GOVERNMENT POLYTECHNIC, NAYAGARH

STRENGTH OF MATERIALS

3 RD SEMESTER, MECHANICAL ENGG.

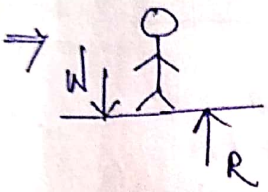
PREPARED BY- RAMYA RASHMI ROUT

* Strength of Materials [Crash Course]

We study about real solids (1) Effect of External force
(2) Effect of mechanical Energy
of bodies at rest / stationary.

→ External forces

- With physical contact (Surface force)
  → point Load, Concentrated Load
- Without physical contact (Body force)
  Ex - Gravity force
  Weight of body.

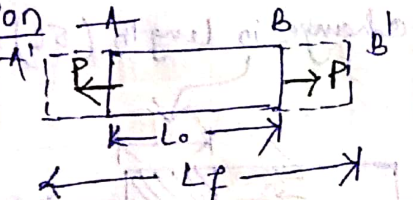
→ 

for rest condition

$$\boxed{\Sigma f_V = 0}$$

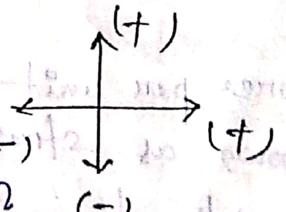
→ LOAD :- External force which is acting on a body.
force can change shape, size and state of motion.
change in shape & size → Deformation.
→ condition of static equilibrium is $\Sigma f_x = 0, \Sigma f_y = 0, \Sigma f_z = 0$
$\Sigma M_x = 0, \Sigma M_y = 0, \Sigma M_z = 0$

1. Elongation



force is moving away from the surface
force will stress the body.

here $\Sigma f_x = 0$, Body is at rest.
$P - P = 0$, so it is in static equilibrium

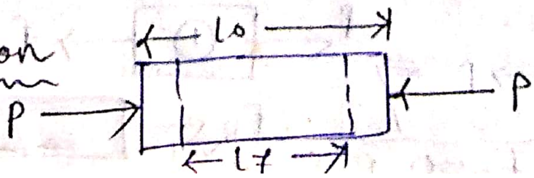


CW (+)
CCW (−)
(sign convention)

$L_f - L_o$ = Change in length
size is increased.

These forces are called tensile force (Effect is elongation)

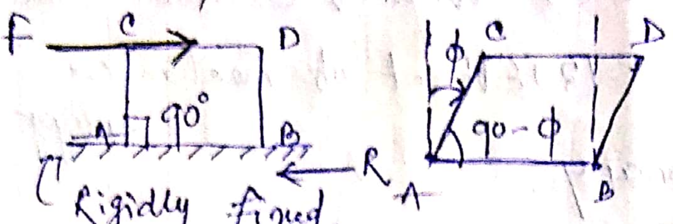
2. Contraction



(force is towards the surface).

These forces are called Compressive forces.

3. Shear / cutting / Tangential force



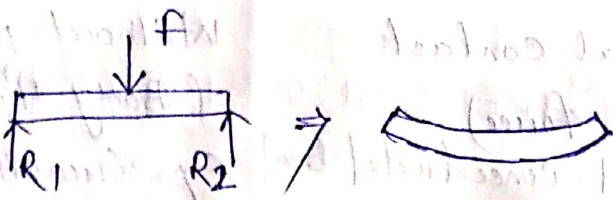
Rigidly fixed.

$\Sigma F_x = 0$

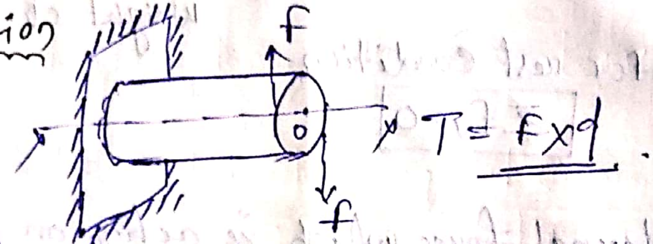
i.e $\boxed{R = V}$

The force which change the shape called shear force

4. Bending :-



5. Twisting / Torsion

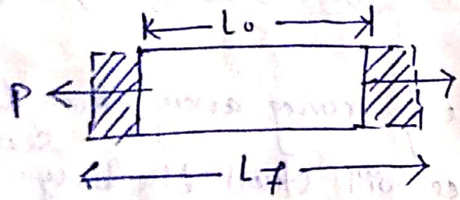


$T = F \times d$

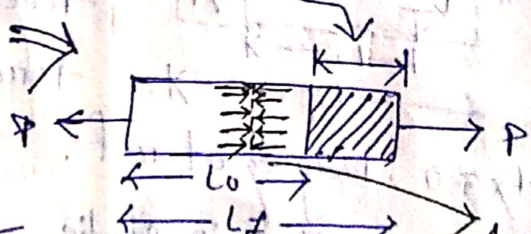
→ Strength of material is study of three 'S'

(i) strength (ii) stiffness (iii) stability

* Stress & strain



change in length $(\delta L) = L_f - L_o$



Internal resisting force.

→ Resisting force per unit area is known as stress

→ To find resisting force we use method of section.

in this method we virtually cut the member so that it will follow condition of static equilibrium.

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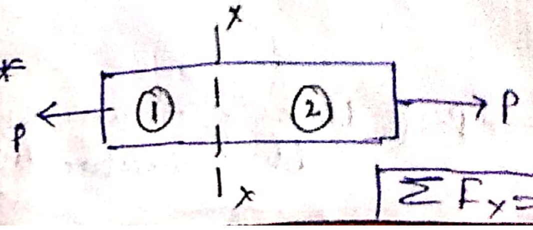
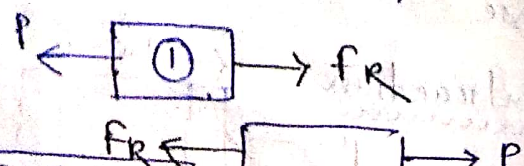
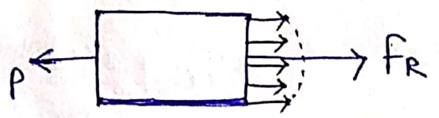




$\boxed{\Sigma F_x = 0, P - F_R = 0}$

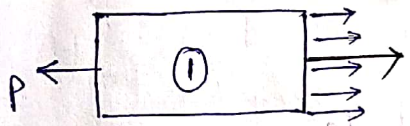
magnitude of resisting force is equal to the external force.



Average stress / Engineering stress :- Resisting force per unit Original area

$$\boxed{\sigma_{avg} = \frac{\text{Resisting force}}{\text{Original area}}}$$

→ Internal resisting force per unit area is called stress.



f_R = Resultant of resisting force.

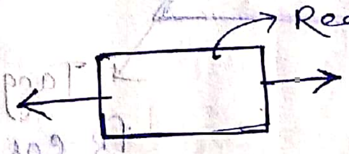
$\boxed{F_R = P}$ $\because \Sigma F_x = 0$ for static equilibrium

Resultant resisting force.

→ Engineering stress $= \dfrac{\text{Resisting force}}{\text{Original area}}$.

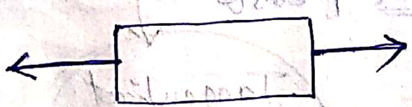
* Strain

Engineering strain



Real Solids (Deformable) / Resistant body.

This body will ~~transfer~~ transfer force will giving some noticiable deformation)



Real Solids (Deformable).

Resistant body [These bodies will transfer force will giving some noticiable deformation]

Strain (Engg. strain) → measure of deformation.

Strain is defined as deformation per unit original dimension

$$\text{strain} = \frac{\text{Deformation (final dimension} - \text{original dimension)}}{\text{Original dimension}}.$$

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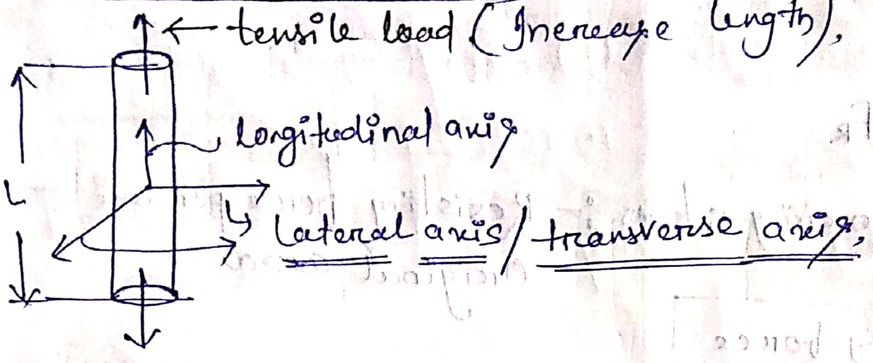
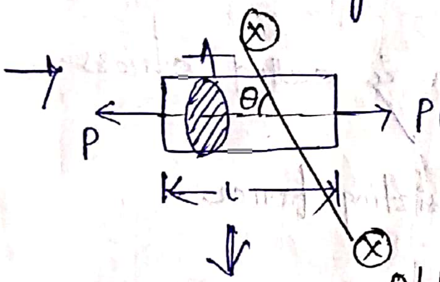
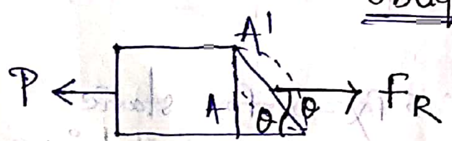
* Tensile stress & Strain



Axial loading



Oblique plane / Oblique section.

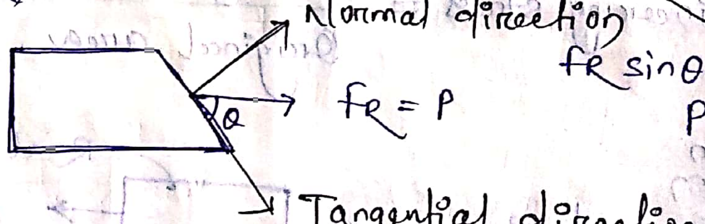


for static equilibrium

$\sum f_x = 0$, i.e $\boxed{P - f_R = 0}$

$\sin\theta = \frac{A}{A'}$

$A' = \frac{A}{\sin\theta}$



Normal Resisting force, $f_R \sin\theta$ or $P\sin\theta$

$f_R \cos\theta$ or $P\cos\theta$

Tangential Resisting force

Normal stress (σ_n)

$\sigma_n = \frac{\text{Normal resisting force}}{\text{Area carrying normal Resisting force } (A')}$

$= \frac{P\sin\theta}{\frac{A}{\sin\theta}} = \left(\frac{P}{A}\right)\sin^2\theta$

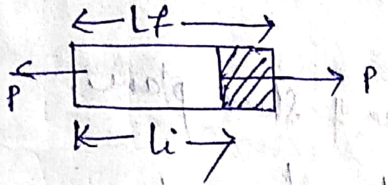
P = force in longitudional direction.

Normal stress on cross-sectional plane (A)

$\sigma = f_R = \frac{P}{A}$ = longitudional stress

f_R → Resisting force.

* tensile stress is one type of Longitudional stress / Normal stress on cross-sectional plane.



