensional flow:-**

Two-dimensional flow is the flow in which velocity is a function of time and 2- space co- ordinates only. For a steady 2- dimensional flow the velocity is a function of two – space co-ordinate only.

$$\text{So, } U = f_1(x,y) ,$$

$$V = f_2(x,y) ,$$

$$W = 0$$

3. **Three-dimensional flow:-**

Three – dimensional flow is the flow in which velocity is a function of time and 3- space co-ordinates only. For steady three- dimensional flow, the velocity is a function of three space co-ordinates only.

So $U = f_1(x, y, z)$

$$V = f_2(x, y, z)$$

$$W = f_3(x, y, z)$$

RATE OF FLOW OR DISCHARGE

It is defined as the quantity of a fluid flowing per second through a section of pipe.

For an incompressible fluid the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second.

For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section.

$$Q = A \cdot V$$

Where A = cross sectional area of the pipe

V = velocity of fluid across the section

Unit:-

1. For incompressible fluid

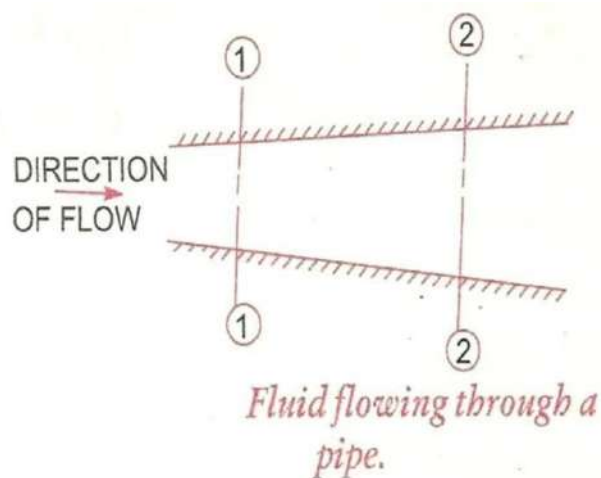
$$\frac{m^3}{sec} \text{ or } \frac{litre}{sec}$$

2. For compressible fluid:

$$\frac{newton}{sec} \text{ (S.I units)}, \frac{kgf}{sec} \text{ (M.K.S units)}$$

EQUATION OF CONTINUITY:-

It is based on the principle of conservation of mass. For a fluid flowing through the pipe at all the cross-section, the quantity of fluid per second is constant.



Let V_1 = average velocity at cross-section 1-1.

ρ_1 = density at cross-section 1-1

A_1 = area of pipe at section 1-1

V_2 = average velocity at cross-section 2-2

ρ_2 = density at cross-section 2-2

A_2 = area of pipe at section 2-2

The rate of flow at section 1-1 = $\rho_1 A_1 V_1$

The rate of flow at section 2-2 = $\rho_2 A_2 V_2$

According to laws of conservation of mass rate of flow at section 1-1 is equal to the rate of flow at section 2-2 ,

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

This is called continuity equation.

If the fluid is compressible, then $\rho_1 = \rho_2$,

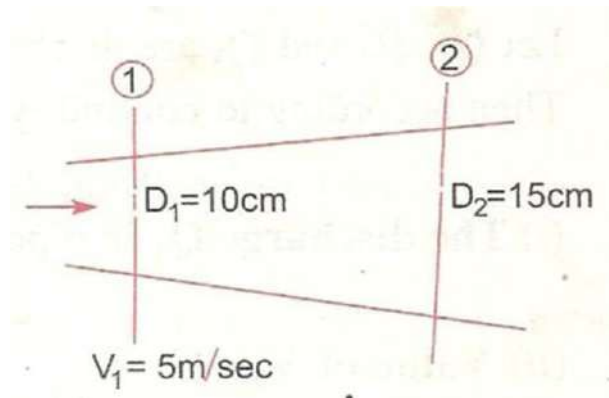
$$\text{so } A_1 V_1 = A_2 V_2$$

“If no fluid is added removed from the pipe in any length then the mass passing across different sections shall be same”

Simple Problems

Problem:-1

The diameters of a pipe at the sections 1 and 2 are 10cm and 15cm respectively. Find the discharge through the pipe if the velocity of the water flowing through the pipe at section 1 is 5m/s. Determine also the velocity at section 2.



Solution. Given :

At section 1,

$$D_1 = 10 \text{ cm} = 0.1 \text{ m}$$

$$A_1 = \frac{\pi}{4} (D_1)^2 = \frac{\pi}{4} (.1)^2 = .007854 \text{ m}^2$$

$$V_1 = 5 \text{ m/s.}$$

At section 2,

$$D_2 = 15 \text{ cm} = 0.15 \text{ m}$$

$$A_2 = \frac{\pi}{4} (.15)^2 = 0.01767 \text{ m}^2$$

(i) Discharge through pipe is given by equation (5.1)

or

$$Q = A_1 \times V_1$$

$$= .007854 \times 5 = \mathbf{0.03927 \text{ m}^3/\text{s. Ans.}}$$

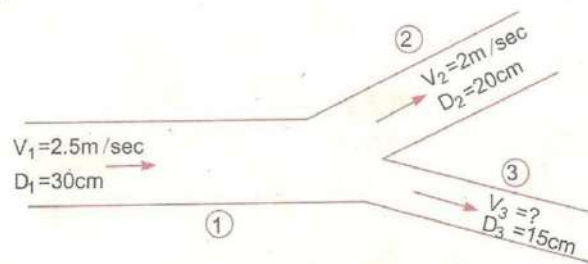
Using equation (5.3), we have $A_1 V_1 = A_2 V_2$

(ii) $\therefore$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{.007854}{.01767} \times 5.0 = \mathbf{2.22 \text{ m/s.}}$$

Problem:-2

A 30cm diameter pipe conveying water branches into two pipes of diameter 20cm and 15cm respectively. If the average velocity in the 30cm diameter pipe is 2.5 m/s, find the discharge in this pipe. Also determine the velocity in 15cm pipe if the average velocity in 20cm diameter pipe is 2m/s

Solution:**Given Data:**

$$D_1 = 30\text{cm} = 0.30\text{m}$$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (0.3)^2 = 0.07068 \text{ m}^2$$

$$V_1 = 2.5 \text{ m/s}$$

$$D_2 = 20\text{cm} = 0.2\text{m}$$

$$A_2 = \frac{\pi}{4} 0.2^2 = 0.0314 \text{ m}^2$$

$$V_2 = 2\text{m/s}$$

$$D_3 = 15\text{cm} = 0.15\text{m}$$

$$A_3 = \frac{\pi}{4} 0.15^2 = 0.01767 \text{ m}^2$$

Let Q_1 , Q_2 , Q_3 are discharges in pipe 1, 2, 3 respectively

$$Q_1 = Q_2 + Q_3$$

The discharge Q_1 in pipe 1 is given as

$$Q_1 = A_1 V_1$$

$$= 0.07068 \times 2.5 \text{ m}^3/\text{s}$$

$$Q_2 = A_2 V_2$$

$$= 0.0314 \times 2.0 \text{ m}^3/\text{s}$$

Substituting the values of Q_1 and Q_2 on the above equation we get

$$0.1767 = 0.0628 + Q_3$$

$$\begin{aligned} Q_3 &= 0.1767 - 0.0628 \\ &= 0.1139 \text{ m}^3/\text{s} \end{aligned}$$

$$\begin{aligned} \text{Again } Q_3 &= A_3 V_3 \\ &= 0.01767 \times V_3 \end{aligned}$$

$$\text{Or } 0.1139 = 0.01767 \times V_3$$

$$\begin{aligned} V_3 &= \frac{0.1139}{0.01767} \\ &= 6.44 \text{ m/s} \end{aligned}$$

Problem:-3

A 25 cm diameter pipe carries oil of sp. Gr. 0.9 at a velocity of 3m/s. At another section the diameter is 20cm. Find the velocity at this section and also mass rate of flow of oil.

Solution. Given :

at section 1,

$$D_1 = 25 \text{ cm} = 0.25 \text{ m}$$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times .25^2 = 0.049 \text{ m}^2$$

$$V_1 = 3 \text{ m/s}$$

at section 2,

$$D_2 = 20 \text{ cm} = 0.2 \text{ m}$$

$$A_2 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}^2$$

$$V_2 = ?$$

Mass rate of flow of oil = ?

Applying continuity equation at sections 1 and 2,

$$A_1 V_1 = A_2 V_2$$

or

$$0.049 \times 3.0 = 0.0314 \times V_2$$

$\therefore$

$$V_2 = \frac{0.049 \times 3.0}{.0314} = \mathbf{4.68 \text{ m/s. Ans.}}$$

Mass rate of flow of oil

$$= \text{Mass density} \times Q = \rho \times A_1 \times V_1$$

Sp. gr. of oil

$$= \frac{\text{Densit of oil}}{\text{Densit of water}}$$

$\therefore$ Density of oil

$$= \text{Sp. gr. of oil} \times \text{Density of water}$$

$$= 0.9 \times 1000 \text{ kg/m}^3 = \frac{900 \text{ kg}}{\text{m}^3}$$

$\therefore$ Mass rate of flow

$$= 900 \times 0.049 \times 3.0 \text{ kg/s} = \mathbf{132.23 \text{ kg/s. Ans.}}$$

Bernoulli's equation:

Statement: It states that in a steady ideal flow of an incompressible fluid, the total energy at any point of flow is constant.

The total energy consists of pressure energy, kinetic energy & potential energy or datum energy. These energies per unit weight are

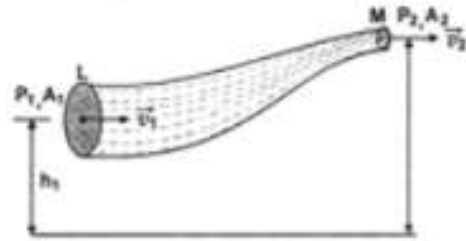
$$\text{Pressure energy} = \frac{P}{\rho g}$$

$$\text{Kinetic energy} = \frac{v^2}{2g}$$

$$\text{Datum energy} = z$$

Mathematically

$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{Constant}$$



Proof: Let us consider the ideal liquid of density ρ flowing through the pipe LM of varying cross-section. Let P_1 and P_2 be the pressures at ends L and M and A_1 and A_2 be the areas of cross-sections at ends L and M respectively. Let the liquid enter with velocity V_1 and leave with velocity v_2 . Let $A_1 > A_2$. By equation of continuity,

$$A_1 v_1 = A_2 v_2$$

Since $A_1 > A_2$,

$$\therefore v_2 > v_1 \quad \text{and} \quad P_1 > P_2$$

Let m be mass of liquid entering at end L in time t . In time t , the liquid will cover a distance of $v_1 t$.

Therefore the work done by pressure on the liquid at end L in time t is

$$\begin{aligned} W_1 &= \text{force} \times \text{displacement} \\ &= P_1 A_1 v_1 t \end{aligned} \quad \dots(1)$$

Since same mass m leaves the pipe at end M in same time t , in which liquid will cover the distance $v_2 t$, therefore work done by liquid against the force due to pressure P_2 is

$$W_2 = P_2 A_2 v_2 t \quad \dots(2)$$

Net work done by pressure on the liquid in time t is,

$$W = W_1 - W_2 = P_1 A_1 v_1 t - P_2 A_2 v_2 t \quad \dots(3)$$

This work done on liquid by pressure increases its kinetic and potential energy.

Increase in kinetic energy of liquid is,

$$\Delta K = \frac{1}{2} m(v_2^2 - v_1^2) \quad \dots(4)$$

According to work-energy relation,

$$P_1 A_1 v_1 t - P_2 A_2 v_2 t = \frac{1}{2} m(v_2^2 - v_1^2) + mg(h_2 - h_1) \quad \dots(6)$$

If there is no source and sink of liquid, then mass of liquid entering at end L is equal to the mass of liquid leaving the pipe at end M and is given by

$$A_1 v_1 \rho t = A_2 v_2 \rho t = m$$

$$\text{or} \quad A_1 v_1 t = A_2 v_2 t = \frac{m}{\rho} \quad \dots(7)$$

From (6) and (7)

$$P_1 \frac{m}{\rho} - P_2 \frac{m}{\rho} = \frac{1}{2} m(v_2^2 - v_1^2) + mg(h_2 - h_1)$$

$$\text{or} \quad P_1 \frac{m}{\rho} + \frac{1}{2} m v_1^2 + m g h_1 = P_2 \frac{m}{\rho} + \frac{1}{2} m v_2^2 + m g h_2$$

$$\text{or} \quad \frac{P}{\rho} + g h + \frac{1}{2} v^2 = \text{Constant}$$

Problem:- 5

Water is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5m above the datum line.

Solution. Given :

Diameter of pipe

$$= 5 \text{ cm} = 0.05 \text{ m}$$

Pressure,

$$p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$$

Velocity,

$$v = 2.0 \text{ m/s}$$

Datum head,

$$z = 5 \text{ m}$$

Total head

$$= \text{pressure head} + \text{kinetic head} + \text{datum head}$$

Pressure head

$$= \frac{p}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m} \quad \left\{ \rho \text{ for water} = 1000 \frac{\text{kg}}{\text{m}^3} \right\}$$

Kinetic head

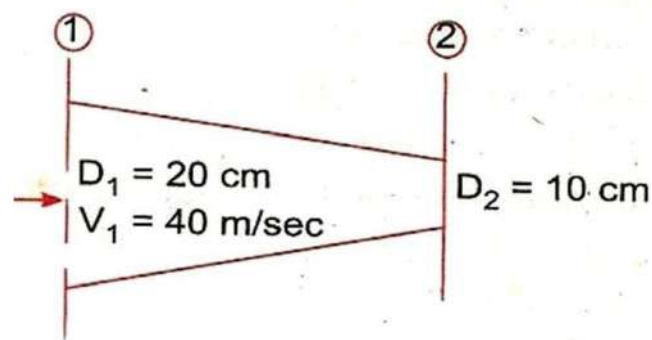
$$= \frac{v^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$$

$\therefore$ Total head

$$= \frac{p}{\rho g} + \frac{v^2}{2g} + z = 30 + 0.204 + 5 = 35.204 \text{ m. Ans.}$$

Problem:- 6

A pipe, through which water is flowing, is having diameters, 20cm and 10cm at the cross sections 1 and 2 respectively. The velocity of water at section 1 is given 4.0 m/s. Find the velocity head at sections 1 and 2 and also rate of discharge.



Solution. Given :

$$D_1 = 20 \text{ cm} = 0.2 \text{ m}$$

∴ Area,

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}^2$$

$$V_1 = 4.0 \text{ m/s}$$

$$D_2 = 0.1 \text{ m}$$

∴

$$A_2 = \frac{\pi}{4} (.1)^2 = .00785 \text{ m}^2$$

(i) Velocity head at section 1

$$= \frac{V_1^2}{2g} = \frac{4.0 \times 4.0}{2 \times 9.81} = \mathbf{0.815 \text{ m. Ans.}}$$

(ii) Velocity head at section 2 = $V_2^2/2g$

To find V_2 , apply continuity equation at 1 and 2

$$\therefore A_1 V_1 = A_2 V_2 \quad \text{or} \quad V_2 = \frac{A_1 V_1}{A_2} = \frac{.0314}{.00785} \times 4.0 = 16.0 \text{ m/s}$$

$$\therefore \text{Velocity head at section 2} = \frac{V_2^2}{2g} = \frac{16.0 \times 16.0}{2 \times 9.81} = \mathbf{83.047 \text{ m. Ans.}}$$

(iii) Rate of discharge

$$\begin{aligned} &= A_1 V_1 \quad \text{or} \quad A_2 V_2 \\ &= 0.0314 \times 4.0 = 0.1256 \text{ m}^3/\text{s} \end{aligned}$$

Application of Bernoulli's equation:

Bernoulli's equation is applied in all problems of incompressible fluid flow where energy consideration are involved. It is also applied to following measuring devices

1. Venturimeter

2. Pitot tube

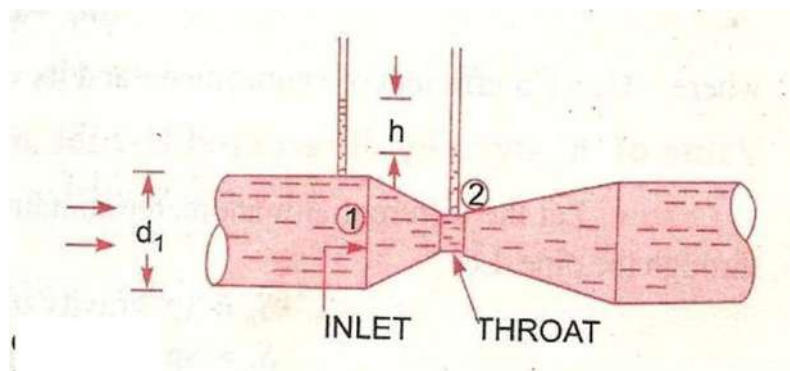
Venturimeter:

A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe it consists of three parts.