Tensile Stress

$\sigma = \frac{P}{A}$ (+) $\because$ Length is increased

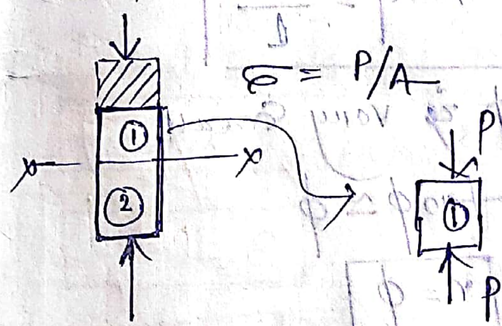
Tensile strain (Longitudional strain).

$$\varepsilon = \frac{\text{Deformation}}{\text{Original Dimension}}$$

$$= \frac{Lf - Li}{Li}$$

$$= \frac{\delta L}{L} \rightarrow \text{change in length}$$
$L \rightarrow$ Original length.

* Compressive stress and strain



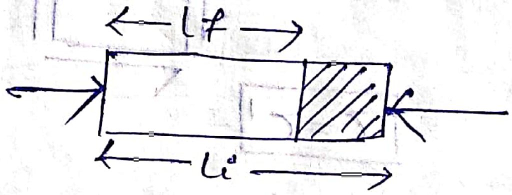
$\sigma = P/A$

$\sigma = \frac{P}{A}$ (Compressive)

$= \frac{P}{A}$ (−)

Strain (compressive) [Longitudional strain]

$$\varepsilon = \frac{\text{Deformation}}{\text{Original Dimension}}$$

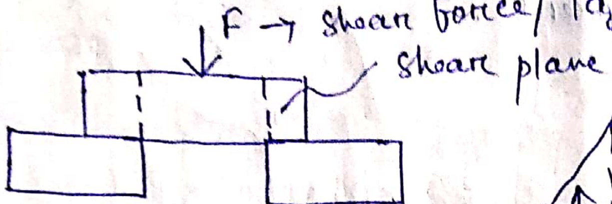


$= \frac{\delta L}{L}$ (−) ($Lf < Li$) ← In compression

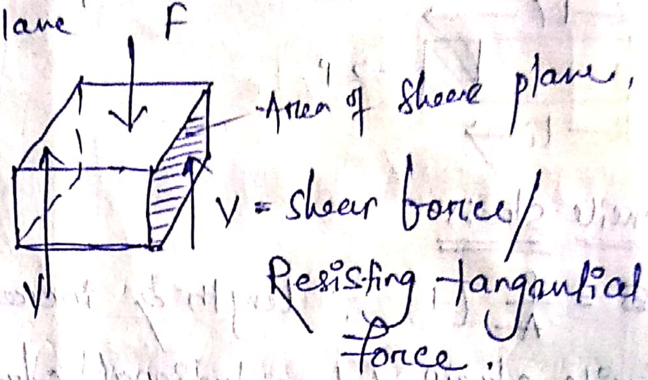
⇒ ↳ Negative

* Normal stress
↓
Longitudional stress
↙ ↘
Tensile   Compressive

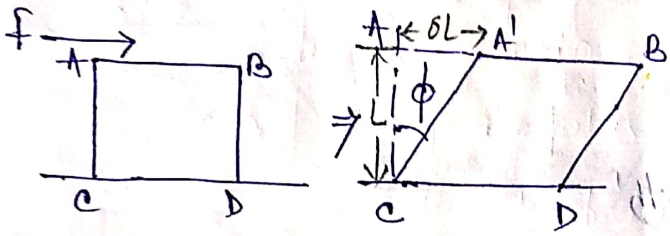
→ Shear stress & strain



(Shearing)



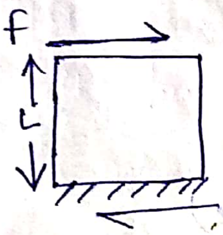
$$\tau = \frac{\text{Resisting shear force}}{\text{Area under shear}}$$



$\boxed{\tan\phi = \frac{\delta L}{L}}$ → slip

→ shear strain (γ)

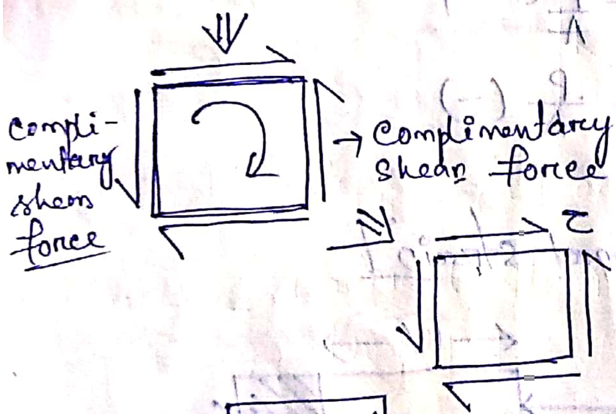
$$\boxed{\gamma = \tan\phi = \frac{\delta L}{L}}$$



* If ϕ is very small

then $\tan\phi \simeq \phi$

So $\boxed{\gamma = \phi}$



$$\boxed{\tau = \tau'}$$

material which resist deformation, → for solids

* **Hooke's Law** :→ (for Elastic materials)

1. material is homogenious & Isotropic
2. Load is applied gradually
3. There is no stress concentration
4. Temperature is constant during loading

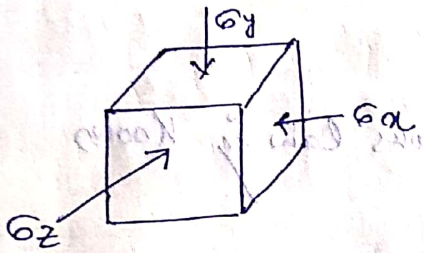
$\boxed{\text{Stress} \propto \text{Strain}}$

→ Normal stress (σ_n) $\propto$ Normal strain (ε_n)

→ Longitudional stress (σ) $\propto$ Longitudional strain (ε)

→ Shear stress (τ) $\propto$ shear strain

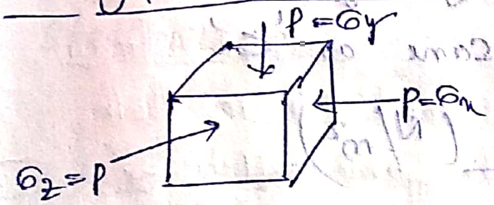
→ Bulk stress (σ_b) $\propto$ Bulk strain (ε_v) or Volumetric strain.



$$\text{Bulk Avg} = \frac{\sigma_x + \sigma_y + \sigma_z}{3}$$

$$\varepsilon_v = \frac{\delta V}{V}$$ δV → change in Volume, V → Original Volume

for hydrostatic condⁿ



$\sigma_b = -P$

↳ Bulk stress

* Bulk stress is a three dimensional stress

* Normal stress

$\boxed{\sigma_n = E \cdot \varepsilon_n}$ E = Young's modulus of Elasticity.

or

$\boxed{\sigma = E \cdot \varepsilon}$

$\boxed{\frac{P}{A} = E \times \frac{\delta l}{L}}$ → strain is dimensionless.

$\frac{N}{m^2}$ = Pa (Pascals)

$1 N/m^2 = 1$ Pascals

Dimension

$$\sigma = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

$$\varepsilon = \frac{L}{L} = M^0L^0T^0$$

→ strain is a dimensionless Quantity.

$10^6 N/m^2 = 1$ megapascal.

$10^9 N/m^2 = 1$ giga pascal

$\boxed{1 N/mm^2 = 1 \text{ mega pascal}}$

→ SI unit of $E = \frac{\sigma}{\varepsilon}$ N/m^2

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$\rightarrow \frac{P}{A} = E \cdot \frac{\delta l}{L}$

δl = deformation/change in length

$\boxed{\frac{PL}{AE} = \delta L}$ or $\boxed{\delta L = \frac{\sigma L}{E}}$

$\boxed{\sigma_n = E \cdot \varepsilon_n} \rightarrow$ Linear equation.

→* The material which follows hooks law is known as linear elastic material.

→ Shear Stress & Strain

$\boxed{\tau = G\gamma}$ G = modulus of Rigidity / modulus of shear

$\boxed{G = \frac{\tau}{\gamma}}$ SI unit of 'G' is same as 'τ' — Pa (N/m^2).

→ Bulk stress & Strain

$\boxed{\sigma_b = K \cdot \varepsilon_v}$ K = Bulk modulus

$\boxed{K = \frac{\sigma_b}{\varepsilon_v}}$ — unit is (N/m^2)

Poisons Ratio :— (μ or $\frac{1}{m}$)

$$\mu = -\frac{\text{Lateral strain } (\varepsilon_d)}{\text{Longitudional strain } (\varepsilon_l)}$$

Lateral strain = $\frac{\text{change in lateral dim}^n}{\text{Original Lateral dim}^n}$

→ -ve sign shows that lateral & longitudional stresses are opposit in Nature.

μ is const. for a material.

$$\mu = -\frac{\varepsilon_d}{\varepsilon_l} = -\frac{\left(\frac{b_f - b}{b}\right)}{\left(\frac{L_f - L}{L}\right)}$$

b = width

from two one should be (-ve) So (-ve)&(-ve) become (+ve)

(μ = +ve) always for Elastic material.

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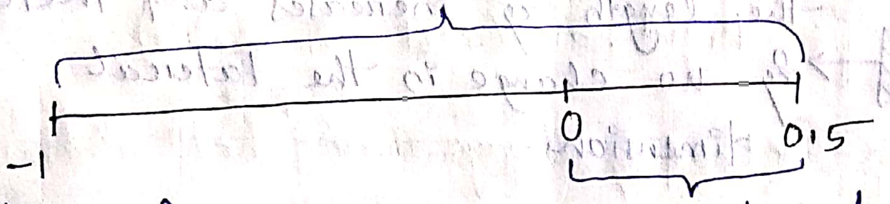
→ for Elastic material.

$$\mu = \left| \frac{\text{Lateral Strain}}{\text{Longitudional Strain}} \right|$$

* $\boxed{-1 \leq \mu \leq 0.5}$ Theoritical poissons ratio range



→ μ value for various materials.

Cork (0) cheese (0.45)

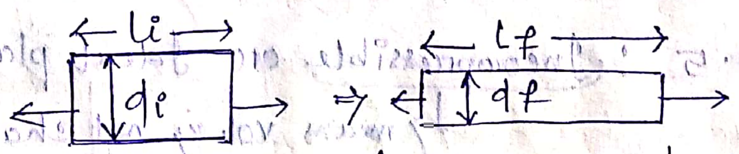
Concrete (0.2) rubber (0.5)

Steel & Copper (0.3), ~~allm~~ aluminium (0.35).

Rubber (0.5)

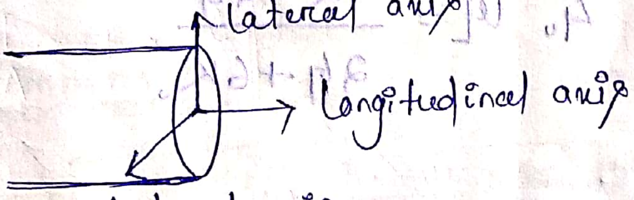
Lecture-4 :-

Poisons ratio :-



length increased and the lateral dimensions are decreased.

$\boxed{\begin{matrix} L_f > L_i \\ d_f < d_i \end{matrix}}$



$$\boxed{\mu = -\frac{\text{Lateral strain}}{\text{Longitudional strain}}}$$

$$\mu = -\frac{\varepsilon_d \; (+)}{\varepsilon_l \; (-)} \text{ (for compr.)}$$

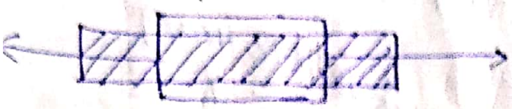
μ = +ve (always for elastic material)

$\boxed{-1 \leq \mu \leq 0.5}$ (generally)

$\boxed{0 \leq \mu \leq 0.5}$ (for Elastic material)

* for (a) $0 < \mu \leq 0.5$:- If we provide tensile force then the length increase and the lateral dimension decrease.

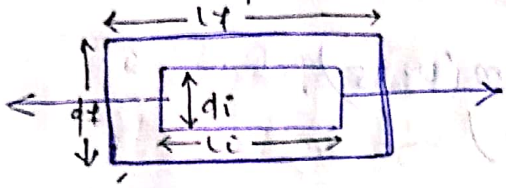
$\Downarrow$ +ve poisons ratio



* for $\mu = 0$:- If we provide tensile force then the length is increases and there is no change in the lateral dimension

* for (b) $-1 \leq \mu \leq 0$:- If we provide tensile force then the length is increases and also the lateral dimensions are increases. i.e lateral strain and longitudinal strain both are of the same nature.

$\Downarrow$ −ve poison's ratio



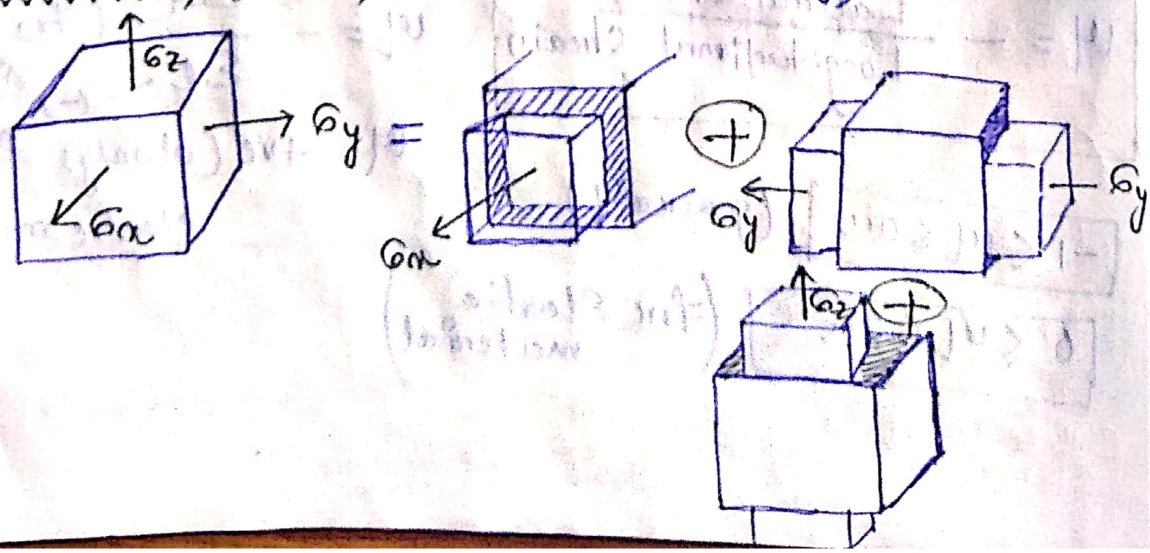
* $\mu = 0.5$: Incompressible or Ideal plastic material.

↳ means vol. is not changing

→ **Relationship between Elastic Constant:**—

1. $E = 2G(1+\mu)$
2. $E = 3K(1-2\mu)$
3. $E = \frac{9KG}{3K+G}$
4. $\mu = \frac{3K-2G}{2G+6K}$

→ **Generalized Hookes Law** :- (3D Loading)



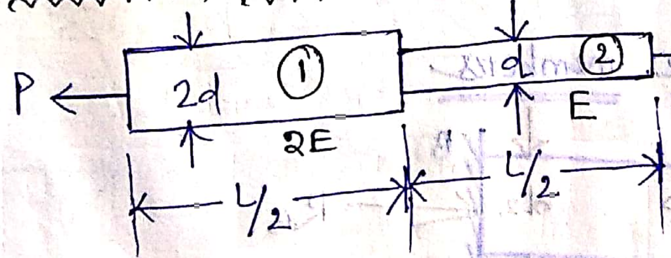
Effect of Stress	strain in x dir^n	strain in y dir^n	strain in z dir^n
σ_x	$\left(\frac{\sigma_x}{E}\right)$	$-\mu\left(\frac{\sigma_x}{E}\right)$	$-\mu\left(\frac{\sigma_x}{E}\right)$
σ_y	$-\mu\left(\frac{\sigma_y}{E}\right)$	$\frac{\sigma_y}{E}$	$-\mu\left(\frac{\sigma_y}{E}\right)$
σ_z	$-\mu\left(\frac{\sigma_z}{E}\right)$	$-\mu\left(\frac{\sigma_z}{E}\right)$	$\left(\frac{\sigma_z}{E}\right)$

$$\left.\begin{aligned} &* \text{ strain in } x \text{ dir}^n\ (\varepsilon_x) = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} \\ &* \text{ Strain in } y \text{ dir}^n\ (\varepsilon_y) = \frac{\sigma_y}{E} - \mu\frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} \\ &* \text{ Strain in } z \text{ dir}^n\ (\varepsilon_z) = \frac{\sigma_z}{E} - \mu\frac{\sigma_x}{E} + \mu\frac{\sigma_y}{E} \end{aligned}\right\} \text{Generalized Hooke's Law}$$

→ Volumetric strain $(\varepsilon_v) = \varepsilon_x + \varepsilon_y + \varepsilon_z$

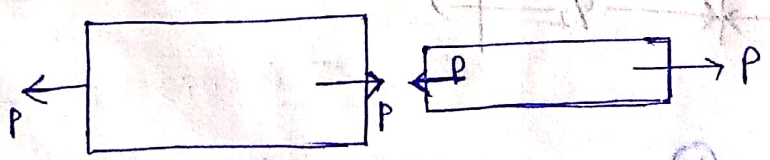
$$\boxed{\varepsilon_v = \frac{\sigma_x+\sigma_y+\sigma_z}{E}(1-2\mu)} \leftarrow \text{Volumetric strain.}$$

* Members in Series



[End to end Connection of members]

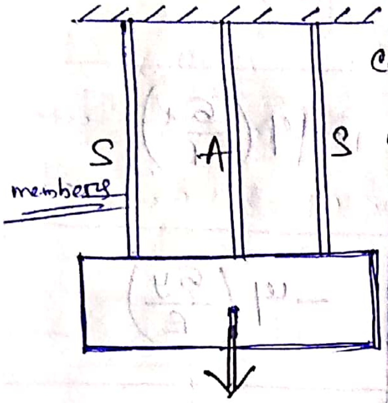
FBD



$$\boxed{(\delta l)_{n\ \text{total}} = \sum_{n=1}^{n} \delta l_n \hookrightarrow \left(\frac{PL}{AE}\right)_n}$$ ← for members in Series,

Members in parallel

S = steel
A = Allumunium



case ① → Rigid bar must remain horizontal
case ② → Rigid bar is tilting

← Rigid bar (Not deformable).

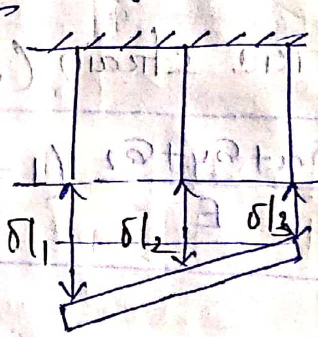
* Load is always shared betn members

→ for case ① i.e rigid bar remain horizontal in this case the members are equally deformed.

i.e $(\delta l_1) = (\delta l)_2 = (\delta l)_3$ ---

Deformation of member's must be same

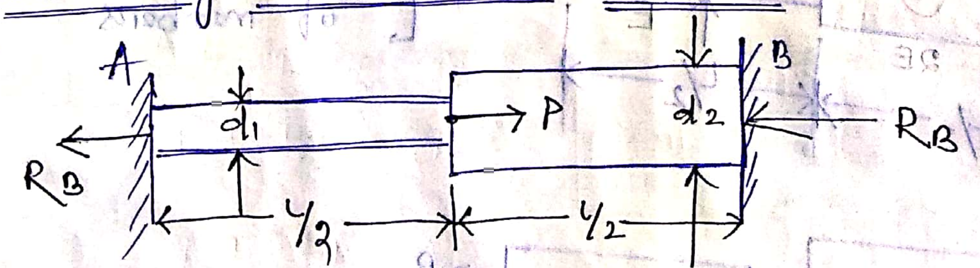
→ for case (2) i.e Rigid bar is tilting, the deformation of members is varying



δl varies linearly from member to member.

* Lecture - 5

Statically indeterminate members



$\Sigma F_x = 0$

$R_A + R_B = P$ —— ①

$\Sigma F_y = 0$

$\Sigma M_A = 0$

here in this question there are two unknowns but we have only one equation

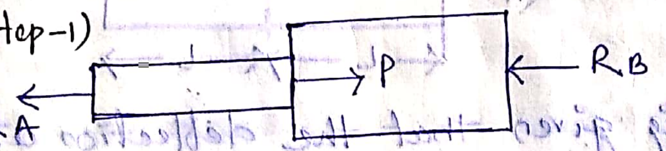
→ mathematically we know that for 'n' number of unknowns we need 'n' number of equations

→ So we can't solve this question with one equation

So this kind of problem is called statically indeterminate.

So here we require another one equation, i.e compartibility equation.

FBD (step-1)



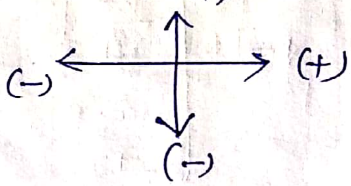
(step - 2)

find $\Sigma F = F_{net}$ (Don't count reactions, only count the loads).

now put the reactions opposit to "fnet".

$R_A + R_B = P$ —— ①

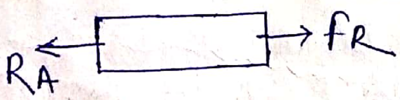
(step - 3) ↳ static equilibrium Eqn.