- I. Short converging part**
- II. Throat**
- III. Diverging part**

Expression for rate of flow through venturimeter:

Consider a venturimeter is fitted in a horizontal pipe through which a fluid flowing



Let d_1 = diameter at inlet or at section (i)-(ii)

P_1 = pressure at section (1)-(1)

V_1 = velocity of fluid at section (1) – (1)

A_1 = area at section (1) – (1) $= \frac{\pi d_1^2}{4}$

D_2, p_2, v_2, a_2 are corresponding values at section 2 applying Bernoulli's equation at sections 1 and 2 we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\therefore \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad \text{or} \quad \frac{p_1 - p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But $\frac{p_1 - p_2}{\rho g}$ is the difference of pressure heads at sections 1 and 2

and it is equal to h

$$\text{So, } h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

Now applying continuity equation at sections 1 & 2 $a_1 v_1 = a_2 v_2$

$$\text{Or } v_1 = \frac{a_2 v_2}{a_1}$$

Substituting this value

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g} \left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{v_2^2}{2g} \left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

$$v_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$$v_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$$Q = a_2 v_2$$

$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

Where Q = Theoretical discharge

Actual discharge will be less than theoretical discharge

$$Q_{\text{act}} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

Where C_d = co-efficient of venturimeter and value is less than 1

Value of 'h' given by differential U-tube manometer:

Case-i:

Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe

Let S_h = Sp. Gravity of the heavier liquid

S_0 = Sp. Gravity of the liquid flowing through pipe

x = difference of the heavier liquid column in U-tube

$$P_A - P_B = gx(\rho_g - \rho_0)$$

$$\frac{P_A - P_B}{\rho_0 g} = x \left(\frac{\rho_g}{\rho_0} - 1 \right)$$

$$h = x \left[\frac{S_h}{S_0} - 1 \right]$$

Case-ii

If the differential manometer contains a liquid lighter than the liquid flowing through the pipe

Where S_1 = Specific gravity of lighter liquid in U-tube nanometre

S_0 = Specific gravity of fluid flowing through in U-tube nanometre

x = Difference of lighter liquid columns in U- tube

The value of h is given by

$$h = x \left[1 - \frac{S_1}{S_0} \right]$$

Case-iii:

Inclined venturimetre with differential U-tube manometre

Let the differential manometer contains heavier liquid

Then h is given as

$$h = \left[\frac{P_1}{\rho g} + z_1 \right] - \left[\frac{P_2}{\rho g} + z_2 \right]$$

$$= x \left[\frac{S_0}{S_1} - 1 \right]$$

Case-iv:

Similarly for inclined venturimetre in which differential manometer contains a liquid which is kighter than the liquid flowing through the pipe. Then

$$h = \left[\frac{P_1}{\rho g} + z_1 \right] - \left[\frac{P_2}{\rho g} + z_2 \right]$$

$$h = x \left[1 - \frac{S_1}{S_0} \right]$$

Limitations:

- Bernoulli's equation has been derived under the assumption that no external force except the gravity force is acting on the liquid. But in actual practice some external forces always act on the liquid when effect the flow of liquid
- If the liquid is flowing in a curved path the energy due to centrifugal force should also be taken into account.

Pitot-tube:

It is a device used for measuring the velocity of flow at any point in a pipe or a channel.

It is based on the principle that if the velocity flow at a point becomes zero, the pressure there is increased due to conversion of the kinetic energy into pressure energy.

The pitot-tube consists of a glass tube, bent at right angles

Consider two points 1 and 2 at the same level. Such a way that 2 is at the inlet of pitot tube and one is far away from the tube

Let P_1 = pressure at point 1

V_1 = velocity of fluid at point 1

P_2 = pressure at 2

V_2 = velocity of fluid at point 2

H = Depth of tube in the liquid

h = Rise of the liquid in the tube above the free surface

Applying Bernoulli's theorem

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{P_1}{\rho g} = H \quad \frac{P_2}{\rho g} = (h + H)$$

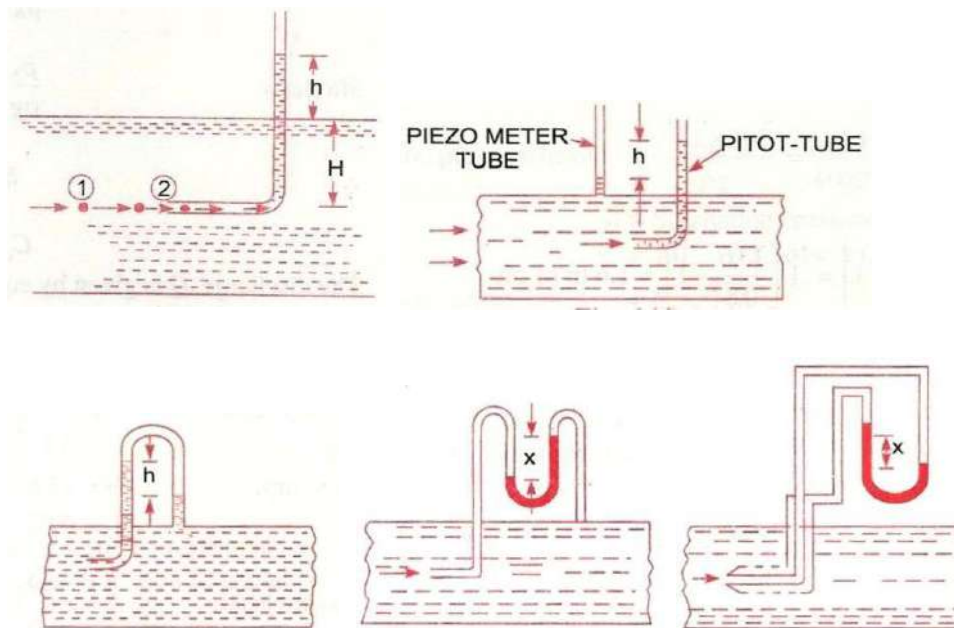
$$H + \frac{V_1^2}{2g} = h + H$$

$$V_1 = \sqrt{2gh}$$

Actual velocity, $V_{act} = C_v \sqrt{2gh}$

C_v = co-efficient of Pitot-tube

Different Arrangement of Pitot tubes



Numerical Problems:

Problem:- 7

Water is flowing through a pipe of 5cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5m above the datum line.

Solution. Given :

Diameter of pipe

$$= 5 \text{ cm} = 0.05 \text{ m}$$

Pressure,

$$p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$$

Velocity,

$$v = 2.0 \text{ m/s}$$

Datum head,

$$z = 5 \text{ m}$$

Total head

$$= \text{pressure head} + \text{kinetic head} + \text{datum head}$$

Pressure head

$$= \frac{p}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m} \quad \left\{ \rho \text{ for water} = 1000 \frac{\text{kg}}{\text{m}^3} \right\}$$

Kinetic head

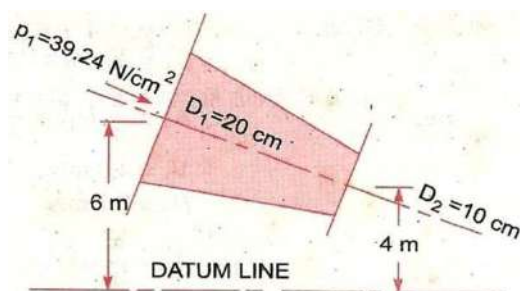
$$= \frac{v^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$$

∴ Total head

$$= \frac{p}{\rho g} + \frac{v^2}{2g} + z = 30 + 0.204 + 5 = 35.204 \text{ m. Ans.}$$

Problem:- 8

The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35lit/s. The section 1 is 6m above datum and section 2 is 4m above datum. If the pressure at section 1 is 39.24 N/cm². Find the intensity of pressure at section 2



Solution:

Given

At section 1,

$$D_1 = 20 \text{ cm} = 0.2 \text{ m}$$

$$A_1 = \frac{\pi}{4} (.2)^2 = .0314 \text{ m}^2$$

$$p_1 = 39.24 \text{ N/cm}^2 \\ = 39.24 \times 10^4 \text{ N/m}^2$$

$$z_1 = 6.0 \text{ m}$$

At section 2,

$$D_2 = 0.10 \text{ m}$$

$$A_2 = \frac{\pi}{4} (0.1)^2 = .00785 \text{ m}^2$$

$$z_2 = 4 \text{ m}$$

$$p_2 = ?$$

Rate of flow,

$$Q = 35 \text{ lit/s} = \frac{35}{1000} = .035 \text{ m}^3/\text{s}$$

Now

$$Q = A_1 V_1 = A_2 V_2$$

 $\therefore$

$$V_1 = \frac{Q}{A_1} = \frac{.035}{.0314} = 1.114 \text{ m/s}$$

and

$$V_2 = \frac{Q}{A_2} = \frac{.035}{.00785} = 4.456 \text{ m/s}$$

Applying Bernoulli's equation at sections 1 and 2, we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\text{or } \frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{p_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

$$\text{or } 40 + 0.063 + 6.0 = \frac{p_2}{9810} + 1.012 + 4.0$$

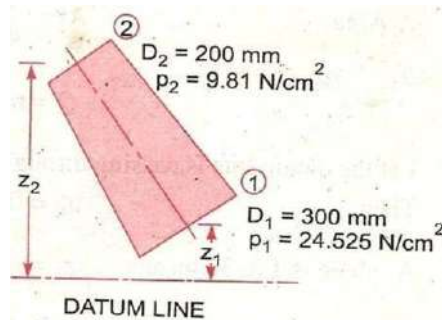
$$\text{or } 46.063 = \frac{p_2}{9810} + 5.012$$

$$\therefore \frac{p_2}{9810} = 46.063 - 5.012 = 41.051$$

$$\therefore p_2 = 41.051 \times 9810 \text{ N/m}^2 \\ = \frac{41.051 \times 9810}{10^4} \text{ N/cm}^2 = \mathbf{40.27 \text{ N/cm}^2}.$$

Problem:- 9

Water is flowing through a pipe having diameter 300mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 9.81N/m^2 . Determine the difference in datum head if the rate of flow through pipe is 40 lit/s



Solution. Given :

Section 1, $D_1 = 300 \text{ mm} = 0.3 \text{ m}$
 $p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2$

Section 2, $D_2 = 200 \text{ mm} = 0.2 \text{ m}$
 $p_2 = 9.81 \text{ N/cm}^2 = 9.81 \times 10^4 \text{ N/m}^2$

Rate of flow = 40 lit/s

or $Q = \frac{40}{1000} = 0.04 \text{ m}^3/\text{s}$

Now

$$A_1 V_1 = A_2 V_2 = \text{rate of flow} = 0.04$$

$$\therefore V_1 = \frac{.04}{A_1} = \frac{.04}{\frac{\pi}{4} D_1^2} = \frac{0.04}{\frac{\pi}{4} (0.3)^2} = 0.5658 \text{ m/s}$$

$$\approx 0.566 \text{ m/s}$$

$$V_2 = \frac{.04}{A_2} = \frac{.04}{\frac{\pi}{4} (D_2)^2} = \frac{0.04}{\frac{\pi}{4} (0.2)^2} = 1.274 \text{ m/s}$$

Applying Bernoulli's equation at (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

$$\text{or } \frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{.566 \times .566}{2 \times 9.81} + z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.274)^2}{2 \times 9.81} + z_2$$

$$\text{or } 25 + .32 + z_1 = 10 + 1.623 + z_2$$

$$\text{or } 25.32 + z_1 = 11.623 + z_2$$

$$\therefore z_2 - z_1 = 25.32 - 11.623 = 13.697 = 13.70 \text{ m}$$

$$\therefore \text{Difference in datum head} = z_2 - z_1 = 13.70 \text{ m. Ans.}$$

Problem:- 10

A horizontal venturimeter with inlet and throat diameters 10cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and throat is 20cm of mercury. Determine the rate of flow. Take $C_d = 0.98$

Solution. Given :

Dia. at inlet, $d_1 = 30 \text{ cm}$

$\therefore$ Area at inlet, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} (30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15 \text{ cm}$

$\therefore$ $a_2 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ cm}^2$

$C_d = 0.98$

Reading of differential manometer $= x = 20 \text{ cm of mercury.}$

$\therefore$ Difference of pressure head is given by (6.9)

or
$$h = x \left[\frac{S_h}{S_o} - 1 \right]$$

where $S_h = \text{Sp. gravity of mercury} = 13.6$, $S_o = \text{Sp. gravity of water} = 1$

$$= 20 \left[\frac{13.6}{1} - 1 \right] = 20 \times 12.6 \text{ cm} = 252.0 \text{ cm of water.}$$

The discharge through venturimeter is given by eqn. (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}} = \frac{86067593.36}{684.4}$$

$$= 125756 \text{ cm}^3/\text{s} = \frac{125756}{1000} \text{ lit/s} = \mathbf{125.756 \text{ lit/s.}}$$

Problem:- 11

An oil of Sp.gr. 0.8 is flowing through a horizontal venturimetre having inlet diameter 20cm and throaty diameter 10 cm. The oil mercury differential manometer shows a reading of 25cm. Calculate the discharge of oil through the horizontal venturimetre. Take $C_d = 0.98$

Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_h = 13.6$

Reading of differential manometer, $x = 25$ cm

$$\begin{aligned}\therefore \text{ Difference of pressure head, } h &= x \left[\frac{S_h}{S_o} - 1 \right] \\ &= 25 \left[\frac{13.6}{0.8} - 1 \right] \text{ cm of oil} = 25 [17 - 1] = 400 \text{ cm of oil.}\end{aligned}$$

Dia. at inlet, $d_1 = 20$ cm

$$\therefore a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$$

$d_2 = 10$ cm

$$\therefore a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$$

$C_d = 0.98$

$\therefore$ The discharge Q is given by equation (6.8)

$$\begin{aligned}\text{or } Q &= C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \\ &= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 400} \\ &= \frac{21421375.68}{\sqrt{98696 - 6168}} = \frac{21421375.68}{304} \text{ cm}^3/\text{s} \\ &= 70465 \text{ cm}^3/\text{s} = \mathbf{70.465 \text{ litres/s. Ans.}}\end{aligned}$$

Problem:- 12

A horizontal venturimeter with inlet and throat diameters 20cm and 10 cm respectively is used to measure the flow of oil of Sp. gr. The discharge of oil through venturimeter is 60lit/s . Find the reading of oil-mercury differential manometer. Take $C_d = 0.98$

Solution. Given :

$$d_1 = 20 \text{ cm}$$

$$\therefore a_1 = \frac{\pi}{4} 20^2 = 314.16 \text{ cm}^2$$

$$d_2 = 10 \text{ cm}$$

$$\therefore a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$$

$$C_d = 0.98$$

$$Q = 60 \text{ litres/s} = 60 \times 1000 \text{ cm}^3/\text{s}$$

Using the equation (6.8), $Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

or
$$60 \times 1000 = 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times h}$$

$$= \frac{1071068.78 \sqrt{h}}{304}$$

or
$$\sqrt{h} = \frac{304 \times 60000}{1071068.78} = 17.029$$

$\therefore h = (17.029)^2 = 289.98 \text{ cm of oil}$

But
$$h = x \left[\frac{S_h}{S_o} - 1 \right]$$

where $S_h = \text{Sp. gr. of mercury} = 13.6$
 $S_o = \text{Sp. gr. of oil} = 0.8$
 $x = \text{Reading of manometer}$

$\therefore 289.98 = x \left[\frac{13.6}{0.8} - 1 \right] = 16x$

$\therefore x = \frac{289.98}{16} = 18.12 \text{ cm.}$

$\therefore \text{Reading of oil-mercury differential manometer} = \mathbf{18.12 \text{ cm.}}$

Problem:-13

A static pitot-tube placed in the centre of a 300 mm pipe line has one orifice pointing upstream and is perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe if the pressure difference between the two orifices is 60mm of water. Take $C_v = 0.98$

Solution. Given :

Dia. of pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$
 Diff. of pressure head, $h = 60 \text{ mm of water} = .06 \text{ m of water}$
 $C_v = 0.98$

Mean velocity, $\bar{V} = 0.80 \times \text{Central velocity}$
 Central velocity is given by equation (6.14)

$$= C_v \sqrt{2gh} = 0.98 \times \sqrt{2 \times 9.81 \times .06} = 1.063 \text{ m/s}$$

$\therefore \bar{V} = 0.80 \times 1.063 = 0.8504 \text{ m/s}$

Discharge, $Q = \text{Area of pipe} \times \bar{V}$

$$= \frac{\pi}{4} d^2 \times \bar{V} = \frac{\pi}{4} (.30)^2 \times 0.8504 = \mathbf{0.06 \text{ m}^3/\text{s. Ans.}}$$

Orifices -

It is a small opening of any cross-section (such as circular, triangular, rectangular etc.) on the side or at the bottom of a tank, through which a fluid is flowing.