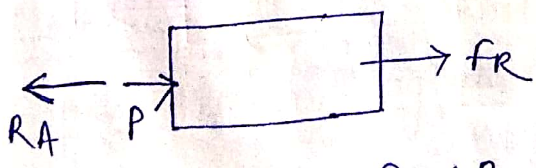
(step-4) put compartibility Eqn.

$$(\delta L)_{total} = \sum_{n=1}^{n=1} \delta L_n = 0$$

$$\left(\frac{PL}{AE}\right)$$



$\Sigma f_x = 0$, so $\boxed{f_R = R_A}$

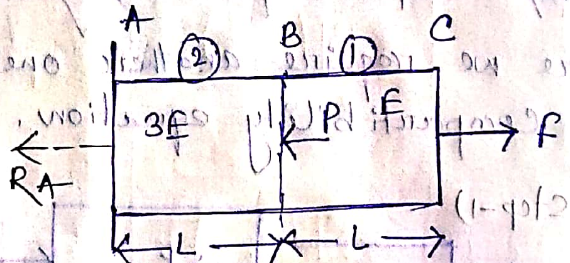


$\Sigma f_x = 0$, $f_R + P - R_A = 0$

$\rightarrow \boxed{f_R = R_A - P}$

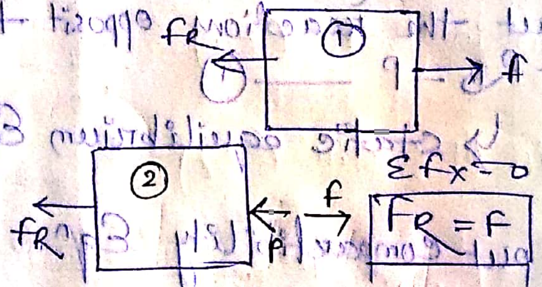
$$(\delta L)_{total} = \left(\frac{f_R \cdot L}{AE}\right)_1 + \left(\frac{f_R \cdot L}{AE}\right)_2 + \cdots \left(\frac{f_R \cdot L}{AE}\right)_n$$

Q. A horizontal bar with a constant crossection is subjected to loading as shown in the figure the youngs modulus for the section AB & BC are 3E and E respectively, the deflection at C to be zero. the ratio P:F is.



here in question it is given that the deflection at 'c' to be zero.

i.e $(\delta l)_{total} = zero$



$\Sigma f_x = 0$

$F - P - f_R = 0$

$\Rightarrow \boxed{F - P = f_R}$

Now $\left(\frac{f_R L}{AE}\right)_1 + \left(\frac{f_R L}{AE}\right)_2 + \ldots = 0$

$\left(\frac{F \cancel{L}}{\cancel{A}\cancel{E}}\right)_1 + \frac{(F-P)\cdot\cancel{L}}{\cancel{A}3\cancel{E}} = 0$

$\Rightarrow F + \frac{F-P}{3} = 0$

$\Rightarrow \frac{3F + F - P}{3} = 0$

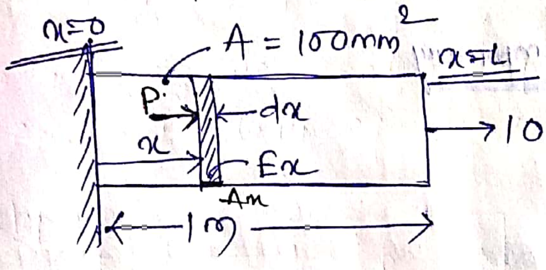
$4F - P = 0$

$P = 4F$

$\Rightarrow \boxed{\frac{P}{F} = 4}$ — Ans

Q. A horizontal bar fixed at one end, has a length of 1 meter, crossectional area is $A = 100\ mm^2$. The Young's modulus $E(x) = 100\, e^{-x}$ gpa (x is in meters). An axial tensile load is applied at free end i.e 10KN. The axial displacement at the free end in mm?

Ans



dl = change in length of element.

$$= \frac{P\, dx}{A_x \cdot E_x}$$

For change in length of bar

$$\boxed{\delta l = \int_{x=0}^{x=L} \frac{P\, dx}{A_x\, E_x}}$$

$$\delta l = \int_0^1 \frac{10\times10^3 N \times dx}{100\times10^{-6} m^2 \times 100 \cdot e^{-x} \times 10^9\ Pa}$$

$$= \int_0^1 \frac{\cancel{10} \times e^{x} dx \times \cancel{10^3}}{\underset{10}{\cancel{100}} \times \cancel{10^3} \times 100}$$

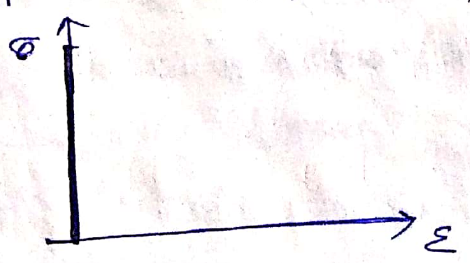
$$= 1.718\times10^{-3} m$$

So $\delta L = 1.718$ mm (Ans).

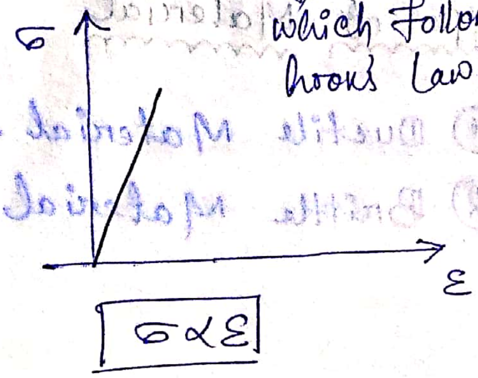
Lecture - 6

Stress Strain Diagram

*Rigid material (hypothetical).

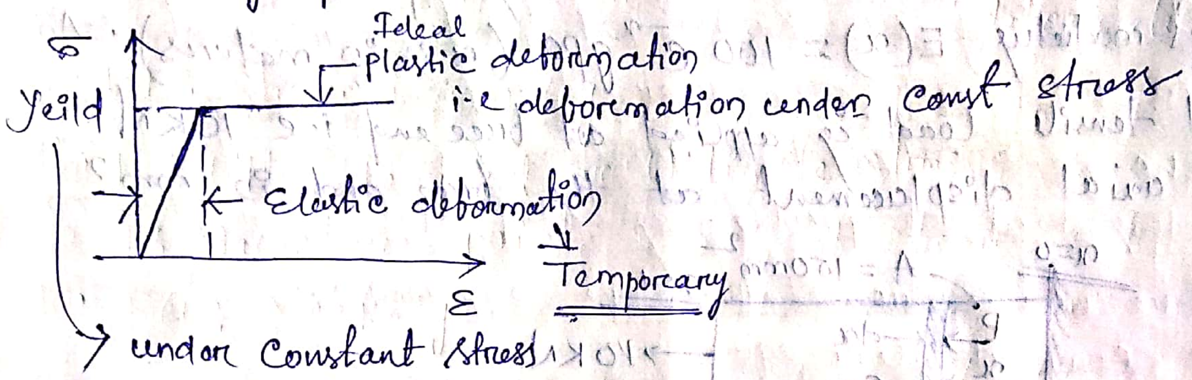


* linear Elastic material. which follows hooks Law.

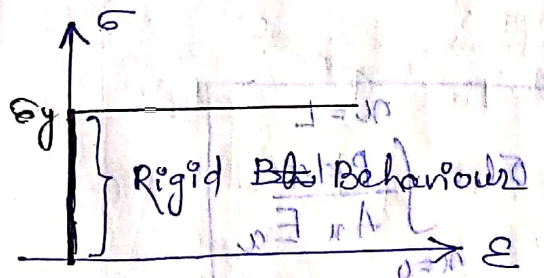


$\boxed{\sigma \propto \varepsilon}$

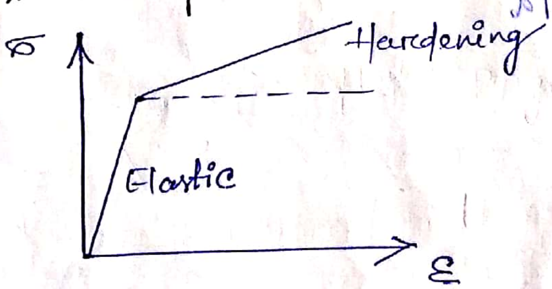
* Ideal plastic or Elastic perfectly plastic



* **Rigid perfect plastic**



* Elastoplastic material with strain hardening



→ **Percentage Elongation**

$$\% \text{ elongation} = \frac{L_f - L_o}{L_o} \times 100\%$$

$$\% \text{ change in Area} = \frac{A_f - A_o}{A_o} \times 100\%$$

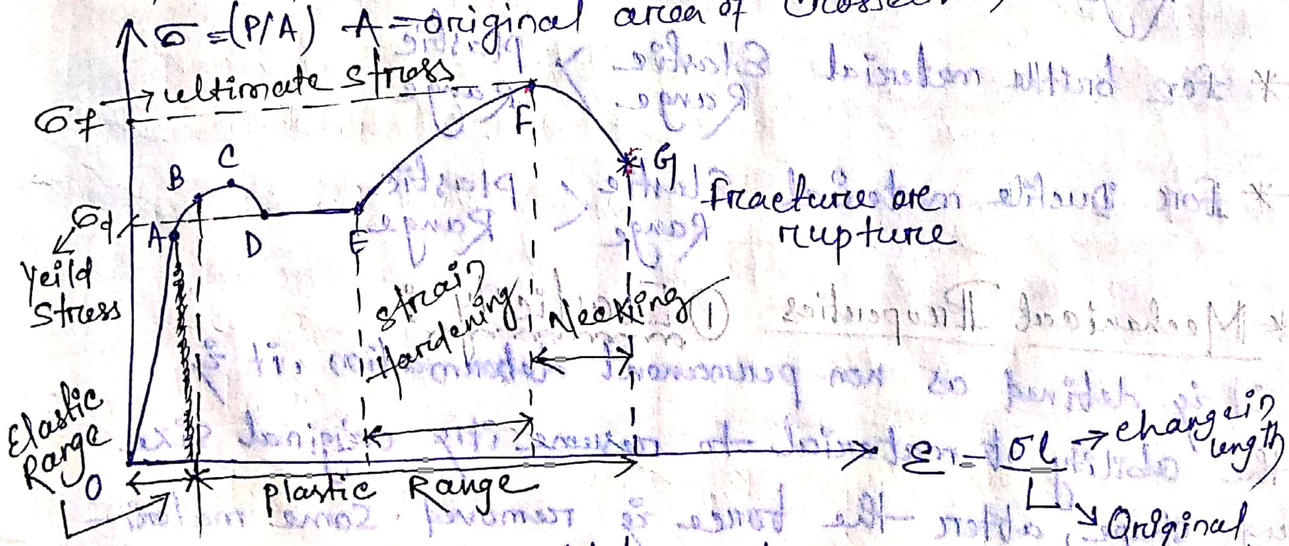
Types of Material

1. Ductile Material.
2. Brittle Material,

Ductile Material	Brittle Material:
→ Solid materials can undergo substantial plastic deformation prior to fracture are called ductile materials	→ Solid materials that exhibit negligible plastic deformation are called brittle materials
→ % Elongation > 5% Also 5% < % Elongation < 15% Are known as intermediate ductile material % Elongation > 15% Are known as completely ductile material.	→ % Elongation < 5%
→ Example of ductile material: mild steel aluminum Copper most plastics.	→ Examples of brittle material: cast Iron, ceramics such as glass, cement, concrete, stone ice.

* Stress-strain diagram for Ductile material.