A mouthpiece is a short length of a pipe which is two to three times its diameter in length, fitted in a tank containing the fluid.

Orifice as well as mouthpieces are used for measuring the rate of flow of fluid.

Classification of orifices:

Small orifice

(If the head of liquid from the centre of orifice is more than five times the depth of orifice)

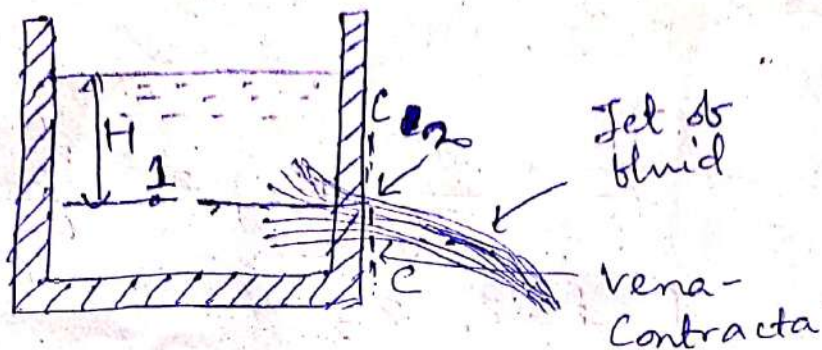
Large orifice

(If the head of liquid from the centre of orifice is ~~more~~ ^{less} than five times the depth of orifice)

The orifices are classified as (i) Circular orifice (ii) triangular orifice (iii) Rectangular orifice (iv) Square orifice. (according to the cross-sectional area)

flow through an orifice :

Consider a tank fitted with a circular orifice in one of its sides.



Let H be the head of liquid above the centre of the orifice.

The liquid flowing through the orifice forms a jet of liquid whose area of cross-section is less than that of orifice.

The area of jet of fluid decreases at section $(C-C)$ called vena-contracta, which is at a distance of half of diameter of the orifice.

At this section the streamlines are straight and parallel to each other and perpendicular to the plane of the orifice. Beyond this section, the jet diverges and is attracted in downward direction by the gravity.

Consider two points 1 & 2. ~~which~~
point 1 is inside the tank and point 2 is at vena-contracta.
Let the flow is steady and at a constant head H .

Applying Bernoulli's equation at point 1 & 2.

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

But $z_1 = z_2$

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g}$$

Now $\frac{P_1}{\rho g} = H$, $\frac{P_2}{\rho g} = 0$ (atmospheric pressure)

v_1 is very small in comparison to v_2 as area of tank is very large as compared to the area of the jet of liquid.

$$H + 0 = 0 + \frac{v_2^2}{2g}$$

$$\Rightarrow v_2 = \sqrt{2gH} \quad (\text{Theoretical velocity})$$

Actual velocity is always less than the theoretical velocity.

Hydraulic Co-efficients

The hydraulic co-efficients are:

- ① Co-efficient of velocity (C_v)
- ② Co-efficient of contraction (C_c)
- ③ Co-efficient of discharge (C_d)

Co-efficient of velocity:

It is defined as the ratio between the actual velocity of a jet of liquid at vena-contracta and the theoretical velocity of jet.

Mathematically

$$C_v = \frac{V}{\sqrt{2gH}}$$

where V = Actual velocity

$\sqrt{2gH}$ = Theoretical velocity

The value of C_v varies from 0.95 to 0.99.
The general value of C_v is 0.98.

Co-efficient of Contraction

It is defined as the ratio of the area of the jet at vena-contracta to the area of the orifice.

$$C_c = \frac{a_c}{a}$$

The value of C_c varies 0.61 to 0.69.
The general value of C_c is 0.64.

Co-efficient of Discharge

It is defined as the ratio of the actual discharge from an orifice to the theoretical discharge from the orifice.

Mathematically

$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual velocity} \times \text{Actual Area}}{\text{Theoretical velocity} \times \text{Theoretical Area}}$$

$$= \frac{\text{Actual velocity}}{\text{Theoretical velocity}} \times \frac{\text{Actual Area}}{\text{Theoretical area}}$$

$$C_d = C_v \times C_c$$

The value of C_d varies from 0.61 to 0.65.

The general value of C_d 0.62

Notch - It is a device used for measuring the rate of flow of a liquid through a small channel or a tank.

Weir - It is a concrete or masonry structure, placed in an open channel over which the flow occurs.

It is generally in the form of vertical wall, with a sharp edge at the top, running all the way across the open channel.

★ The notch is generally made of metallic plate while weir is made of concrete or masonry structure.

Classification of Notches and Weirs :-

The notches are classified as :

① According to the shape of the opening

(a) Rectangular notch

(b) Triangular notch

(c) Trapezoidal notch

(d) Stepped notch

② According to the effect of the sides on the nappe :

(a) Notch with end contraction

(b) Notch without end contraction or suppressed notch.

Weirs are classified according to the shape of the opening, the shape of the crest.

① According to the shape of the opening:

(a) Rectangular weir

(b) Triangular weir

(c) Trapezoidal weir

② According to the shape of crest:

(a) Sharp-crested weir

(b) Broad-crested weir

(c) Narrow-crested weir

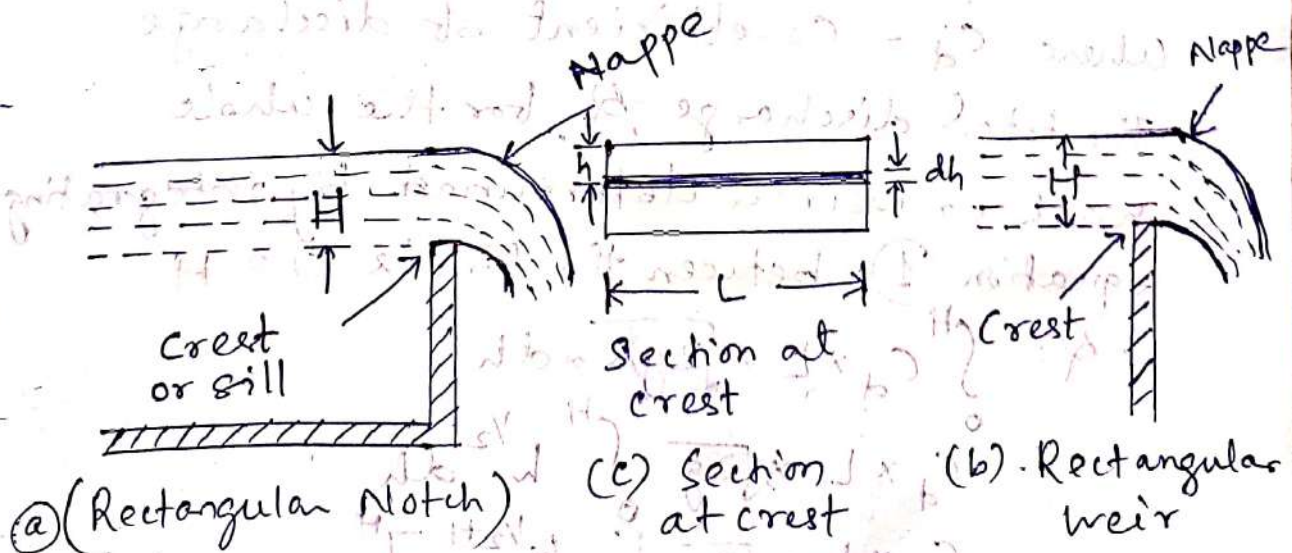
(d) Ogee-shaped weir

③ According to the effect of sides on the emerging nappe:

(a) Weir with end contraction

(b) Weir without end contraction

Discharge over a Rectangular notch or weir:



Consider a rectangular notch or weir provided in a channel carrying water.

Let H = Head of water over the crest

L = Length of the notch or weir

For finding the discharge of water flowing over the weir or notch, consider an elementary horizontal strip of water of thickness dh and length L at a depth h from the free surface of water.

The area of strip = $L \times dh$

The theoretical velocity of water flowing through strip = $\sqrt{2gh}$

The discharge dQ , through strip is,

$$dQ = C_d \times \text{Area of strip} \times \text{Theoretical velocity} \\ = C_d \times L \times dh \times \sqrt{2gh} \quad \text{--- (1)}$$

where C_d = Co-efficient of discharge

The total discharge, Q , for the whole notch or weir is determined by integrating equation (1) between the limits 0 & H

$$\begin{aligned} \therefore Q &= \int_0^H C_d \times L \times \sqrt{2gh} \times dh \\ &= C_d \times L \times \sqrt{2g} \int_0^H h^{1/2} dh \\ &= C_d \times L \times \sqrt{2g} \left[\frac{h^{1/2+1}}{1/2+1} \right]_0^H \\ &= C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H = \left[\frac{2}{3} C_d \times L \times \sqrt{2g} [H]^{3/2} \right] \end{aligned}$$

Problem on Rectangular notch —

- 1) Find the discharge of water flowing over a rectangular notch of 2 m length when the constant head over the notch is 300 mm. Take $C_d = 0.60$.

Solution :

Given data :

Length of the notch $= L = 2.0 \text{ m}$

Head over notch $= H = 300 \text{ mm} = 0.3 \text{ m}$

$C_d = 0.60$

$$\begin{aligned} \text{Discharge, } Q &= \frac{2}{3} C_d \times L \times \sqrt{2g} [H]^{3/2} \\ &= \frac{2}{3} \times 0.60 \times 2 \times \sqrt{2 \times 9.81} \times [0.3]^{3/2} \\ &= 0.582 \text{ m}^3/\text{sec}. \end{aligned}$$

Problem on Rectangular Weir :

- 1) Determine the height of a rectangular weir of length 6 m to be built across a rectangular channel. The maximum depth of water on the upstream side of the weir is 1.8 m and discharge is 2000 lit/sec. Take $C_d = 0.6$ and neglect end contractions.

Solution :

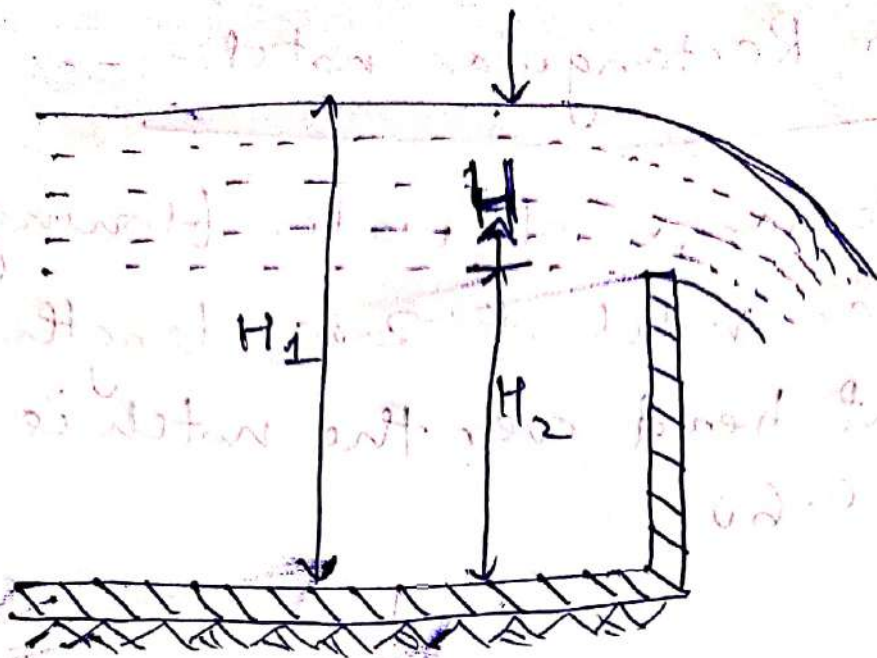
Given data :

Length of weir, $L = 6 \text{ m}$

$C_d = 0.60$

Depth of weir, $H_1 = 1.8 \text{ m}$

Discharge $Q = 2000 \text{ lit/sec} = 2 \text{ m}^3/\text{sec}$



Let H is the height of water above the crest of water and $H_2 =$ height of weir.

The discharge over the weir, $Q =$

$$\frac{2}{3} C_d \times L \times \sqrt{2g} H^{3/2}$$

$$\Rightarrow 2 = \frac{2}{3} \times 0.60 \times 6 \times \sqrt{2 \times 9.81} \times H^{3/2}$$

$$\Rightarrow 2 = \frac{2}{3} \times 0.60 \times 6 \times \sqrt{2 \times 9.81} \times H^{3/2}$$

$$\Rightarrow H^{3/2} = \frac{2}{10.623}$$

$$\Rightarrow H = \left(\frac{2}{10.623} \right)^{2/3} = 0.328 \text{ m}$$

$$\therefore \text{Height of water, } H_2 = H_1 - H$$

$$= 1.8 - 0.328 = 1.472 \text{ m}$$

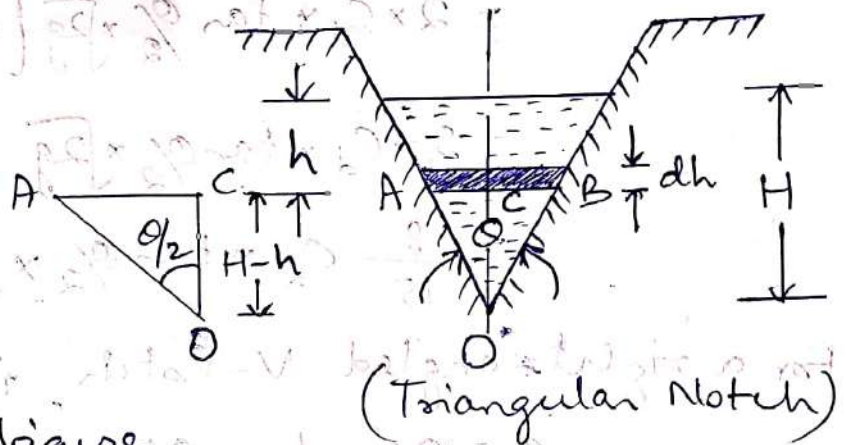
Discharge over a triangular notch or weir:

The expression for the discharge over a triangular notch is derived as:

Let, H = head of water above the V-notch

θ = angle of notch

Consider a horizontal strip of water of thickness dh at a depth of h from the free surface of water.



From the figure,

$$\tan \frac{\theta}{2} = \frac{AC}{OC} = \frac{AC}{(H-h)}$$

$$\therefore AC = (H-h) \tan \frac{\theta}{2}$$

$$\text{width of strip} = AB = 2AC = 2(H-h) \tan \frac{\theta}{2}$$

$$\therefore \text{Area of strip} = 2(H-h) \tan \frac{\theta}{2} \times dh$$

The theoretical velocity of water through strip = $\sqrt{2gh}$

$$\therefore \text{Discharge (Q) through the strip} =$$

$$dQ = C_d \times \text{Area of strip} \times V_{th}$$

$$= C_d \times 2(H-h) \tan \frac{\theta}{2} \times dh \times \sqrt{2gh}$$

$$= 2C_d (H-h) \tan \frac{\theta}{2} \times \sqrt{2gh} \times dh$$

$$\begin{aligned}
 \therefore \text{Total Discharge, } Q &= \int_0^H 2 C_d (H-h) \tan \frac{\theta}{2} \sqrt{2gh} \, dh \\
 &= 2 C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \int_0^H (H-h) h^{1/2} \, dh \\
 &= 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \int_0^H (H h^{1/2} - h^{3/2}) \, dh \\
 &= 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \left[\frac{H h^{3/2}}{3/2} - \frac{h^{5/2}}{5/2} \right]_0^H \\
 &= 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \left[\frac{2}{3} H \cdot H^{3/2} - \frac{2}{5} H^{5/2} \right] \\
 &= 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \left[\frac{2}{3} H^{5/2} - \frac{2}{5} H^{5/2} \right] \\
 &= 2 \times C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \left[\frac{4}{15} H^{5/2} \right] \\
 &= \frac{8}{15} C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2} \quad \text{--- (1)}
 \end{aligned}$$

For a right-angled V-notch, i.e. $C_d = 0.60$

$$\theta = 90^\circ, \tan \frac{\theta}{2} = 1$$

$$\text{Discharge } Q = \frac{8}{15} \times 0.60 \times 1 \times \sqrt{2 \times 9.81} \times H^{5/2} \quad \text{--- (2)}$$

$$Q = 1.417 H^{5/2}$$

Problem on Triangular notch -

- 1) Find the discharge over a triangular notch of angle 60° when the head over the V-notch is 0.3 m . Assume $C_d = 0.60$.

Solution - Given data :

Angle of V-notch, $\theta = 60^\circ$

Head over notch, $H = 0.3 \text{ m}$

$C_d = 0.60$

Discharge, Q , over a V-notch =

$$Q = \frac{8}{15} C_d \times \tan \frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{8}{15} \times 0.6 \times \tan \frac{60^\circ}{2} \times \sqrt{2 \times 9.81} \times (0.3)^{5/2}$$

$$= 0.818 \times 0.049 = 0.040 \text{ m}^3/\text{sec}$$

2) A rectangular channel 2.0 m wide has a discharge of 250 litres per second, which is measured by a right-angled V-notch weir. Find the position of the apex of the notch from the bed of the channel if maximum depth of water is not to exceed 1.3 m. Take $C_d = 0.62$

Solution - Given data :

width of rectangular channel, $L = 2 \text{ m}$; $C_d = 0.62$

$\theta = 90^\circ$ (right angled V-notch)

$$Q = 250 \text{ lit/sec}$$

$$= 0.25 \text{ m}^3/\text{sec}$$

Depth of water in channel = 1.3 m

Let, the height of water over V-notch = H

The rate of flow or discharge through

$$\text{V-notch} = Q = \frac{8}{15} \times C_d \times \sqrt{2g} \times \tan \frac{\theta}{2} \times H^{5/2}$$

$$\Rightarrow 0.25 = \frac{8}{15} \times 0.62 \times \sqrt{2 \times 9.81} \times \tan \frac{90^\circ}{2} \times H^{5/2}$$

$$\Rightarrow 0.25 = \frac{8}{15} \times 0.62 \times 4.429 \times 1 \times H^{5/2}$$

$$\Rightarrow H^{5/2} = \frac{0.25 \times 15}{8 \times 0.62 \times 4.429} = 0.17$$

$$\Rightarrow H = (0.17)^{2/5} = (0.17)^{0.4} = 0.493 \text{ m}$$

∴ Position of apex of the notch from the bed of channel

$$= (\text{depth of water in channel}) - (\text{height of water over V-notch})$$

$$= 1.3 - 0.493 = 0.807 \text{ m}$$

6th Chapter (Flow through Pipes)

Definition of Pipe :-

It is a hollow cylinder of metal, wood or other material used for the conveyance of water, gas, steam, petroleum etc.

Loss of energy in pipes :

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as :

Energy Losses

1. Major Energy Losses

This is due to friction and it is calculated by the following formulae :

- (a) Darcy-Weisbach formula
- (b) Chezy's formula

2. Minor Energy losses

This is due to

- (a) Sudden expansion of pipe.
- (b) Sudden contraction of pipe
- (c) Bend in pipe
- (d) Pipe fittings etc.
- (e) An obstruction in pipe.

Loss of energy (or head) due to friction :

(a) Darcy-Weisbach formula :

The loss of head (or energy) in pipes due to friction is calculated from Darcy-Weisbach equation $\rightarrow$

$$h_f = \frac{4 f L V^2}{d \times 2g}$$

Where h_f = loss of head due to friction.

f = Co-efficient of friction which is a function of Reynolds number

$$= \frac{16}{Re} \text{ for } Re < 2000 \text{ (Viscous flow)}$$

$$= \frac{0.079}{Re^{1/4}} \text{ for } Re \text{ (varying from 4000 to } 10^6)$$

L = Length of pipe

V = Mean velocity of flow

d = diameter of pipe.

(b) Chezy's formula for loss of head due to friction in pipes. :-

for loss of head due to friction in pipes =

$$h_f = \frac{f'}{fg} \times \frac{P}{A} \times L \times V^2 \quad \text{--- (1)}$$

Where h_f = loss of head due to friction

A = Area of cross-section of pipe

V = Mean velocity of flow

P = Wetted perimeter of pipe

L = Length of pipe.

The ratio of $\frac{A}{P}$ ($\frac{\text{Area of flow}}{\text{Wetted Perimeter}}$) = m is called hydraulic mean depth or

hydraulic radius

$$\therefore \text{Hydraulic mean depth, } m = \frac{A}{P} = \frac{\frac{\pi d^2}{4}}{\pi d}$$

$$\therefore \boxed{m = \frac{d}{4}}$$

Substituting $\frac{A}{P} = m$ or $\frac{P}{A} = \frac{1}{m}$ in equation (1), we get

$$h_f = \frac{f'}{fg} \times L \times V^2 \times \frac{1}{m}$$

$$\Rightarrow V^2 = h_f \times \frac{fg}{f'} \times m \times \frac{1}{L} = \frac{fg}{f'} \times m \times \frac{h_f}{L}$$

$$\Rightarrow V = \sqrt{\frac{fg}{f'} \times m \times \frac{h_f}{L}}$$

$$\Rightarrow V = \sqrt{\frac{fg}{f'}} \sqrt{m \frac{h_f}{L}} \quad \text{--- (2)}$$

Let $\sqrt{\frac{fg}{f'}} = C$, where C is Chezy's constant and $\frac{h_f}{L} = i$, where i is

loss of head per unit length of pipe.

Substituting the value of $\sqrt{\frac{fg}{f'}}$ and $\sqrt{\frac{h_f}{L}}$

in equation (2), we get:

$$\boxed{V = C \sqrt{mi}} \quad \text{--- (3)}$$

The equation (3) is known as Chezy's formula.

Problems on Darcy formula and Chezy's formula :-

- 1) Find the head lost due to friction in a pipe of diameter 300 mm and length 50 m through which water is flowing at a velocity of 3 m/sec using (i) Darcy formula (ii) Chezy's formula for which $C = 60$.
Take ν for water = 0.01 stoke.

Solution - Given data:

Diameter of pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$

Length of pipe, $L = 50 \text{ m}$

Velocity of flow $V = 3 \text{ m/sec}$

Chezy's constant $C = 60$

Kinematic viscosity $\nu = 0.01 \text{ stoke}$

$$= 0.01 \text{ cm}^2/\text{sec}$$

$$= 0.01 \times 10^{-4} \text{ m}^2/\text{sec}$$

(i) Darcy formula is
$$h_f = \frac{4 \times f \times L \times V^2}{d \times 2g}$$

where 'f' = coefficient of friction is a function of Reynolds number, Re

$$Re = \frac{V \times d}{\nu} = \frac{3 \times 0.30}{0.01 \times 10^{-4}} = 9 \times 10^5$$

$$\therefore f = \frac{0.079}{Re^{1/4}} = \frac{0.079}{(9 \times 10^5)^{1/4}} = 0.00256$$

$$\therefore \text{Head lost, } h_f = \frac{4 \times 0.00256 \times 50 \times 3^2}{0.3 \times 2.0 \times 9.81}$$

$$= 0.782 \text{ m}$$

(ii) Chezy's formula

$$V = C \sqrt{m i}$$

Where $C = 60$, $m = \frac{d}{4} = \frac{0.30}{4} = 0.075 \text{ m}$.

$V = 3 \text{ m/sec}$, $L = 50 \text{ m}$

$$\therefore 3 = 60 \sqrt{0.075 \times i}$$

$$\Rightarrow i = \left(\frac{3}{60} \right)^2 \times \frac{1}{0.075} = 0.033$$

But $i = \frac{h_f}{L} = \frac{h_f}{50}$

Equating the two values of i , we get

$$0.033 = \frac{h_f}{50}$$

$$\Rightarrow h_f = 50 \times 0.033 = 1.665 \text{ m}$$

2) Find the diameter of pipe of length 2000 m when the rate of flow of water through the pipe is 200 lit/sec and the head lost due to friction is 4 m. Take the value of $C = 50$ in Chezy's formula.

Solution:

Given data:

Length of pipe, $L = 2000 \text{ m}$.

Discharge, $Q = 200 \text{ lit/sec}$
 $= 0.2 \text{ m}^3/\text{sec}$

Head lost due to friction, $h_f = 4 \text{ m}$.

Value of Chezy's Constant, $C = 50$

Let the diameter of pipe $= d$

Velocity of flow, $V = \frac{\text{Discharge}}{\text{Area}}$

$$= \frac{Q}{\frac{\pi d^2}{4}} = \frac{0.2}{\frac{\pi d^2}{4}}$$

$$= \frac{0.2 \times 4}{\pi d^2}$$

Hydraulic mean depth, $m = \frac{d}{4}$

Loss of head per unit length,

$$i = \frac{h_f}{L} = \frac{4}{2000} = 0.002$$

Chezy's formula, $V = C \sqrt{m i}$

Substituting the values of V , m , i

and C , we get

$$\frac{0.2 \times 4}{\pi d^2} = 50 \sqrt{\frac{d}{4} \times 0.002}$$

$$\Rightarrow \sqrt{\frac{d}{4} \times 0.002} = \frac{0.2 \times 4}{\pi d^2 \times 50} = \frac{0.005}{d^2}$$

Squaring both sides, $\frac{d}{4} \times 0.002 = \frac{0.005^2}{d^4}$

$$\Rightarrow \frac{d}{4} \times 0.002 = \frac{0.000025}{d^4}$$

$$\Rightarrow d^5 = \frac{4 \times 0.000025}{0.002} = 0.05$$

$$\Rightarrow d = \sqrt[5]{0.05} = 0.55 \text{ m}$$

Hydraulic gradient and Total Energy line:-

It is very useful in the study of flow of fluids through pipes.

Hydraulic Gradient Line (HGL) :-

It is defined as the line which gives the sum of pressure head ($P/\rho g$) and datum head (Z) of a flowing fluid in a pipe with respect to some reference line.

Total Energy Line (TEL) :-

It is defined as the line which gives the sum of pressure head $\frac{P}{\rho g}$, datum head Z and kinetic head $\frac{V^2}{2g}$ of a flowing fluid in a pipe with respect to some reference line.

It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe.

7th Chapter (Impact of Jets)

The liquid comes out in the form of a jet from the outlet of a nozzle, which is fitted to a pipe through which the liquid is flowing under pressure.

The impact of jet means the force exerted by the jet on a plate which is stationary or moving.

This force is obtained from Newton's second law of motion or from Impulse-momentum equation:

- ① The force exerted by the jet on a plate, ~~when~~ (stationary plate), when
 - (a) Plate is vertical to the jet
 - (b) Plate is inclined to the jet
 - (c) Plate is curved.
- ② The force exerted by the jet on a moving plate, when
 - (a) Plate is vertical to the jet
 - (b) Plate is inclined to the jet
 - (c) Plate is curved.

Force exerted by the jet on a stationary (fixed) vertical plate: —

Consider a jet of water coming out from the nozzle, strikes a flat vertical plate.

Let, V = velocity of the jet

d = diameter of the jet

a = area of cross-section of the

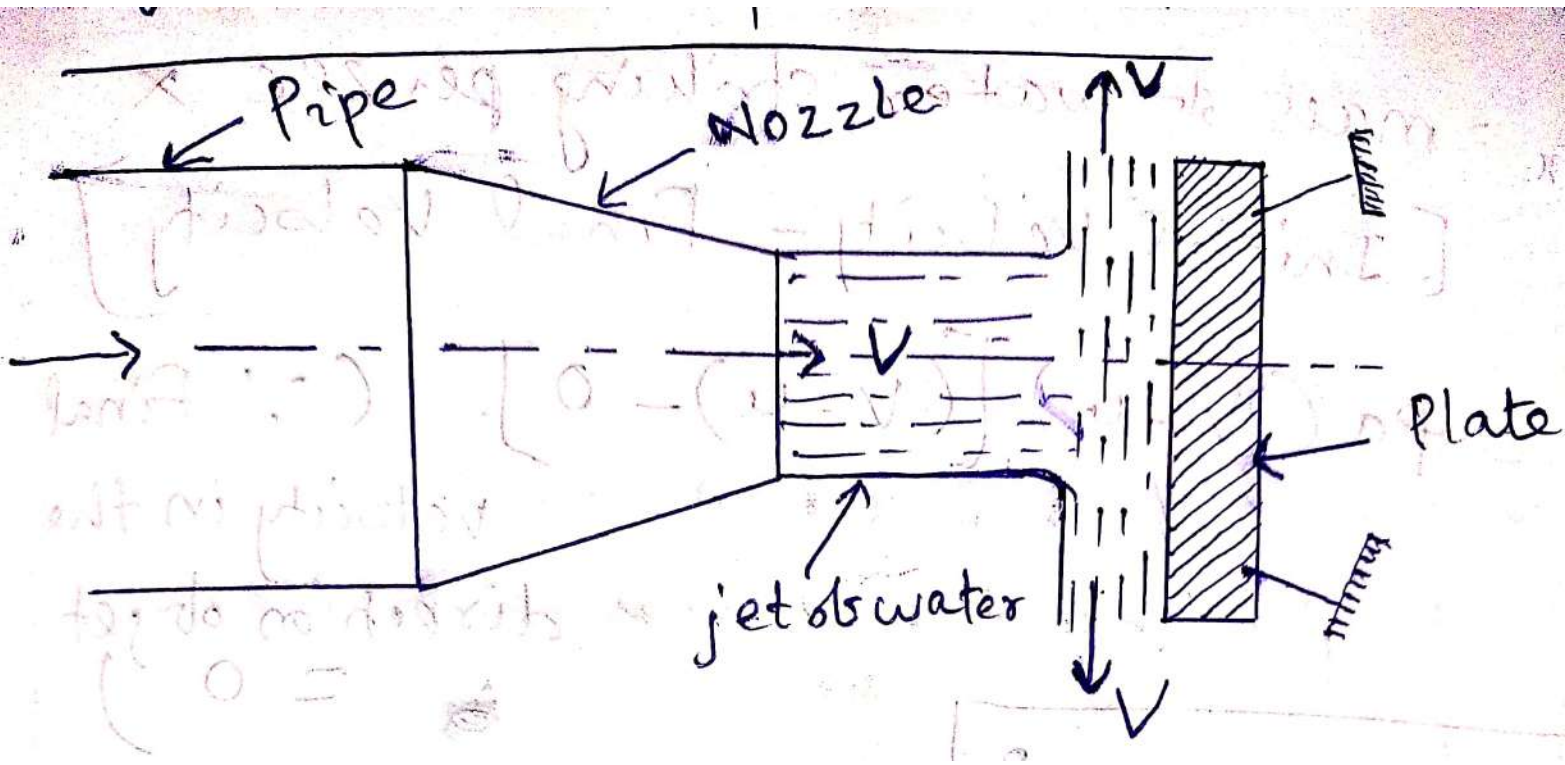
$$\text{jet} = \frac{\pi}{4} d^2$$

The jet after striking the plate, moves along the plate.

But the plate is at right angles to the jet.

Hence the jet after striking, deflects through 90° .

So the component of the velocity of jet in the direction of jet, after striking is zero.



∴ The force exerted by the jet on the plate in the direction of jet,

$F_x =$ Rate of change of momentum in the direction of force

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{(\text{mass} \times \text{Initial velocity} - \text{mass} \times \text{Final velocity})}{\text{Time}}$$

$$= \frac{\text{mass}}{\text{time}} [\text{Initial velocity} - \text{Final velocity}]$$

$$= (\text{mass/sec}) \times (\text{velocity of jet before striking} - \text{velocity of jet after striking})$$

$$= \rho a v [v - 0] \quad \left(\because \text{mass/sec} = \rho \times a v \right)$$

$$\boxed{F_x = \rho a v^2}$$

* If the force exerted by the jet on the plate is calculated, then

$$\cancel{F_x \text{ Initial}} \quad F_x = \frac{\text{mass}}{\text{time}} (\text{Initial velocity} - \text{Final velocity})$$

If the force exerted on the jet is calculated, then

$$F_x = \frac{\text{mass}}{\text{time}} (\text{Final velocity} - \text{Initial velocity})$$

Force exerted by a jet on moving vertical flat plate

The jet of water strikes a flat vertical plate moving with a uniform velocity away from the jet.