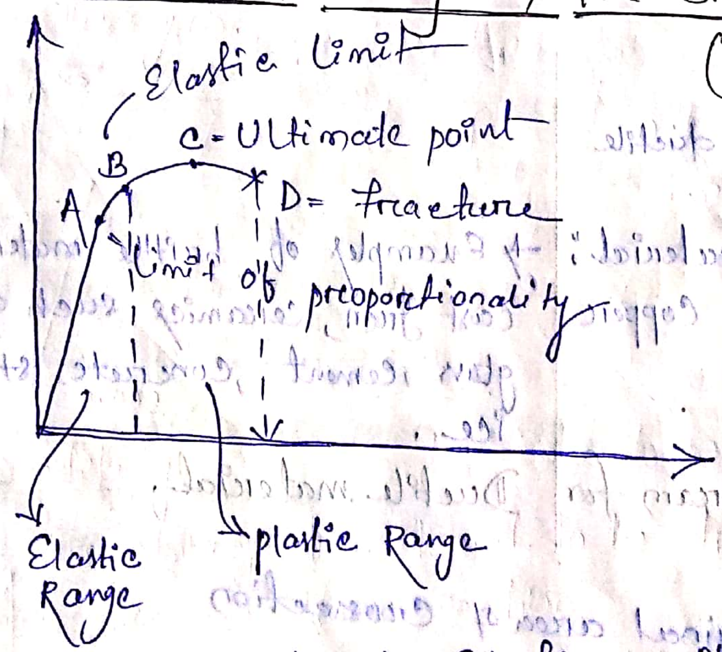
Mild Steel



1. OA : Linear behaviour/deformation (Initial Linearity)
2. A : Limit of proportionality.
3. B : Elastic limit.
4. C : Upper yeild point

5. D: Lower yeild point
6. DE: Deformation under const. stress / Ideal plastic deformation
7. EF: strain hardening
8. F: Ultimate point or Ultimate stress
9. G: Fracture point
10. FG: Necking

* **Stress-strain Diagram for Brittle Material.** (CAST IRON)



* for brittle material Elastic Range > Plastic Range

* For Ductile material Elastic Range < Plastic Range

* **Mechanical Properties** ① **Elasticity:**
It is defined as non-permanent deformation. it is the ability of material to resume its original size and shape, after the force is removed. Some materials are more elastic than others, but there is no material which is perfectly elastic throughout the entire range of stress i.e up to rupture.
Elasticity is mostly measured in terms of stress or elastic limit.

Elastic limit is the maximum stress that a material is capable to withstand, without permanent deformation. The more elastic material will have higher value of elastic limit.

2. Plasticity :-

It is the ability of material to be permanently deformed without fracture even after the removal of force. All materials are plastic to some extent. Plasticity is other extreme opposit of elasticity.

The plasticity of a material is it's ability to undergo some degree of permanent deformation without repture. plastic deformation will take place only after the elastic range has been exceed.

A general expression of plastic action would involve the time rate of strain since the plastic state.

3. Strength (maximum stress which a material can withstand without failure)

Strength may be defined as the capacity of metal to withstand load. It is expressed as force per unit area of cross-section. The material has to withstand different types of load such as tensile, bending, torsion, compressive and shear load. Hence, strength is known as tensile bending, torsion, compressive and shear strength.

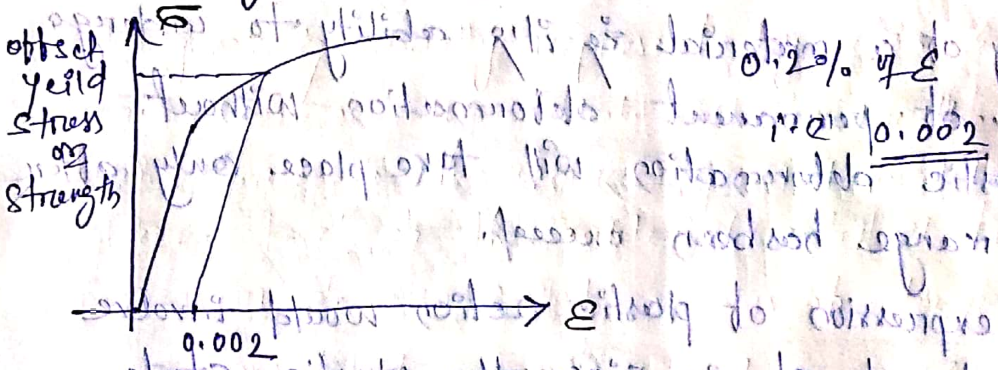
Tensile, torsion, bending and shear strength are greater in ductile materials while the compression strength is lower. for brittle material like, stone, brick and cast Iron, the ultimate compression strength will be much higher than the ~~tsl~~ ultimate tensile strength.

→ Ductile materials are weak in shear.

→ Brittle materials are weak in tension.

→ S_{ut} = ultimate strength (Brittle material).
S_{yt} = Yeild strength (Ductile materials).

* for some materials we can't notice the upper yeild point and the lower yeild point Ex - Allumunium, for these cases we take 0.2% of strain and draw a parallel line with the linear curve of the diagram. the point where the line cut the curve is called offset yeild stress or strength.

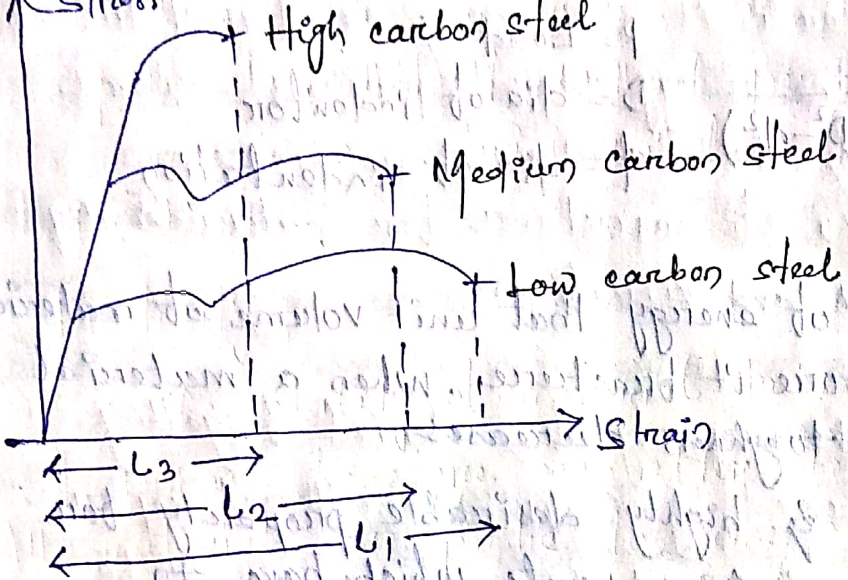


4) **Ductility**

It is the ability of material to be drawn into small section from a large section. It is also expressed as plastic deformation at breaking point. It's value is expressed as elongation i.e. percentage elongation is most widely used to measure ductility and its units are same as strain.

Any treatment which increases strength, decreases the ductility. The strength and ductility of material are appreciably affected by temperature. As temperature increases, the strength and elastic limit decreases and ductility increases.

Ex:- copper, alumunium, nickel, and lead are ductile metals.



Ductility is degree of plastic deformation. more plastic deformation means more ductility
→ Ductility is ability of material to undergo large reduction in crossectional area under action of tensile force.
→ Ductility decreases as the strength increases

5. Brittleness
The property fracturing a material without warning are appreciable deformation is called brittleness.
Brittleness is opposit to ductility
→ Most of brittle materials have high compressive strength, Usually, tensile strength of brittle materials is only a fraction of the compressive strength
→ Cast Iron, Concrete and glass are brittle materials
• Cast Iron shows a little deformation before rupture
• Materials having less than 5% elongation are considered as brittle.

6. Hardness
Hardness of a material is defined as the resistance of a material to ~~sea~~ scratch, wear or penetration of it's surface by harder bodies. Hardness is mostly measured by determining the resistance to penetration (indentation) by different methods as Brinell, Rockwell and Vickers hardness tests.

$$\text{BHN or HB} = \frac{2P}{\pi D\left(D-\sqrt{D^2-d^2}\right)}$$

P = Load
D = dia of indentor
d = dia of indentation.

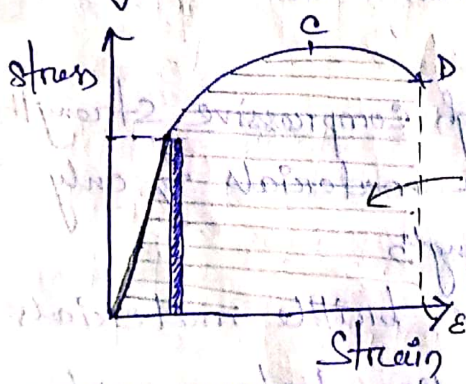
7. Toughness

It is the amount of energy that unit volume of material can absorb before it fracture. when a material is heated it's toughness decreases.

→ Thus toughness is highly desirable property for structural and machine parts which have to withstand shock and vibration.

→ Toughness is usually represent by area under stress strain curve. The total area under the stress-strain curve is the work expanded in deforming one cubic meter of the material untill it fracture. This work is also called modulus of toughness.

Brass, mild steel, manganese and wrought iron are tough materials.



Modulus of Toughness i.e Area under stress strain diagram.

[stress-strain diagram of a ductile material]

$dA = \sigma \cdot d\varepsilon$

$$\boxed{A = \int_0^{\varepsilon_f} \sigma \, d\varepsilon}$$

Unit $\frac{J}{m^3} = \frac{N\text{-}m}{m^3} \cdot \frac{N}{m^2}$ modulus of Toughness

(energy per unit volume upto fracture. i.e Toughness

7. Toughness :-

It is the amount of energy that material can absorb, before it fracture.
When a material is heated, it's toughness decreases.

→ The total area under stress-strain curve is the work expanded in deforming one cubic meter of the material until it fractures. This work is also called modulus of toughness.

→ Toughness is usually represented by area under a load v/s Deformation curve.
Brass, mild steel, manganese and wrought iron are tough materials.

8. Stiffness

→ Stiffness is the property of material which enables material to resist deformation.
The material which suffers less deformation under load, has high degree of stiffness. The greater the stress required to produce a given strain, the stiffer is the material.

→ The materials which follow hook's law, their stiffness is measured by young's modulus of elasticity.
The materials which do not follow hooke's law, their stiffness varies with stress.

→ The term flexibility is opposit to stiffness. The overall stiffness or flexibility is the function of dimensions, shape and characteristics of the material.

9. Resilience :-

It is measured by the amount of energy absorbed by material within elastic limit. This property is important in materials used for springs.

→ The maximum energy which can be stored in a body upto elastic limit is called proof resilience.

proof resilience per unit volume is called modulus of resilience. Thus the energy stored per unit volume at elastic limit is the modulus of resilience.

→ Resilience is also of importance for materials requi[red] to bear shocks and vibrations.

10. Malleability

→ Malleability is the property of material of getting permanently deformed by compression without rupture

→ Malleability requires that the material should be plastic but not so much dependent on strength

→ The common metals which are malleable:
Gold, Silver, Alumunium, copper, Tin, platinum, Lead Zinc, Iron, and Nickel.

11. Fatigue The failure of metal under alternating stress is known as fatigue.

12. Creep

The slow and progressive deformation of a material with time at constant stress is called creep.

→ After creep sets in, it continues until sufficient strain has occurred so that a necking down and a reduction of cross-sectional area occurs. After this and until rupture, the rate of deformation increases because there is less area to support the load.

→ The phenomenon of creep is observable in metals, ionic and covalent crystals, and amorphous metals such as glass and polymers. Metals generally, exhibit creep at high temperature, whereas plastic rubbers and similar amorphous materials are very temperature-sensitive to creep.

creep is extremely structure sensitive and is much more affected by grain size, micro structure and previous strain history, for instance, cold work and many other factors.

Lecture-7

* True stress and strain :-

Engineering stress and strain based on original dimension.
True stress and strain based on instantaneous dimension.