Let V = Velocity ~~of jet~~ of the jet

a = Area of cross-section of the jet

u = Velocity of the flat plate.

In this case, the jet does not strike the plate with a velocity V , but it strikes with a relative velocity which is equal to the absolute velocity of jet of water minus the velocity of plate.

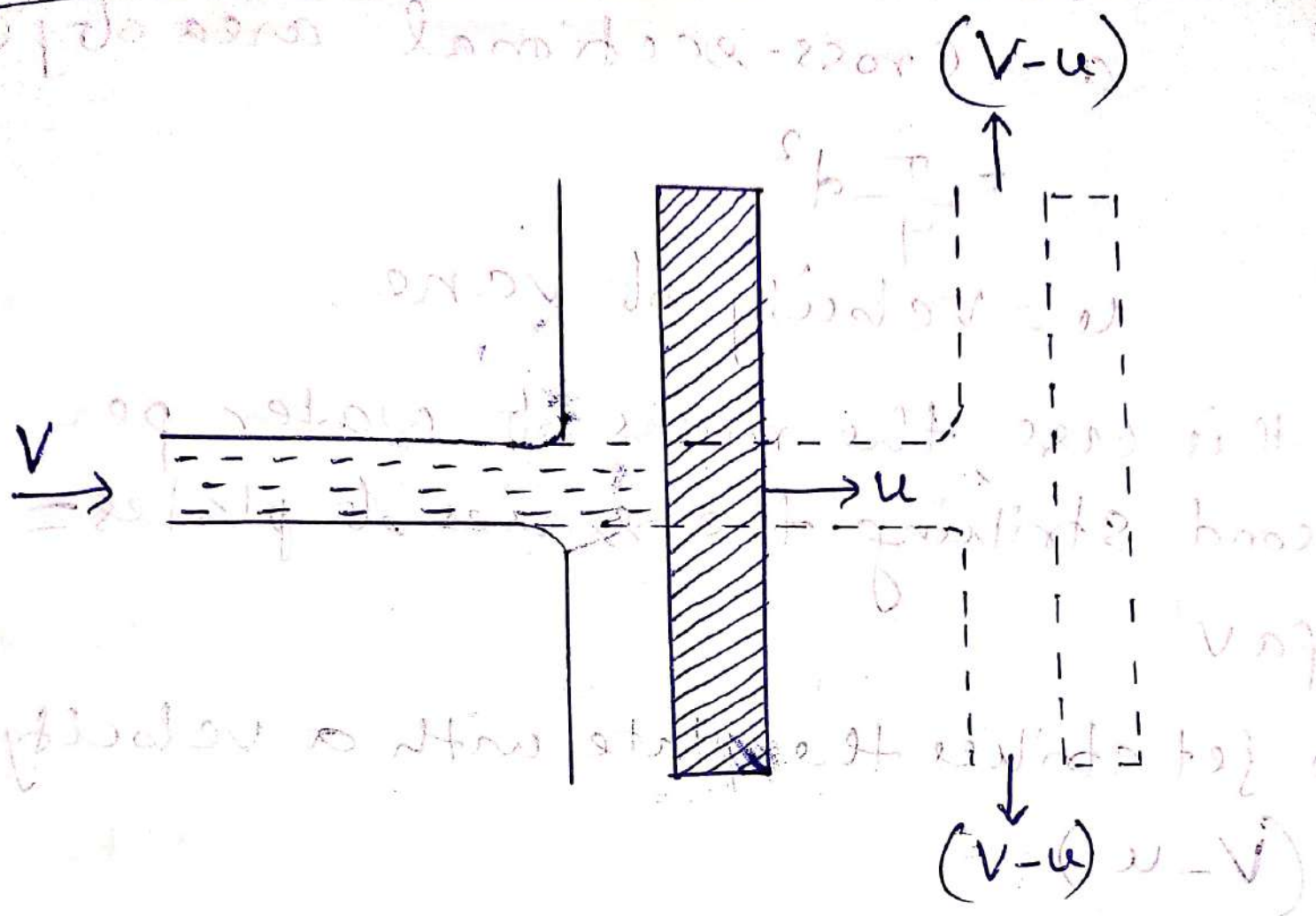
∴ the relative velocity of the jet with respect to plate = $V - u$

Mass of water striking the plate per sec

$$= \rho \times \text{Area of jet} \times \text{velocity}$$

with which jet strikes the plate

$$= \rho a \times [V - u]$$



∴ Force exerted by the jet on the moving plate in the direction of jet,

$$F_x = \text{mass of water striking per sec} \times [\text{Initial velocity} - \text{Final velocity}]$$

$$= \rho a (v-u) [(v-u) - 0] \quad (\because \text{Final velocity in the direction of jet} = 0)$$

$$F_x = \rho a (v-u)^2 \quad \text{--- ①}$$

In this case, the work done per second by the jet on the moving plate

$$= \text{Force} \times \frac{\text{Distance in the direction of force}}{\text{time}}$$

$$= F_x \times u$$

$$W = \rho a (v-u)^2 \times u \quad \text{--- ②}$$

~~from the~~ In the equation ②, the value of $\rho = 1000 \text{ kg/m}^3$ for water

The unit of W is Nm/sec .

Force exerted by a jet of water on series of vanes:

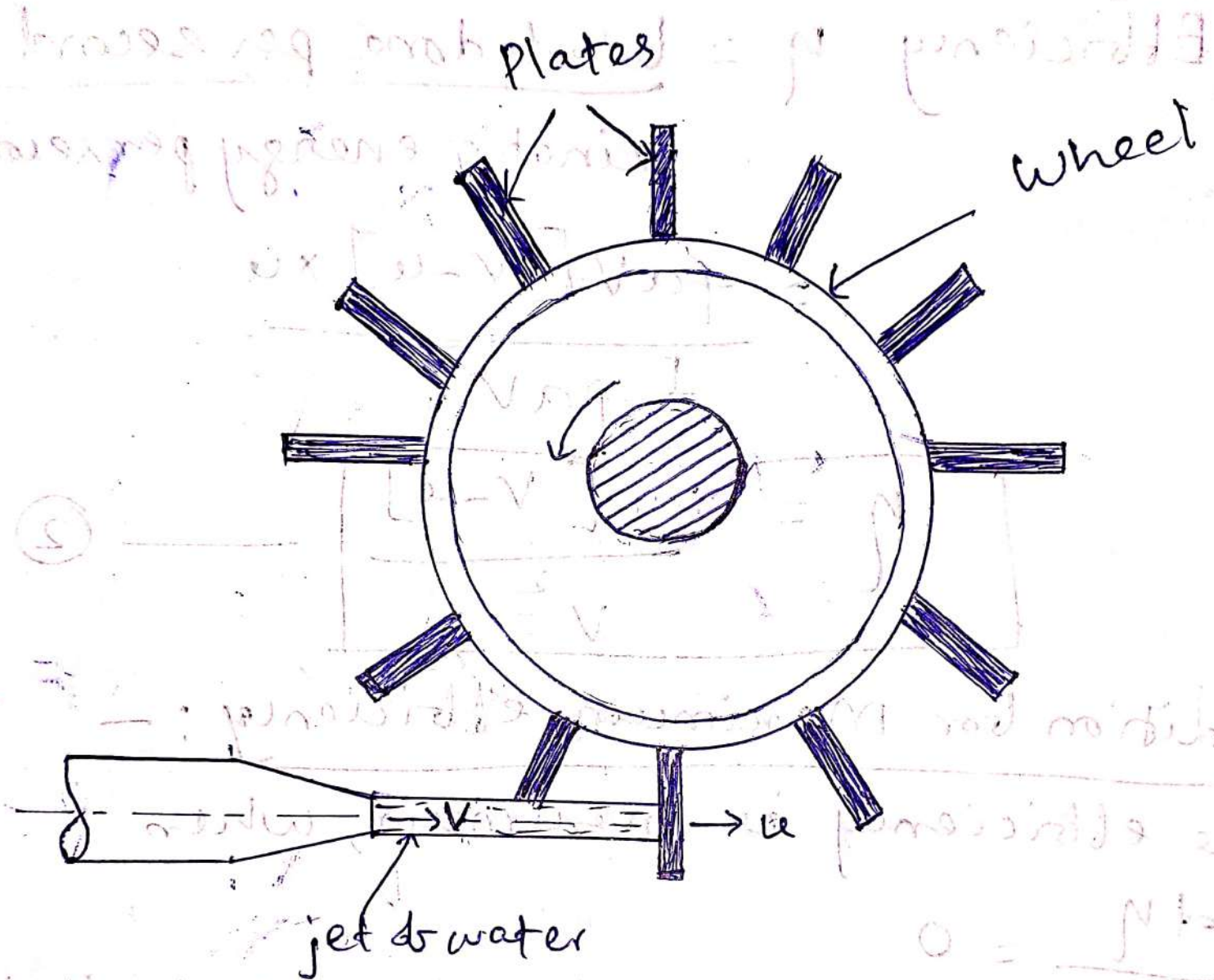
In this case, a large number of plates are mounted on the circumference of a wheel at a fixed distance apart.

The jet strikes a plate and due to the force exerted by the jet on the plate, the wheel starts moving.

The 2nd plate mounted on the wheel appears before the jet, which again exerts the force on the 2nd plate.

So each plate appears before the jet successively and the jet exerts force on each plate.

The wheel starts moving at a constant speed.



Force exerted on a series of Radial Curved Vanes :-

Consider a series of radial curved vanes mounted on a wheel.

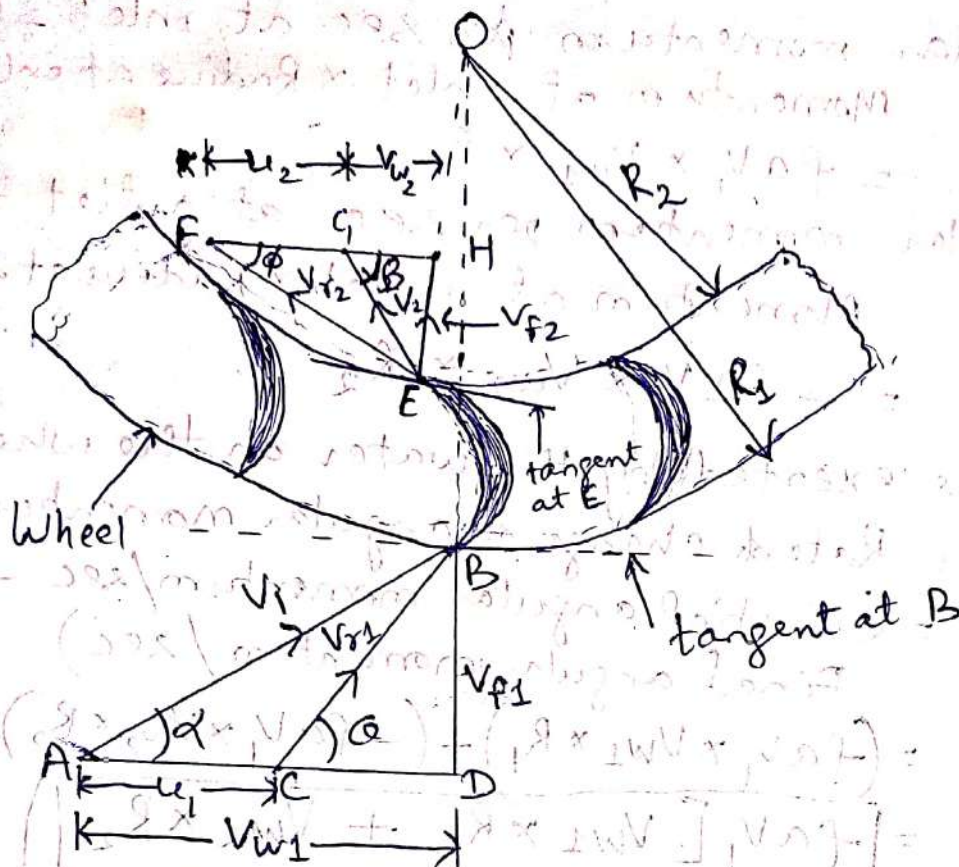
The jet of water strikes the vanes and the wheel starts rotating at a constant angular speed.

Let R_1 = Radius of wheel at inlet of the vane.

R_2 = Radius of wheel at the outlet of the vane.

ω = Angular speed of the wheel

$$\therefore u_1 = \omega R_1 \quad \text{and} \quad u_2 = \omega R_2$$



From the velocity triangles at inlet and outlet,
 The mass of water striking per sec. for a series
 of vanes = Mass of water coming out from nozzle
 per sec. = $\rho a V_1$

where a = Area of jet, V_1 = Velocity of jet.

Momentum of water striking the vanes in the
 tangential direction per sec. at inlet =

$\rho a V_1$ = Mass of water per sec. $\times$ Component of V_1
 in the tangential direction

$$= \rho a V_1 \times V_{w1} \quad (\because V_1 \cos \alpha = V_{w1})$$

Momentum of water at outlet per sec.

$= \rho a V_1 \times$ Component of V_2 in the tangential direction

$$= \rho a V_1 \times (-V_2 \cos \beta) = -\rho a V_1 \times V_{w2} \quad (\because V_2 \cos \beta = V_{w2})$$

-ve sign indicates the velocity V_2 at outlet
 is in opposite direction.

$$\begin{aligned}\text{Angular momentum per sec. at inlet} &= \\ &\text{Momentum at inlet} \times \text{Radius at inlet} \\ &= \rho a V_1 \times V_{w1} \times R_1\end{aligned}$$

$$\begin{aligned}\text{Angular momentum per sec. at outlet} &= \\ &\text{Momentum at outlet} \times \text{Radius at outlet} \\ &= -\rho a V_2 \times V_{w2} \times R_2\end{aligned}$$

$$\text{Torque exerted by the water on the wheel} =$$

$$T = \text{Rate of change of angular momentum}$$

$$= (\text{Initial angular momentum/sec} - \text{Final angular momentum/sec})$$

$$= (\rho a V_1 \times V_{w1} \times R_1) - (-\rho a V_2 \times V_{w2} \times R_2)$$

$$= \rho a V_1 [V_{w1} \times R_1 + V_{w2} \times R_2]$$

$$\text{Work done per sec. on the wheel} =$$

$$\text{Torque} \times \text{Angular velocity} = T \times \omega$$

$$= \rho a V_1 [V_{w1} \times R_1 + V_{w2} \times R_2] \times \omega$$

$$= \rho a V_1 [V_{w1} \times R_1 \times \omega + V_{w2} \times R_2 \times \omega]$$

$$= \rho a V_1 [V_{w1} u_1 + V_{w2} u_2] \quad (\because u_1 = \omega R_1 \text{ and } u_2 = \omega R_2)$$

If the angle β is an obtuse angle, then

$$\text{work done per sec.} = \rho a V_1 [V_{w1} u_1 - V_{w2} u_2]$$

$$\therefore \text{The work done per sec. on the wheel} = \rho a V_1 [V_{w1} u_1 \pm V_{w2} u_2]$$

If the discharge is radial at outlet, then

$$\beta = 90^\circ \text{ and the work done} = \rho a V_1 [V_{w1} u_1]$$

$$(\because V_{w2} = 0)$$

Efficiency of the Radial Curved Vane :-

The work done per sec. on the wheel is the output of the system.

The Initial kinetic energy per sec of the jet is input.

$$\begin{aligned}\therefore \text{Efficiency } \eta &= \frac{\text{work done per sec.}}{\text{kinetic energy per sec.}} \\ &= \frac{\rho a V_1 [V_{w1} u_1 \pm V_{w2} u_2]}{\frac{1}{2} (m/\text{sec}) \times V_1^2} \\ &= \frac{\rho a V_1 [V_{w1} u_1 \pm V_{w2} u_2]}{\frac{1}{2} \rho a V_1 \times V_1^2} \\ &= \boxed{\frac{2 [V_{w1} u_1 \pm V_{w2} u_2]}{V_1^2}}\end{aligned}$$

Work done per sec. on the wheel = Change in K.E per sec. of the ~~wheel~~ jet.

$$= (\text{Initial K.E per sec.} - \text{Final K.E per sec.})$$

$$= \left(\frac{1}{2} m V_1^2 - \frac{1}{2} m V_2^2 \right)$$

$$= \frac{1}{2} m (V_1^2 - V_2^2) = \boxed{\frac{1}{2} (\rho a V_1^2) (V_1^2 - V_2^2)}$$

$$(\because m/\text{sec} = \rho a V_1)$$

$$\therefore \text{Efficiency } \eta = \frac{\text{work done per sec. on the wheel}}{\text{Initial K.E per sec. of the jet}}$$

$$= \frac{\frac{1}{2} \rho a V_1^2 (V_1^2 - V_2^2)}{\frac{1}{2} (\rho a V_1^2) \cdot V_1^2}$$

$$= \frac{V_1^2 - V_2^2}{V_1^2} = \boxed{1 - \frac{V_2^2}{V_1^2}}$$

From the above equation, the efficiency is maximum when V_2 is minimum.

(CHAPTER -4) PNEUMATIC CONTROL SYSTEM

Elements –filter-regulator-lubrication unit

FRL Unit: Filter, Regulator, & Lubricator - How They Work

Filter, regulator, and lubricator (FRL) compressed air systems are used to deliver clean air, at a fixed pressure, and lubricated (if needed) to ensure the proper pneumatic component operation and increase their operational lifetime.

The air supplied by compressors is oftentimes contaminated, over-pressurized, and non-lubricated meaning that an FRL unit is required to prevent damage to equipment. Filters, regulators, and lubricators can be bought individually or as a package (as seen in Figure 1) depending on what is needed to ensure the proper air specifications are being met for downstream equipment.

It is recommended to install these devices if you:

- Use pneumatic tools and equipment;
- Are installing an HVAC system;
- Require clean air to be delivered to your facility or workplace;
- Require compliance to ISO, OSHA, ASHRA or other air quality standards;
- Want to improve the service life, safety and reliability of your air system.



What do FRL units do?

An FRL unit is comprised of a filter (F), regulator (R), and a lubricator (L). They are often used as one unit to ensure clean air in a pneumatic system but can also be used individually. Having a proper FRL unit installed in a pneumatic system provides higher reliability of the components downstream, reduced power waste due to over pressurization, and increased component lifetime. The three components work together to do the following:

- Filters remove water, dirt and other harmful debris from an air system. This is often the first step in improving the air quality.

- Regulators adjust and control the air pressure of a system to ensure that down-line components do not exceed their maximum operating pressures. This is the second step in the FRL system.
- Lubricators reduce the internal friction in tools or equipment by releasing a controlled mist of oil into the compressed air. This is often done last and/or right before the component needs lubrication.

Pneumatic filter selection



Pneumatic filter

- Filters remove water, dirt and other harmful debris from an air system (Figure 2). The type and size of contaminants present in the system and the air requirements for components will ultimately affect what micron size and bowl material is needed for the filter.
 - Common applications generally only require a filter rated between 5-40 microns. However, ISO 8157 goes down to 0.1 micron and for special applications, like medical or pharmaceutical the specifications can be as low as .001 micron.
 - The rating means that it doesn't allow bigger particles through. For example, if you have a 20 micron filter it will allow particles smaller than 20 microns to pass through. It should be noted that filters experience a small pressure drop across the inlet and outlet ports because of the flow restriction.
 - A 0.1 micron filter will create a larger pressure drop than a 40 micron filter and will require more regular maintenance due to the easy build-up of contaminants. Therefore, do not oversize your filter by selecting the finest possible micron size.
 - It will lead to higher cost for the component, a larger pressure drop, and more maintenance time. Instead, select a filter that will remove only the smallest contaminant specific to your system.
 - The bowl material and drainage type are also important. The bowl comes into contact with the contaminants and houses the filtered particles. The pressure, temperature, and chemicals present affect the bowl material selection.
 - Filters also require drainage, which can be either accomplished by the filter as an automatic, semi-automatic, or manual drainage system or a condensate drain can be attached to the outlet to remove the filtered contaminants.
- **Automatic:** An automatic drain is a 2/2-way valve that closes when the system is pressurized. It has a float system in it that rises when the system is de-pressurized or when

liquid accumulates, and this rise in the float causes the drain to open. Advised when the equipment is in continual use, requires frequent drainage, or in a hard to reach location.

- **Semi-automatic:** A semi-automatic drain automatically drains the system upon depressurization. It can drain the system when pressurized, but only by a manual process. If the system is not always under pressure, it is recommended to have a semi-automatic drainage filter.
- **Manual:** The filter can be manually drained when the system is depressurized. Not advised for a hard-to-reach location, if it requires frequent drainage, and if the system isn't regularly depressurized.
- **Condensate Drain:** A condensate drain can be attached to the outlet of the filter to accomplish the drainage. However, proper timing of the open/closing needs to be set.

PRESSURE CONTROL VALVE

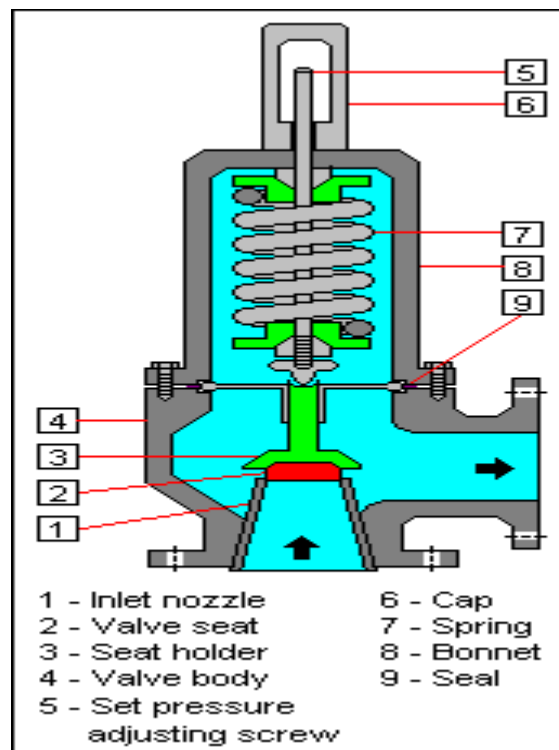
- The valves enable the regulation of system pressure to adjust the force on a hydraulic piston rod or the torque on a hydraulic motor shaft. Pressure relief valves are used to set the maximum pressure in the circuit and protect it from overloading.



Pressure Relief Valves

- Most pneumatic and hydraulic power systems are designed to operate within a defined pressure range. This range is a function of the forces the actuators in the system must generate to do the required work.
- Without controlling these forces, the power components and expensive equipment could get damaged. Relief valves make it possible to avoid this hazard.
- They are the safeguards that limit maximum pressure in a system by diverting excess gases when pressure gets too high. The pressure at which a relief valve first opens to allow fluid to flow through is known as cracking pressure.
- When the valve is bypassing its full rated flow, it is in a state of full-flow pressure.

- The difference between full-flow and cracking pressure is sometimes known as the pressure differential, or the pressure override.
- In some cases, this pressure override is not objectionable. It can be a disadvantage if it wastes power via gas lost through the valve prior to reaching the maximum setting.
- This can allow maximum system pressure to exceed the ratings of the other components.



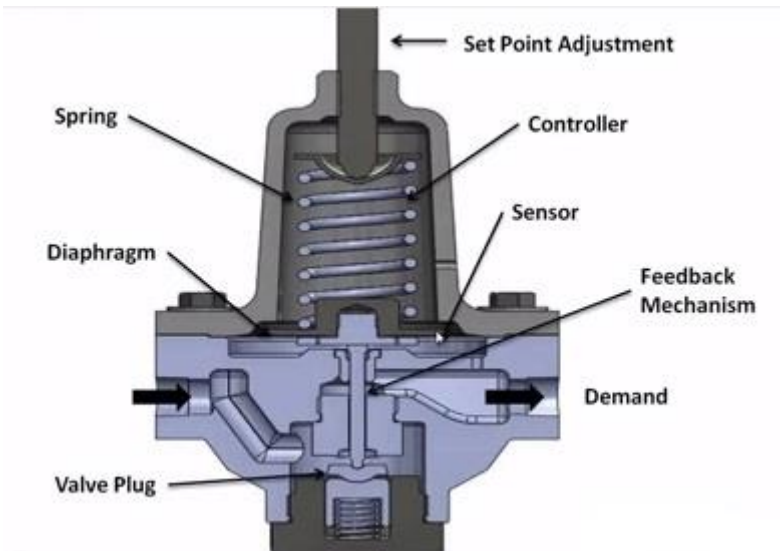
Pressure-Reducing Valves

The most practical components for maintaining lower pressure in a pneumatic system are pressure-reducing valves. Pressure-reducing valves are usually open two-way valves that close when subjected to sufficient downstream pressure. There are subcategories of pressure-reducing valves: direct acting and pilot operated.

Direct-acting valves are pressure-reducing valves that limit the maximum pressure available in the secondary circuit regardless of pressure changes in the main circuit.

- This assumes the workload generates no backflow into the reducing valve port, in which case the valve will close.
- The pressure-sensing signal comes from the secondary circuit. The valve operates in reverse from a relief valve because they are normally closed and sense the pressure from the inlet.
- When outlet pressure reaches the valve setting, the valve closes except for a small quantity of gas that bleeds from the low-pressure side of the valve, usually through an orifice in the spool.
- The spool in a pilot-operated, pressure-reducing valve is balanced hydraulically by downstream pressure at both ends. The pilot valve relieves enough gas to position the spool so that flow through the main valve equals the requirements of the reduced-pressure circuit.

- If no flow is required during the cycle, the main valve closes. Leakage of high-pressure gas into the reduced-pressure section of the valve then returns to the reservoir through the pilot-operated relief valve.
- This type of valve generally has a wider range of spring adjustments than direct-acting valves and provides better repetitive accuracy. However, in hydraulic applications, oil contamination can block flow to the pilot valve, and the main valve will fail to close properly.



DIRECTIONAL CONTROL VALVES

Directional control valves perform only three functions:

- stop fluid flow
- allow fluid flow, and
- change direction of fluid flow.

These three functions usually operate in combination.

- The simplest directional control valve is the 2-way valve. A 2-way valve stops flow or allows flow. A water faucet is a good example of a 2-way valve. A water faucet allows flow or stops flow by manual control.
- A single-acting cylinder needs supply to and exhaust from its port to operate. This requires a 3-way valve. A 3-way valve allows fluid flow to an actuator in one position and exhausts the fluid from it in the other position. Some 3-way valves have a third position that blocks flow at all ports.
- A double-acting actuator requires a 4-way valve. A 4-way valve pressurizes and exhausts two ports interdependently. A 3-position, 4-way valve stops an actuator or allows it to float. The 4-way function is a common type of directional control valve for both air and hydraulic circuits. A 3-position, 4-way valve is more common in hydraulic circuits.

3/2-Way Pneumatic Valve - How They Work



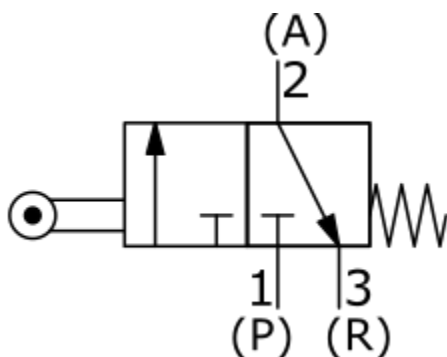
- A 3/2-way valve has three ports and two positions that can be driven pneumatically, mechanically, manually or electrically via a solenoid valve.
- They are used, for example, to control a single-action cylinder, driving pneumatic actuators, blow-off, pressure release and vacuum applications.
- A valve is used to fill the cylinder, and also to exhaust it afterwards, so that a new working stroke can be realized. Therefore, a valve with two ports would not be adequate. Venting requires a third port.
- There are two kinds of 3/2 valves: mono-stable and bi-stable. Mono-stable 3/2-way valves can also be normally closed or normally open, just like 2/2-way valves.

Circuit function of 3 way air valves

The 3/2-way pneumatic valve has three connection ports and two states. The three ports are:

- inlet (P, 1),
- outlet (A, 2)
- exhaust (R, 3)

The two states of the valve are open and closed. When the valve is open, air flows from the inlet (P, 1) to the outlet (A, 2). When the valve is closed, air flows from the outlet (A, 2) to the exhaust (R, 3). A valve that is closed in a non-actuated state is normally closed (N.C.), the opposite is called normally open (N.O.).



Circuit function of a monostable, normally closed 3/2 way valve

- Most valves are mono-stable and return to their default position when not actuated, this is achieved with a spring mechanism. Bi-stable 3/2-way valves retain their position during power loss and require a separate action to switch the valve state.
- Therefore, they cannot be designated as Normally Closed or Normally Open. Bi-stable pneumatic solenoid valves typically have a coil at each position and are pulse operated.

Summarized, the different functions of the 3/2-way valve are:

- 3/2-way mono-stable NC
- 3/2-way mono-stable NO
- 3/2-way bi-stable

The circuit functions can be shown with valve symbols. For the three above-mentioned functions, the symbol of an indirectly operated solenoid valve is shown below. You can find detailed information about other pneumatic valve symbols and their explanation in our valve symbol article.

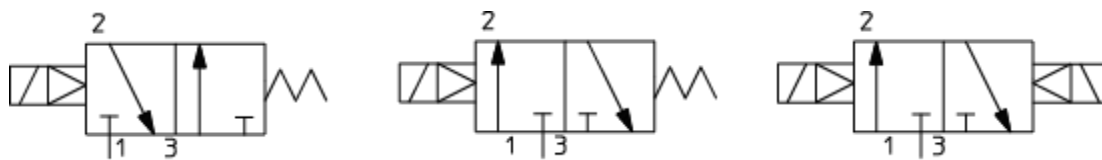


Figure 3: Symbols of 3/2-way pneumatic solenoid valves, from left to right: normally open, monostable (left), normally closed, monostable (center), bi-stable (right).

3/2-way valves can be actuated by different means such as:

- pneumatically
- manually
- mechanically
- electrically (solenoid valve)

Furthermore, the valves can be directly operated or indirect operated. With the indirect operation, the valve uses the inlet pressure to help switching the valve state.

3/2-way valves are available in several designs. The sealing mechanism of the valves can be a poppet or a spool. The valve's main parts are the following: housing, seals, poppet (or spool) and an actuator

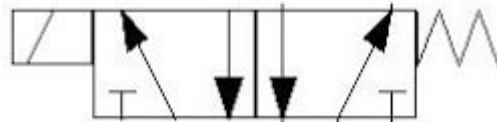
With direct operated valves, the spool or poppet is moved directly by the actuator. Several types of actuators are possible:

- Solenoid (coil)
- Push button

- Lever
- Foot pedal, etc.

5/2 DCV

5 Port 2 Position Valve Working Principle



A valve is a device that regulates the flow of fluid (gases, liquids, fluidized solids, or slurries) by opening and closing or partially obstructing passageways.

A 5/2 way directional valve from the name itself has 5 ports equally spaced and 2 flow positions. It can be used to isolate and simultaneously bypass a passageway for the fluid which for example should retract or extend a double-acting cylinder.

There are a variety of ways to have this valve actuated. A solenoid valve is commonly used, a lever can be manually twisted or pinched to actuate the valve, an internal or external hydraulic or pneumatic pilot to move the shaft inside, sometimes with a spring return on the other end so it will go back to its original position when pressure is gone or a combination of any of the mention above.

In the Illustration given, a single solenoid is used and a spring return is installed in the other end. The inlet pressure is connected to (P)1. (A)2 could possibly be connected to one end of the double-acting cylinder where the piston will retract while (B)4 is connected to the other end that will make the piston extend.

The normal position when the solenoid is de-energized is that the piston rod is blocking (B)4 and pressure coming from (P)1 passes through (A)2 that will make the cylinder normally retracted.

When the solenoid is energized, the rod blocks (A)2 and pressure from (P)1 passes through (B)4 and will extend the cylinder and when the solenoid is de-energized, the rod bounces back to its original position because of the spring return. (E)3 and (E)5 is condemned or used as exhaust.