→ Relation between true stress & Engg. stress.

at $t = \Delta t$ (Pi) Load $= P_i$

$A_i = A_f$

L_f

$$\boxed{\sigma_{true} = \frac{P}{A_f}}$$

for Ideal plastic deformation

$\delta V = 0,\ \epsilon_v = 0$

$K = \frac{\sigma_b}{\epsilon_v} = \infty$

$E = 3K(1-2\nu)$

$\Rightarrow \boxed{\nu = 0.5}$ for ideal plastic deformation

$A_0 L_0 = A_f L_f = A_i L$

$$\sigma_{true} = \frac{P}{A_f} = \frac{P}{A_0} \times \frac{A_0}{A_f} = \sigma \times \frac{L_f}{L_0} \quad \{ L_f - L_0 = \delta L \}$$

$$\sigma_{true} = \sigma \times \frac{L_0 + \delta L}{L_0} = \sigma \times \left(1 + \frac{\delta L}{L_0}\right)$$

$$\boxed{\sigma_{true} = \sigma(1+\varepsilon)}$$ ← This ~~the~~ is the relationship betn true stress and Engg. stress.

* True strain

$$\boxed{\varepsilon_{true} = \ln\left(\frac{L_f}{L_0}\right)}$$

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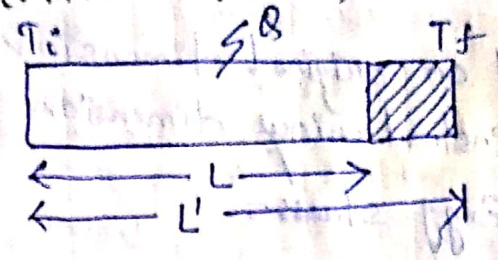
Rel betn true strain & engg. strain :

$$\varepsilon_{true} = \ln\left(\frac{L_0 + \delta L}{L_0}\right)$$

$$\boxed{\varepsilon_{true} = \ln(1+\varepsilon)}$$

→ Thermal stress & strain :—



$\Delta T = T_f - T_i$

[Space No restriction]
↓
Thermal expansion without the restriction is known as free expansion.

$\delta_{th} = L' - L$

Thermal Strain (ε_{th})

$$\varepsilon_{th} = \frac{\delta L_{th}}{L}$$

$\Rightarrow \varepsilon_{th} \propto \Delta T$

$\varepsilon_{th} = \alpha \Delta T$ —— (1)

α = Co-efficient of thermal expansion / Linear Expansion.

α = per deg °c or per Kelvin.

If $\Delta T = 1$ in eqn (1)

$\varepsilon_{th} = \alpha$

* During free expansion the stress are thermal stress = 0

$\varepsilon_{th} = \alpha \Delta T$

free expansion of bar is

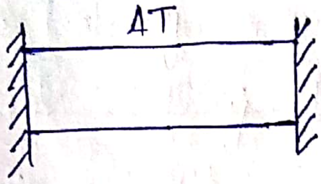
$$\boxed{\delta l_{th} = \alpha \Delta T L}$$

Thermal Stress (σ_{th})

Conditions

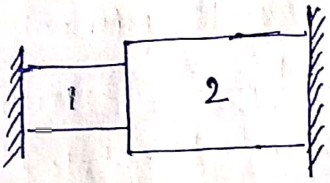
1. Temperature is being changed.
2. The expansion is restricted, either fully or partially.

1. Fully restricted



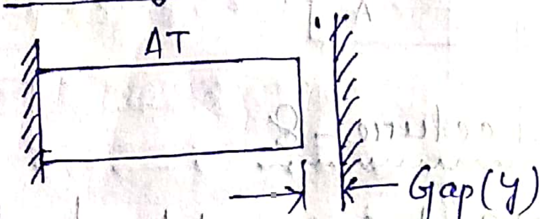
$\delta L_{total} = 0$

$\alpha \Delta T L - \left(\frac{PL}{AE}\right) = 0$



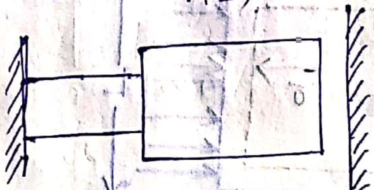
$$\sum_{n=1}^{n=n} (\alpha \Delta T L) - \sum_{n=1}^{n=n} \left(\frac{PL}{AE}\right) = 0$$

2. partially Restricted



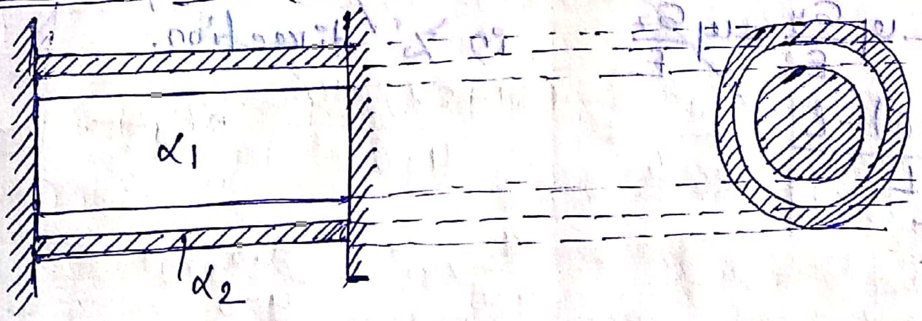
$\delta L_{total} = Gap$

$\alpha \Delta T L - \left(\frac{PL}{AE}\right) = gap$

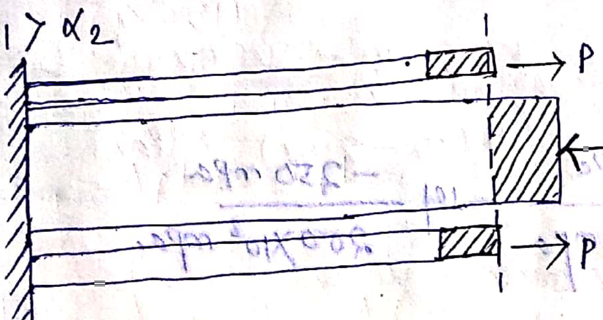


$$\sum_{n=1}^{n=n} (\alpha \Delta T L) - \sum_{n=1}^{n=n} \left(\frac{PL}{AE}\right) = Gap$$

3. Circular bar inside a tube.



$\alpha_1 > \alpha_2$



for Rod $(\alpha \Delta T L)_1 - \left(\frac{PL}{AE}\right)$ (final length)

Tube $(\alpha \Delta T L)_2 + \left(\frac{PL}{AE}\right)$ (final length)

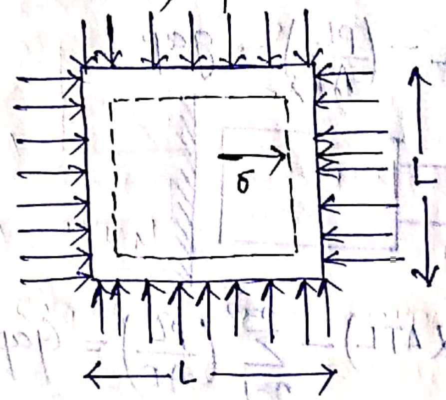
$$(\alpha \Delta T L)_1 - \left(\frac{PL}{AE}\right)_1 = (\alpha \Delta T L)_2 + \left(\frac{PL}{AE}\right)_2$$

Remember ~~$\alpha_1 = \alpha_2$~~, $\alpha_1 > \alpha_2$

$$\boxed{\sigma_{th} = \frac{P}{A}}$$

Lecture - 8

Q3. A square plate of dimension $L \times L$ is subjected to a uniform pressure load $P = 250$ mpa



$P = 250$ mpa
$E = 200$ GPa
$L = 2$m
$\delta = 0.001$m

Here put generalized hook's law.

$$\varepsilon_x = \frac{\sigma_x}{E} - \mu\frac{\sigma_y}{E} - \cancel{\mu\frac{\sigma_z}{E}}$$

$\to 0$ ∵ There is no load/stress in 'z' direction.

$$\varepsilon_x = \frac{(L - 2\delta) - L}{L}$$

$$= \frac{-2\delta}{L}$$

$$= -\frac{2 \times 0.001}{2}$$

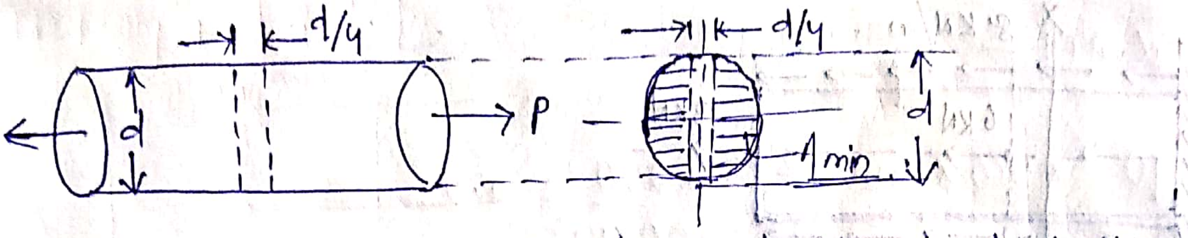
Now

$$\frac{-2\times 0.001}{2} = \frac{-250\text{ mpa}}{200\times 10^3\text{ mpa}} - \mu\frac{-250\text{ mpa}}{200\times 10^3\text{ mpa}}$$

$$\Rightarrow \frac{-2\times 0.001}{2} = \frac{1}{200\times 10^3}\left[-250 + \mu \times 250\right]$$

$$\Rightarrow \mu = 0.2 \text{ (Ans)}$$

Q. A solid bar of circular cross section has a hole of ~~dima~~ diameter D/4 drilled laterally through the centre of the bar as shown in the figure



i) What is the relationship for allowable load that can the bar can carry in tension.

$$A_{min} = \frac{\pi}{4}d^2 - \frac{d \times d}{4}$$

At the minimum area the stress is maximum.

$$= \frac{\pi d^2}{4} - \frac{d^2}{4}$$

$$= d^2\left(\frac{\pi - 1}{4}\right)$$

$\rightarrow \sigma_{max} \leq \sigma_{allow}$

for Limiting condⁿ $\sigma_{max} = \frac{P_{max}}{A_{min}} = \sigma_{allow}$

$$P_{max} = \left(\frac{\pi-1}{4}\right) d^2 \sigma_{allow}$$

$$= 0.535398\, d^2 \times \sigma_{allow}$$

$$\simeq 0.54\, d^2\, \sigma_{allow} \quad \text{Ans}$$

(ii) If the bar is made of brass, $d = 40mm$, $\sigma_{allow} = 80 mpa$

What is P_{allow} =

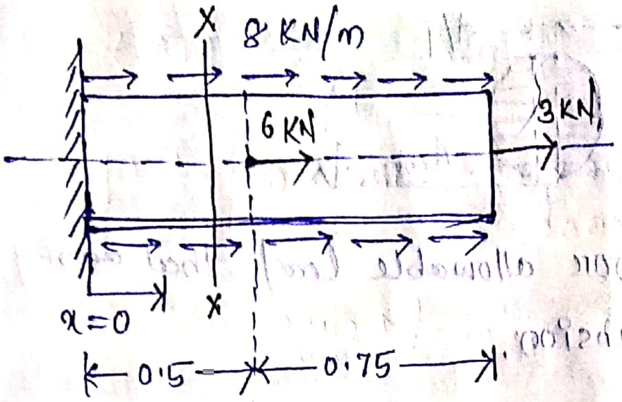
$$P_{max} = P_{allow} = 0.54 \times d^2 \times \sigma_{allow}$$

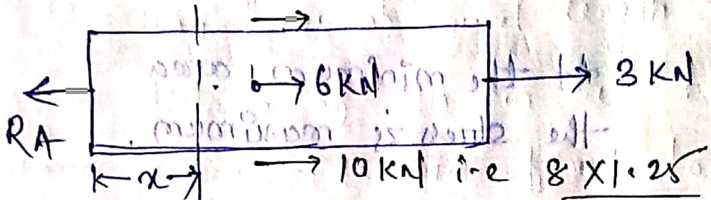
$$= 0.54 \times 40^2 mm \times 80\ N/mm^2$$

$$= 69120\ N$$

$$= 69.12\ KN \simeq 70\ KN$$

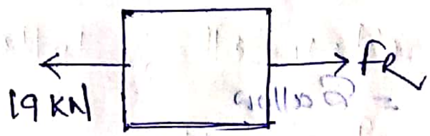
Q. A bar has cross-sectional area of $400 \times 10^{-6} m^2$. If it is subjected to uniform axial distributed load along its length and two concentrated load as shown in the figure. The average normal stress in the bar as function of x, from $x = 0$ to $x \leq 0.5m$.