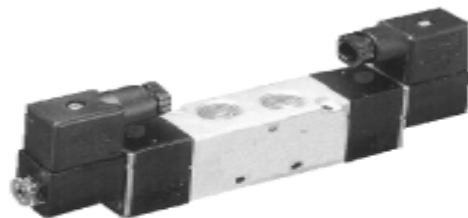
5/3 DCV

What is a 5 Way 3 Position Pneumatic Valve?

The 5/3 or 4/3 series are directional-control valves. The 5/3 and 4/3 body designs allow compressed air to flow to one port of a double-acting air actuator while simultaneously allowing air to exhaust from the other port on the same air actuator at the same time.

By shifting the internal flow paths of the valve, the 5/3 and 4/3 air valve sends compressed air alternatively to each of the two actuator ports and exhaust from the other, thus allowing the double-acting air cylinder to function.

The valve shown in the following image has 5 airports. It may be a 5/3, or it may be a 5/2 configuration. You cannot tell the difference from looking at the valve body. The valve schematic, which is typically shown on the side of the valve, is the only way you can determine if the spool is two-position (a 5/2) or three-position, (a 5/3) unless it is identified as such by the vendor.



**A 5/2 double solenoid air valve.
The two actuator ports are visible.
On the opposite side are three
ports; 2 exhaust & 1 supply**

What is a 3 Position Valve?

The “extra” position inside a 5/3 or 4/3 air valve means that the internal spool can be shifted to a center position. The typical spool movement is end to end inside the valve. With a two-position valve, the spool shifts from the end, across the middle, and to the other end. In the three-position body style, the spool can be positioned to stop in the middle location to accomplish a specific goal.

Each of the three spool positions is selected to accomplish the desired result in the action of the air cylinder.

Since valves with three-position spools are more expensive than their two-position counterparts, the selection of a three-position valve will be deliberate. The circuit designer

will have a particular scenario in mind for the action of the air cylinder when the valve that controls it is shifted, and that circuit will require the selection of a specific three-position valve to accomplish the goal.

A 5/3 or 4/3 valve will normally have two internal spring actuators that, when the valve is not being operated by an external valve actuator, shift that valve spool to the center position automatically. It is normally when the 5/3 or 4/3 valve is “at rest” that the third of the three positions come into play.

Flow-Control Valves

1. Flow-control valves include simple orifices to sophisticated closed-loop electrohydraulic valves that automatically adjust to variations in pressure and temperature.
2. The purpose of flow control in a hydraulic system is to regulate speed. All the devices discussed here control the speed of an actuator by regulating the flow rate. Flow rate also determines rate of energy transfer at any given pressure.
3. The two are related in that the actuator force multiplied by the distance through which it moves (stroke) equals the work done on the load. The energy transferred must also equal the work done. Actuator speed determines the rate of energy transfer (i.e., horsepower), and speed is thus a function of flow rate.
4. Directional control, on the other hand, does not deal primarily with energy control, but rather with directing the energy transfer system to the proper place in the system at the proper time. Directional control valves can be thought of as fluid switches that make the desired "contacts." That is, they direct the high-energy input stream to the actuator inlet and provide a return path for the lower-energy oil.
5. It is of little consequence to control the energy transfer of the system through pressure and flow controls if the flow stream does not arrive at the right place at the right time.
6. Thus, a secondary function of directional control devices might be defined as the timing of cycle events. Because fluid flow often can be throttled in directional-control valves, some measure of flow rate or pressure control can also be achieved with them.

PNEUMATIC THROTTLE VALVE

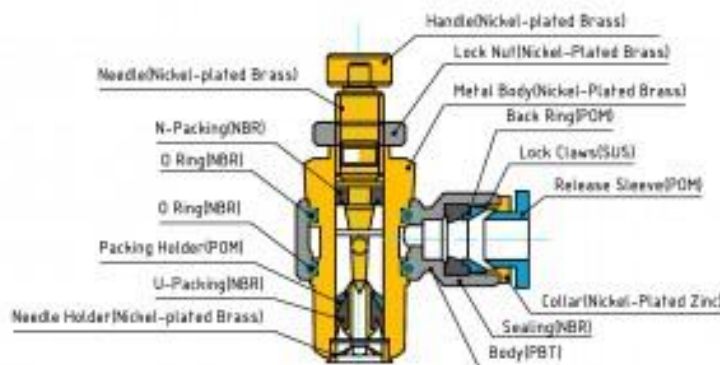
Pneumatic speed controller/Pneumatic throttle valve is used for controlling the operation speed of a driving device and the movement of machines such as cylinder, pneumatic finger, etc. The flow rate of air from A side to B side can be control, whereas air entering from B side to A side is not under control.



FEATURES

- Mainly installed in the air actuator.
- Working Pressure: 0-150 Psi.
- Working Temperature: 32 -140°F / 0-60°C.

CASE IN USE



METER OUT:

The flow rate of air entering from the thread side can be controlled, whereas air entering from joint side comes out from the thread side at the same flow rate (not controlled).

METER IN:

The flow rate of air entering from the joint side can be controlled, whereas air entering from thread side come out from joint side at the same speed (not controlled).

FLAT TYPE:

The way to control of flow or control flow upon piping in accordance with, the signal on the body. Air flows from each side of valve.

Pneumatics Symbols

DIN ISO1219-1, 03/96. Graphic symbols for pneumatic equipment.

Circuit symbols are used through this catalogue and on the labels of most SMC Pneumatic products.

There are several symbol systems and conventions in use around the world, most officially recognised by standards bodies. Commonly used is ISO1219-1.

The symbols found in this catalogue generally conform to the Japanese Industrial Standard (JIS) in many cases, there is no difference between JIS and ISO circuit symbols.

The situation also occurs when SMC develop new product systems for which an ISO or JIS symbol does not exist. Examples include the MQZ high power cylinder or the AV series air operated soft start / release valve. In this situation either a composite symbol showing a representative circuit is used, or the nearest standard symbol is modified by SMC.

For assistance a table below shows both ISO symbols, which may differ from JIS symbols in this catalogue, and common ISO/JIS/SMC Symbols.

Volume 1

Symbol	Description
	Directional control valve 2/2-way valve, closed normal position
	Directional control valve 2/2-way valve, open normal position
	Directional control valve 3/2-way valve, closed normal position
	Directional control valve 3/2-way valve, open normal position
	Directional control valve 3/3-way valve, closed neutral position
	Directional control valve 4/2-way valve
	Directional control valve 4/3-way valve, closed neutral position
	Directional control valve 4/3-way valve, exhaust neutral position
	Directional control valve 5/2-way valve
	Directional control valve 5/3-way valve, closed neutral position

Symbol	Description
	Directional control valve 5/3-way valve, exhaust neutral position
	Directional control valve 5/3-way valve, open neutral position
	Manual Control General
	Manual Control Lever
	Manual Control Pedal
	Mechanical Control Plunger
	Mechanical Control Spring
	Mechanical Control Roller
	Solenoid with one effective winding
	Solenoid with two windings acting in opposition
	Combined Control by solenoid and pilot valve
	Shuttle valve
	Pneumatic-Electric-Relay
	Pneumatic indicator
	Pressure Control Valve Air operated
	Mechanical Component Detent

Pneumatic circuits

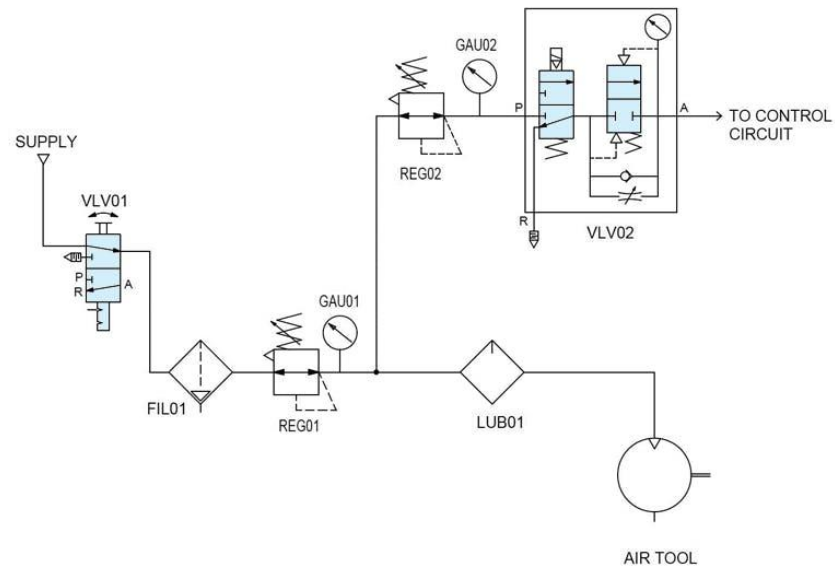
A pneumatic circuit is an interconnected set of components that convert compressed gas (usually air) into mechanical work. In the normal sense of the term, the circuit must include a compressor or compressor-fed tank.

4 Basic Pneumatic Circuits

The following four pneumatic circuits can be used for air preparation, double-acting cylinders, continuous cycling and hand control applications. They can also be subsystems in larger circuits.

Air Preparation

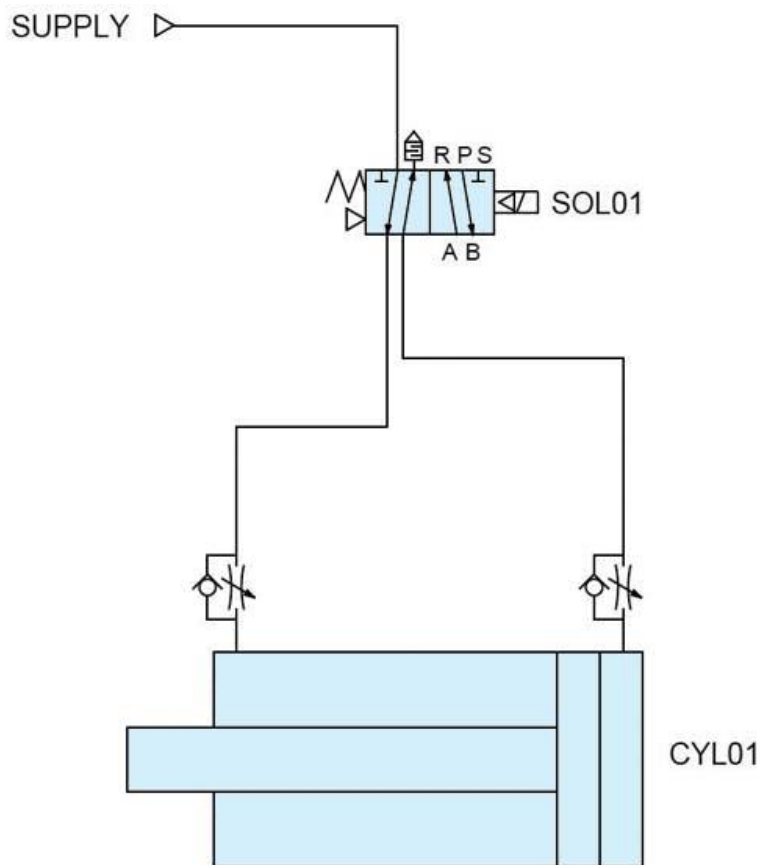
Before compressed air is used in a pneumatic device, it must be properly prepared so that it does not damage components. Here is a schematic (below) for a pneumatic device that prepares compressed air coming from a single source.



- Putting the manual shut-off valve or pneumatic isolation/lockout valve first makes it easier to maintain the FRL and it protects downstream equipment when depressurizing the system for maintenance.
- For safety, operators should be able to lock the valve in the off position. If it is necessary to have clean, dry air flowing through the valve, the valve can be mounted after the FRL.

Double-Acting Cylinder

The schematic below shows a common automation application: using a 4-way solenoid valve (SOL01) to extend and retract a double-acting cylinder (CYL01). Triangles at each side of the symbol indicate it is a pilot-activated, single-solenoid, spring-return valve.



Double-acting cylinder circuits are common on PLC-controlled machines.

- Filtered air feeds the solenoid valve, which is usually energized by a 24 V dc PLC output. This activates the valve and lets air leave through port B and flow freely through the flow control to extend the cylinder rod and plunger to the left.
- Air on the left side of the cylinder is forced out through its flow control to the valve's port A, and then goes to port R and exits through a muffler to reduce exhaust noise.
- Pilot valves need only a small amount of air to efficiently move a large valve spool. However, valves require a minimum operating pressure, typically about 20 psi, to move the spool.
- A spring on the left side pushes the valve spool to the right to maintain its normal off or resting state. With the valve off, air flows out of port A and freely through the adjustable flow control to the left side of the cylinder (CYL01), making it retract.
- As the cylinder retracts, air on the right-side leaves through an adjustable flow-control device. As the device's check valve closes, air in the flow section can be adjusted to throttle the cylinder retraction.
- The flow-controlled air then goes through the valve's port B and leaves at port S through a muffler.

Direct Control of Single Acting Cylinder

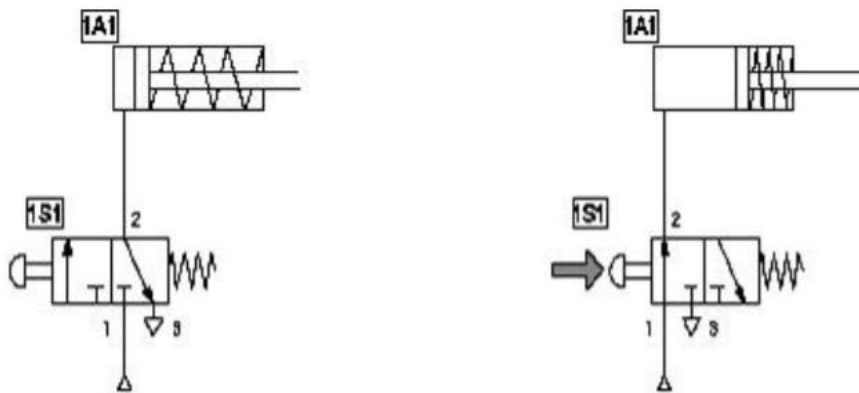


Figure 4.1 Direct Control of Single Acting Cylinder

- Pneumatic cylinders can be directly actuated by actuation of final control valve, manually or electrically in small cylinders as well as cylinders which operates at low speeds where the flow rate requirements are less.
- When the directional control valve is actuated by push button, the valve switches over to the open position, communicating working source to the cylinder volume.
- This results in the forward motion of the piston. When the push button is released, the reset spring of the valve restores the valve to the initial position [closed].
- The cylinder space is connected to the exhaust port thereby piston retracts either due to spring or supply pressure applied from the other port.

Indirect Control of Single Acting Cylinder

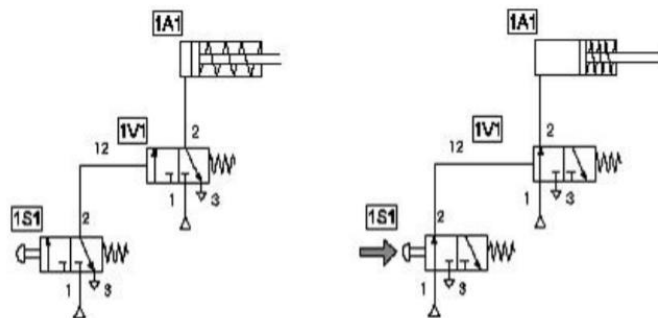


Figure 4.2: Indirect Control of Single and Double Acting Cylinders

- Large cylinders as well as cylinders operating at high speed are generally actuated indirectly as the final control valve is required to handle large quantity of air. In the case of pilot operated valves, a signal input valve [3/2 way N.C type, 1S1] either actuated manually or mechanically is used to generate the pilot signal for the final control valve.
- The signal pressure required can be around 1-1.5 bar. The working pressure passing through the final control valve depends on the force requirement [4-6 bar].
- Indirect control permits the processing of input signals. Single piloted valves are rarely used in applications where the piston has to retract immediately on taking out the set pilot signal, suitable for large single-acting cylinders.

CHAPTER – 5 (HYDRAULIC CONTROL SYSTEM)

Hydraulic system

A hydraulic system is a drive technology where a fluid is used to move the energy from e.g. an electric motor to an actuator, such as a hydraulic cylinder. The fluid is theoretically incompressible and the fluid path can be flexible in the same way as an electric cable.

Advantages of the hydraulic system

Talking about the advantages and disadvantages of hydraulic systems, let's first discuss the advantages. Even though it seems that the hydraulic system only relies on fluid and pressure to be distributed in all directions, this working principle is proven to be able to lift heavy materials. Here are ten advantages of a hydraulic system:

1. Any hydraulic motion is independent of the load as long as the fluid is not subject to resistance and a flow control valve can be used.
2. Precision and flexible.
3. Can move large power using only relatively small components.
4. Can move freely when lifting large loads.
5. Easy to use and friendly control system. Beginners who are just learning to operate heavy equipment with a hydraulic system will find it easy to understand.
6. The operation is smooth, does not cause unnecessary noise and vibration on the machine.
7. The design and design is simple.
8. Can be rotated in the reverse direction (reversible).
9. Make it easy to move heavy material, one lift, or move goods up to tons of weight.
10. The service life is quite long so it can save the cost of purchasing new equipment.

Weaknesses of the hydraulic system

It is incomplete to discuss the advantages and disadvantages of hydraulic systems without showing the shortcomings of the working principle of this system. After the advantages, now we will describe the weaknesses of the hydraulic system. Even though there are shortcomings, it doesn't mean that this hydraulic system is bad. It's just that, every working principle on heavy equipment must have advantages and limitations.

1. Hydraulic systems require intensive and periodic maintenance.
2. Systems frequently require parts with a very high degree of precision.
3. A small leak in the hydraulic pipeline will be fatal to the transfer of power. This causes
4. the risk of accidents increases.
5. The high pressure received by the fluid can also cause work accidents if the power is too high and the pipeline is unable to withstand the power delivered by the fluid.

Hydraulic accumulators-

- A hydraulic accumulator is a pressure storage reservoir in which an incompressible hydraulic fluid is held under pressure that is applied by an external source of mechanical energy.

- The external source can be an engine, a spring, a raised weight, or a compressed gas.^[note 1] An accumulator enables a hydraulic system to cope with extremes of demand using a less powerful pump, to respond more quickly to a temporary demand, and to smooth out pulsations.
- It is a type of energy storage device.
- Compressed gas accumulators, also called hydro-pneumatic accumulators, are by far the most common type.

Pressure control valves

The valves enable the regulation of system pressure to adjust the force on a hydraulic piston rod or the torque on a hydraulic motor shaft. Pressure relief valves are used to set the maximum pressure in the circuit and protect it from overloading.

Pressure-control valves are found in virtually every hydraulic system, and they assist in a variety of functions, from keeping system pressures safely below a desired upper limit to maintaining a set pressure in part of a circuit. Types include relief, reducing, sequence, counterbalance, and unloading. All of these are normally closed valves, except for reducing valves, which are normally open. For most of these valves, a restriction is necessary to produce the required pressure control. One exception is the externally piloted unloading valve, which depends on an external signal for its actuation.

Relief valves

Most fluid power systems are designed to operate within a present pressure range. This range is a function of the forces the actuators in the system must generate to do the required work. Without controlling or limiting these forces, the fluid power components (and expensive equipment) could be damaged. Relief valves avoid this hazard. They are the safeguards which limit maximum pressure in a system by diverting excess oil when pressures get too high.

Cracking pressure and pressure override —The pressure at which a relief valve first opens to allow fluid to flow through is known as *cracking pressure*. When the valve is bypassing its full rated flow, it is in a state of *full-flow pressure*. The difference between full-flow and cracking pressure is sometimes known as *pressure differential*, also known as *pressure override*.

Pressure-reducing valves

The most practical components for maintaining secondary, lower pressure in a hydraulic system are pressure-reducing valves. Pressure-reducing valves are normally open, 2-way valves that close when subjected to sufficient downstream pressure. There are two types: direct acting and pilot operated.

Fluid Power Pump-

Fluid power is the use of fluids under pressure to generate, control, and transmit power. Fluid power is subdivided into hydraulics using a liquid such as mineral oil or water, and pneumatics using a gas such as air or other gases. Compressed-air and water-pressure systems were once used to transmit power from a central source to industrial users over extended geographic areas; fluid power systems today are usually within a single building or mobile machine.

What is a gear pump?

A gear pump is a type of positive displacement (PD) pump. It moves a fluid by repeatedly enclosing a fixed volume using interlocking cogs or gears, transferring it mechanically using a cyclic pumping action. It delivers a smooth pulse-free flow proportional to the rotational speed of its gears.

How does a gear pump work?

Gear pumps use the actions of rotating cogs or gears to transfer fluids. The rotating element develops a liquid seal with the pump casing and creates suction at the pump inlet. Fluid, drawn into the pump, is enclosed within the cavities of its rotating gears and transferred to the discharge. There are two basic designs of gear pump: *external* and *internal* (Figure 1).

External Gear Pump

An external gear pump consists of two identical, interlocking gears supported by separate shafts. Generally, one gear is driven by a motor and this drives the other gear (the *idler*). In some cases, both shafts may be driven by motors. The shafts are supported by bearings on each side of the casing.

1. As the gears come out of mesh on the inlet side of the pump, they create an expanded volume. Liquid flows into the cavities and is trapped by the gear teeth as the gears continue to rotate against the pump casing.
2. The trapped fluid is moved from the inlet, to the discharge, around the casing.
3. As the teeth of the gears become interlocked on the discharge side of the pump, the volume is reduced and the fluid is forced out under pressure.

No fluid is transferred back through the centre, between the gears, because they are interlocked. Close tolerances between the gears and the casing allow the pump to develop suction at the inlet and prevent fluid from leaking back from the discharge side (although leakage is more likely with low viscosity liquids).

External gear pump designs can utilise spur, helical or herringbone gears.

Internal gear pump

An internal gear pump operates on the same principle but the two interlocking gears are of different sizes with one rotating inside the other. The larger gear (the *rotor*) is an internal gear i.e. it has the teeth projecting on the inside. Within this is a smaller external gear (the *idler* – only the rotor is driven) mounted off-centre. This is designed to interlock with

the rotor such that the gear teeth engage at one point. A pinion and bushing attached to the pump casing holds the idler in position. A fixed crescent-shaped partition or spacer fills the void created by the off-centre mounting position of the idler and acts as a seal between the inlet and outlet ports.

1. As the gears come out of mesh on the inlet side of the pump, they create an expanded volume. Liquid flows into the cavities and is trapped by the gear teeth as the gears continue to rotate against the pump casing and partition.
2. The trapped fluid is moved from the inlet, to the discharge, around the casing.
3. As the teeth of the gears become interlocked on the discharge side of the pump, the volume is reduced and the fluid is forced out under pressure.

Internal gear pump designs only use spur gears.

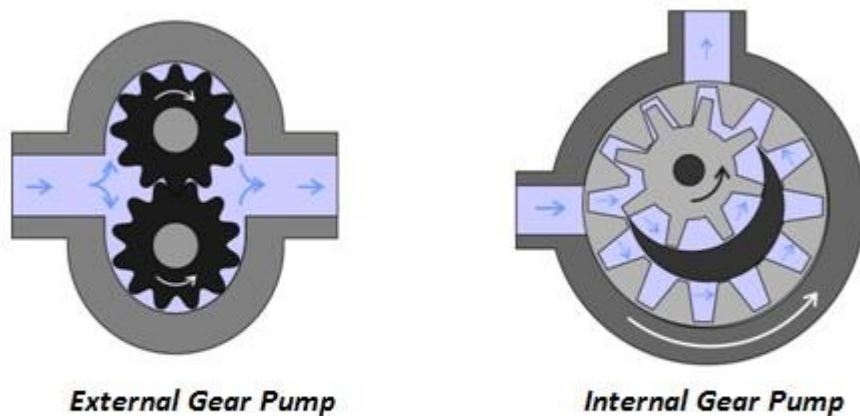
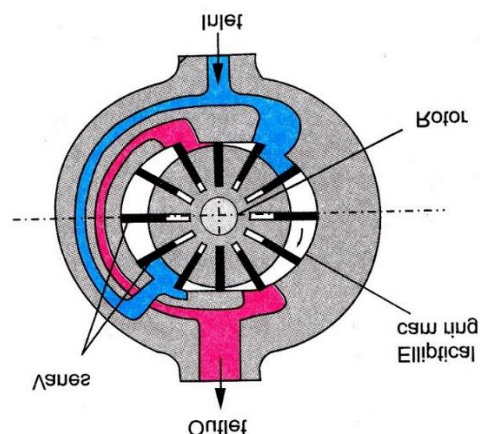


Figure 1. Gear pump designs (arrows indicate the direction of the pump and liquid)

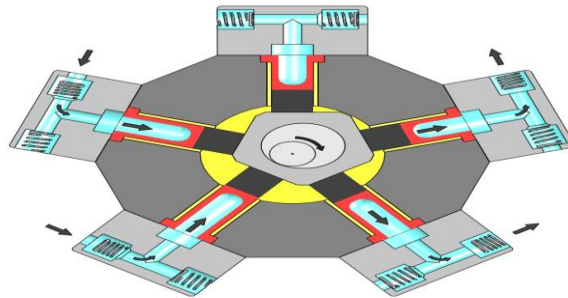
What is a vane pump?



Vanes or blades fit within the slots of the impeller. As the rotor rotates (yellow arrow) and fluid enters the pump, centrifugal force, hydraulic pressure, and/or pushrods push the vanes to the walls of the housing.

Radial piston pumps

A radial piston pump is a form of hydraulic pump. The working pistons extend in a radial direction symmetrically around the drive shaft, in contrast to the axial piston pump.



These kinds of piston pumps are characterized by the following advantages:

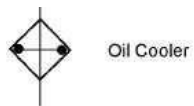
- high efficiency
- high pressure (up to 1,000 bar or 14000psi)
- low flow and pressure ripple (due to the small dead volume in the workspace of the pumping piston)
- low noise level
- very high load at lowest speed due to the hydrostatically balanced parts possible
- no axial internal forces at the drive shaft bearing
- high reliability

These kinds of piston pumps are characterized by the following advantages:

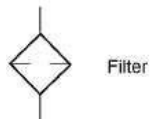
- high efficiency
- high pressure (up to 1,000 bar or 14000psi)
- low flow and pressure ripple (due to the small dead volume in the workspace of the pumping piston)
- low noise level
- very high load at lowest speed due to the hydrostatically balanced parts possible
- no axial internal forces at the drive shaft bearing
- high reliability

ISO Symbols for hydraulic components

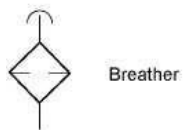
FLUID POWER SYMBOLS			
a) Energy Conversion Elements			
Elements	Description	Symbol	
Hydraulic Pumps Conversion of Mech. energy to hyd. energy.	a) With one directional flow	Displacement Fixed 	Variable
	b) With two directional flow		
Hydraulic Motor Conversion of hyd. energy to Mech. energy.	a) With one directional flow		
	b) With two directional flow		
	c) Limited rotation motor		
Pump / Motor	Components which can operate both as Pump and Motor		



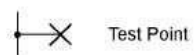
Oil Cooler



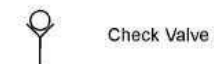
Filter



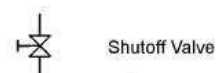
Breather



Test Point



Check Valve



Shutoff Valve



Pump with Motor



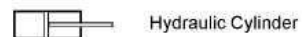
Pump



Fan Drive Motor



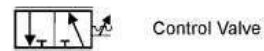
Accumulator



Hydraulic Cylinder



Brake Piston



Control Valve



Pressure Switch



Warning Switch

TYPES OF ACTUATORS

- Based on the source of Input Power actuators are classified in to three groups :

1. Pneumatic Actuators.

- These utilize pneumatic energy provided by the compressor and transforms it into mechanical energy by means of pistons or turbines.

2. Hydraulic Actuators.

- These Transform the energy stored in reservoir into mechanical energy by means of suitable pumps.

3. Electric Actuators.

- Electric actuators are simply electro-mechanical devices which allow movement through the use of an electrically controlled systems of gears



Hydraulic circuits

Hydraulic circuits transmit and control power from a mechanical input to a mechanical output by means of liquids, mostly oils. Power is transmitted hydrostatically, where high pressures make static forces dominate over dynamic forces, and energy is transmitted mostly through static pressure at low flow velocities.

Direct control of single acting cylinder

The single-acting cylinder is normally controlled by a **three-port valve**, for example a pneumatic solenoid valve. One port connects to the source of compressed air, the second port is used to supply/vent air to the cylinder and the third port is an exhaust port.

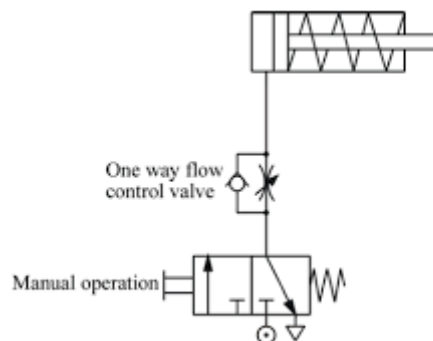
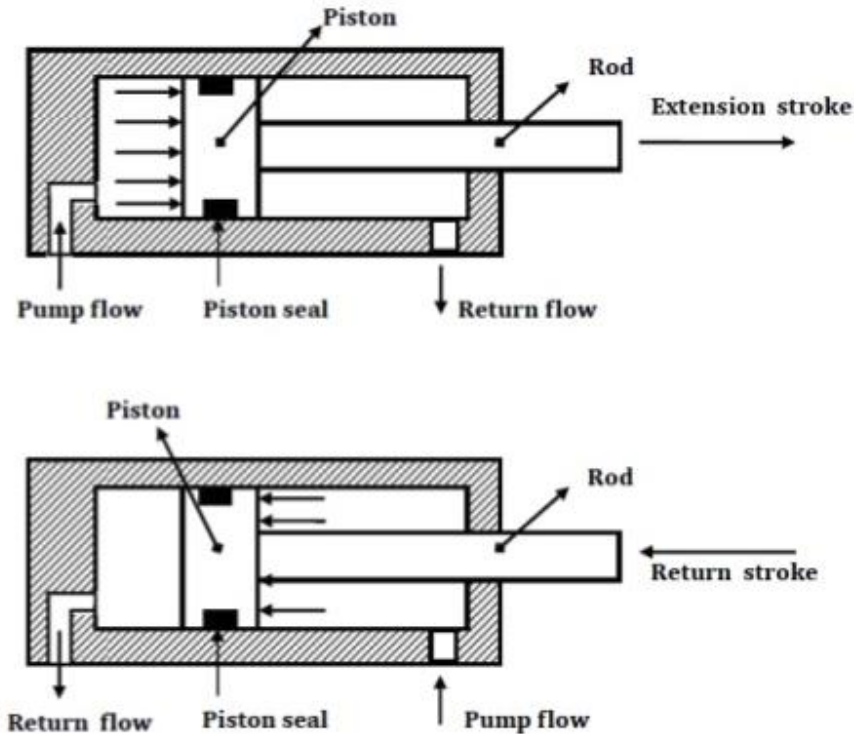


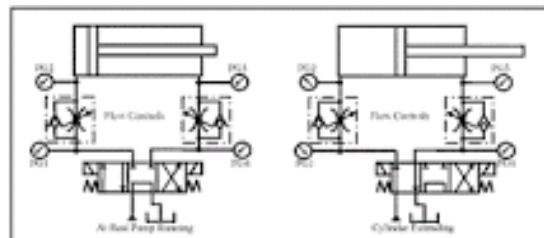
Fig. 28 Direct control of a single acting cylinder

DOUBLE ACTING CYLINDERS



- Double-acting cylinders have a port at each end and move the piston forward and back by alternating the port that receives the high-pressure air, necessary when a load must be moved in both directions such as opening and closing a gate.
- Air pressure is applied alternately to the opposite ends of the piston.

What is meter in and meter out circuit in hydraulics?



Meter-in flow control circuit represents the controlling of fluid flow just before fluid enters to the actuator with the help of flow control valve. ... We can see here the bypass check valve that will force the fluid to flow through the adjustable orifice before fluid enters to the actuator i.e. hydraulic cylinder here.

Comparison of hydraulic and pneumatic system

Hydraulic System –

- Fluid is used for working of system.
- Produce more power.
- Pressure produce from 100 bar up to 700 bar or even more.
- Working fluid is costly as compared to pneumatic system.

- Process is not Clean.
- System with high pressure is difficult to operate.
- Maintenance is not easier.
- Higher initial and operating cost.

Pneumatic system –

- Air or gas is used for working of system.
- Produce less power.
- Pressure limited upto 10 bar
- Working Fluid available in cheap rate
- This process is clean.
- Easy to operate.
- Maintenance is easier and quicker.
- Low initial and operating costs