RA = 19 KN

(Reaction from support.



$f_R + 8x - 19 = 0$

$\rightarrow f_R = 19 - 8x$ KN.

Stress: $\sigma_x = \dfrac{f_R \times 10^3}{400 \times 10^{-6}} = \dfrac{(19 - 8x) \times 10^9}{400}$ N/m²

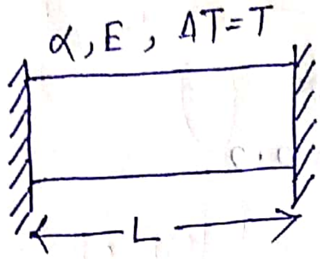
$= \dfrac{19}{400} \times 10^9 - \dfrac{8x}{400} \times 10^9$

$= 47.5 \times 10^6 - 20x \times 10^6$

$= (47.5 - 20x)$ MPa.

Lecture-9 :-

An elastic bar of length l uniform crossectional area A, Co-efficient of thermal expansion α, Young's modulus 'E'. is fixed at two ends, If the length of the bar is doubled the axial stress would be.



$\delta l = 0$ (∵ both side fixed).

$$\alpha \Delta T L - \frac{PL}{AE} = 0$$

$$\alpha \Delta T \cancel{L} = \frac{P\cancel{L}}{AE} \rightarrow \sigma$$

$$\boxed{\sigma = \alpha \Delta T E}$$ (σ is independent of L)

σ will remain unchanged.

Q. 200 mm long stress free rod at room temperature is held between two immovable rigid walls, temperature of the rod is uniformly raised by 250 If the Youngs modulus is 200 GPa & coefficient of thermal expansion 1×10^{-5}. The magnitude of longitudinal stress develop is ?

Ans

$$\sigma = \alpha \Delta T E$$

$$= 1 \times 10^{-5} \times 250 \times 200 \times 10^{3}$$

$$\boxed{\sigma_{th} = 500 \text{ MPa}}$$

Q. A circular metalic rod of 250 mm is placed betn two rigid immovable walls. In the figure the rod is in perfect contact with the wall in the left side & there is a gap of 0.2 mm betn the rod and the wall on the right side. If the temperature of the rod is increased by 200°C the axial stress developed in the rod is ?

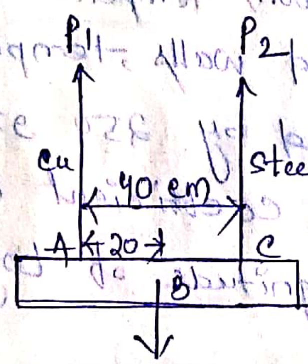
$\alpha \Delta T L - \frac{PL}{AE} = 0.2$

$10^{-5} \times 200 \times 250 - \frac{\sigma \times 250}{200 \times 10^3} = 0.2$

$\Rightarrow \sigma = 240\ N/mm^2$ or (mpa)

Q7. Two bars of equal length are suspended vertically at a distance of 40 cm as shown in the figure below. There upper ends are fixed to the ceiling while their lower end support a rigid horizontal bar which carries a central load of midway between the wires. details of the two wires are given. What is the ratio of elongation.

Ans



Cu Area = 4 cm^2

Steel Area = 2 cm^2

$P_1 + P_2 = W$

$\Sigma M_A = 0 \quad -P_2 \times 0.4 + W \times 0.2 = 0$

$P_2 = \frac{W \times 0.2}{0.4} = \frac{W}{2}$

$P_1 = W/2$

$\rightarrow \Delta_{Cu} = \frac{P_1 L_1}{A_1 E_1} = \frac{500 \times 4000\ cm}{4 \times 1 \times 10^6}$

$\Delta steel = \frac{P_2 L_2}{A_2 E_2} = \frac{500 \times 400\ cm}{2 \times 2 \times 10^6}$

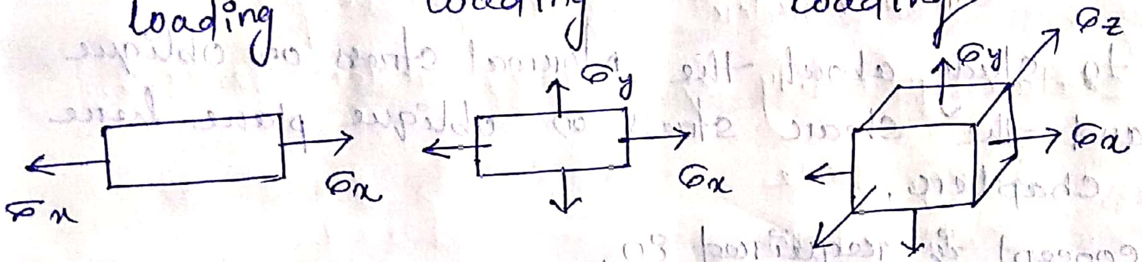
$\frac{A_c}{A_s} = 1$ so the bar is going to remain in the horizontal position.

Q.

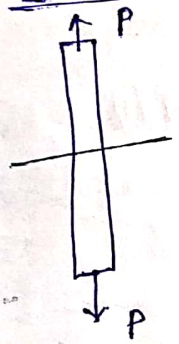
Principal Stress & Strain / complex stress or strain

→ Axial Loading

- Uniaxial Loading
- Biaxial Loading
- Tri-axial Loading



1. Uniaxial Loading



Longitudional stress

$\sigma = P/A$

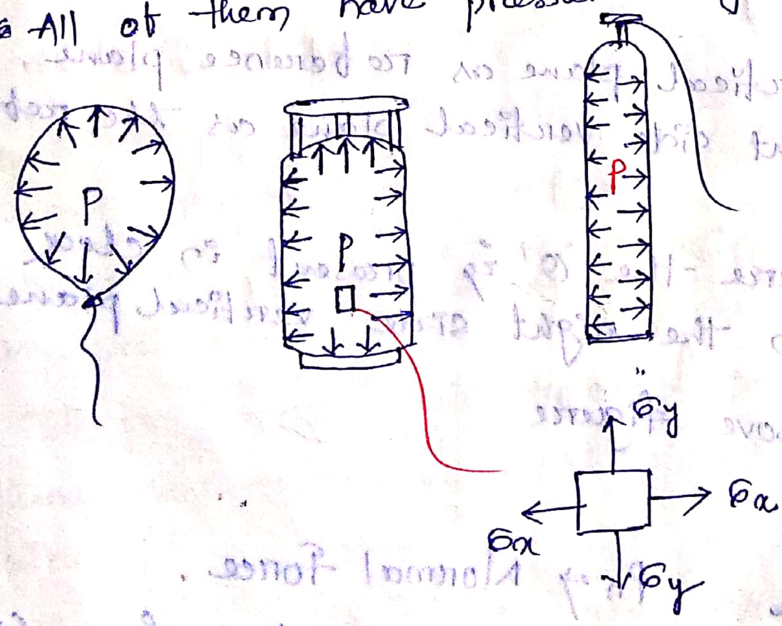
Longitudinal strain

$\varepsilon = \frac{\delta L}{L}$

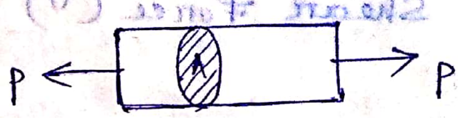
2. Biaxial Loading

Ex- Inflated ballon, Gas cylinder, cylinders for welding

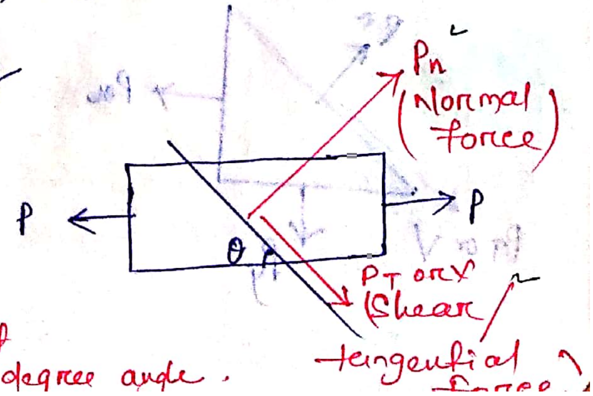
All of them have pressurised gas.



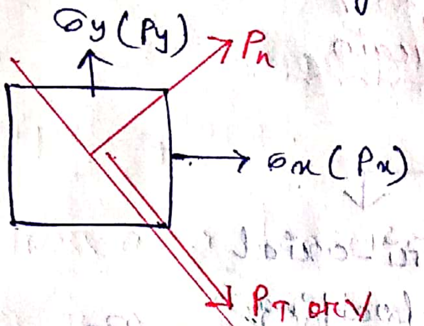
For uniaxial loading



⇒ If we cut a line at θ degree angle.



for bi-axial loading also.

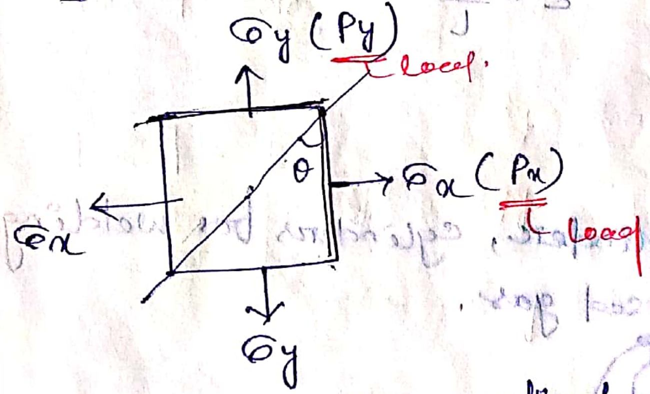


We have to study about the Normal stress on oblique plane and the shear stress on oblique plane here in this chapter.

→ This concept is required in.

Thin cylindrical pressure vessel
Theories of failure.
Combined bending & torsion.

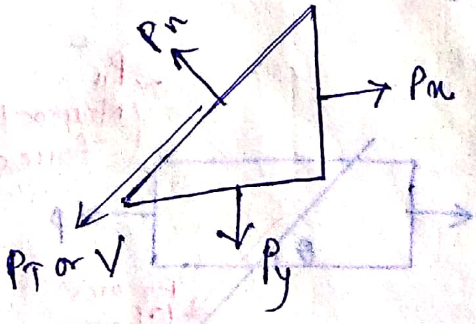
* principal stress and strain



→ Always take the vertical plane as reference plane. mainly take the right side vertical plane as the reference plane.

→ In the above figure the 'θ' is present in clock wise direction from the right side vertical plane.

→ If we cut the above figure



P_n → Normal force.

P_T or V → Tangential force (T or Shear force (V)

Let Area = 'A' of oblique plane.

$\sigma_n = \frac{P_n}{A}$, $\tau = \frac{V}{A}$ → In the oblique plane present in θ degree angle.

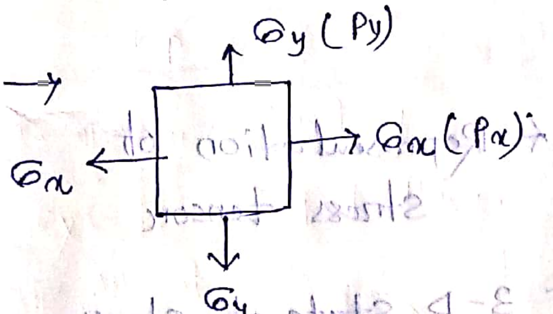
↓ Normal Stress ↓ Shear Stress.

→ At different value of θ we will get different value of σ_n and τ. At different oblique plane area (A) of plane will change.

→ θ = location of oblique plane w.r.t reference plane.

principal plane :- There will be an oblique plane at which shear stress is zero (τ=0). Oblique plane at which τ=0, such plane is known as principal plane.

→



• for biaxial loading there are two mutually perpendicular principal plane.

→ Location of principal plane is written as θ_p w.r.t reference plane.

→ Normal stress acting on principal plane is known as principal stress.

→ There are two type of principal stress. There are two principal plane i.e mutually perpendicular to each other.

→ The plane which carries greatest value of σ_n is called major principal plane.

→ The plane which carries lowest value of σ_n is called minor principal plane.

→ Major principal stress present in major principal plane and minor principal stress present in minor principal plane.

* Stress as a tensor

Stress is a second order tensor quantity.

→ Any scalar quantity is a zero order tensor.

→ stress — magnitude
— direction
— plane of application.

→ Vectors are first order tensor quantity.

→ Stress is a second order tensor quantity.

Stress tensor

$$[\tau] = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix}$$

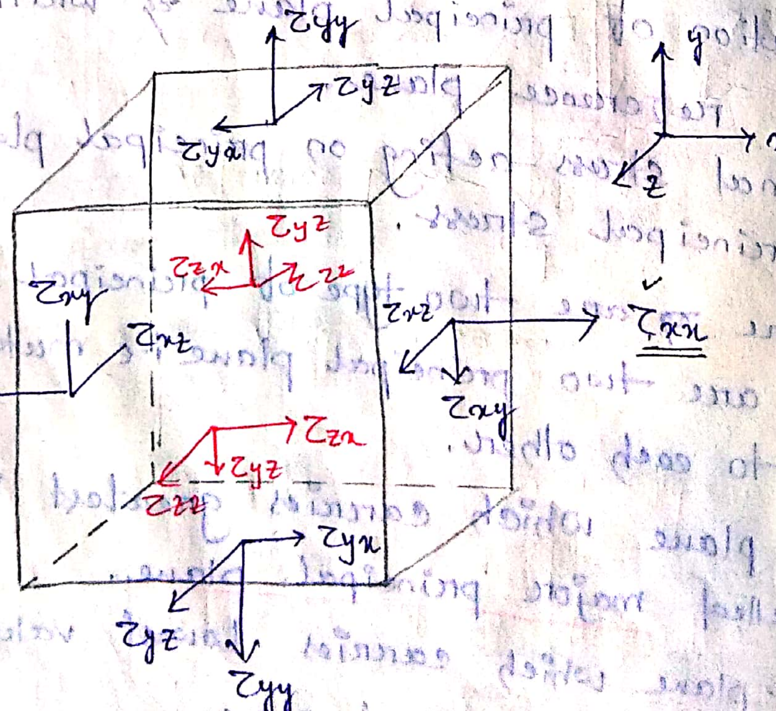
← Representation of stress tensor

← 3-D state of stress

τ_{xy} ① ②

① The plane on which stress is acting

② Dirn in which stress is acting



τ_{xx} ↓ ↳ x direction. } called as normal stresses.

x-plane

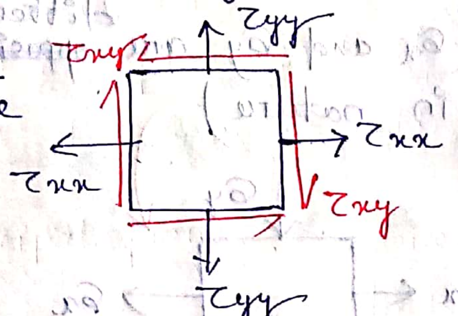
→ $\tau_{xx} = \sigma_x$

$\tau_{yy} = \sigma_y$

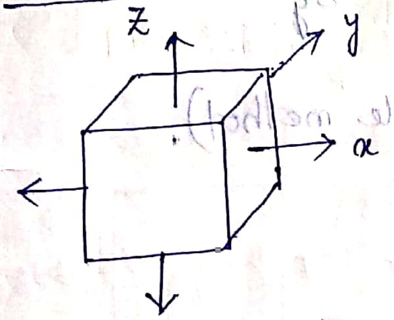
$\tau_{zz} = \sigma_z$

→ complimentary shear stress

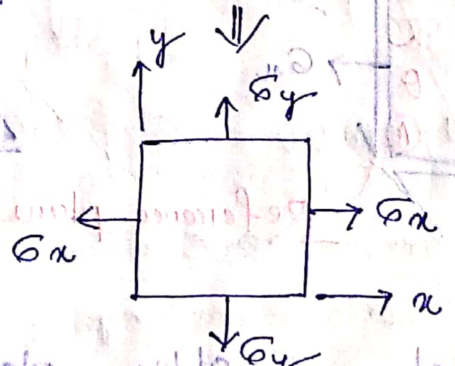
XY plane



* 3- Dimensional State of stress

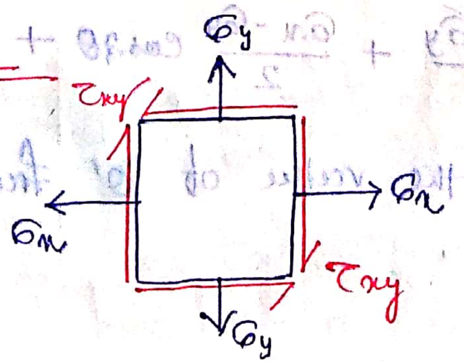


→ When in two direction stress is present and in one direction the stress is less or negligible then it become 2- Dimensional state of stress.



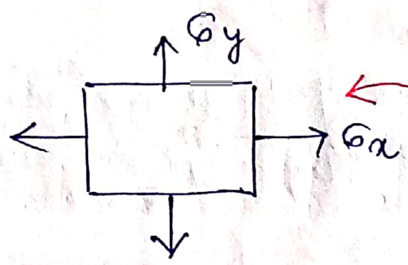
This is also called as plane stress

Complex stress



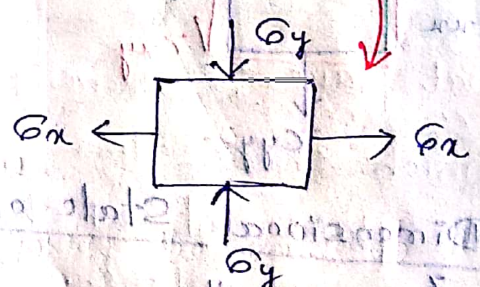
→ Complex stress is also a plane stress. in complex str the normal stress and also the shear stress prese

→ plane stress or 2-D



→ Like stresses (σ_x & σ_y are same in Nature)

→ Unlike stresses (σ_x and σ_y are ~~opposit~~ different in nature)

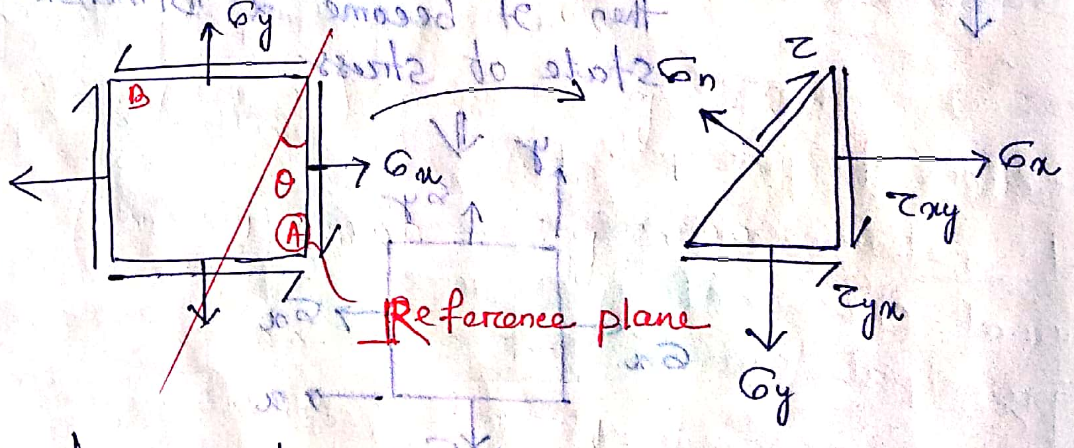


Analysis of state of stress

→ Analytical method

→ Graphical method. (Mohr's circle method).

* Analytical Method



① Normal stress on Oblique plane.

$$\sigma_n \text{ or } \sigma_\theta = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta.$$

Always measure the value of 'θ' from the reference plane.

ii) Shear stress on oblique Plane.

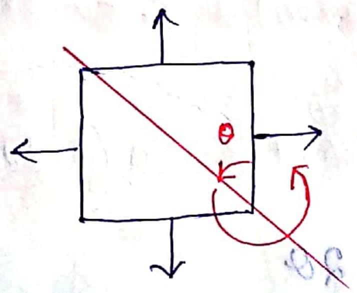
$$\tau \text{ or } \tau_\theta = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta .$$

* θ = (+ve) if measured clockwise from vertical plane.

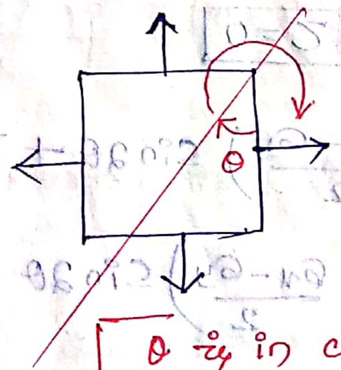
  θ = (−ve) if measured Anticlockwise from vertical plane.

* If in the question only statement is given no diagram is there then assume θ is +ve i.e. clockwise from the vertical reference plane.

→ σ_x is (+ve) when tensile & (−ve) when compressive.
  σ_y is (+ve) when tensile & (−ve) when compressive.



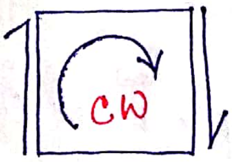
θ is in anticlock wise from the reference plane



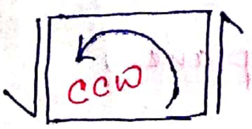
θ is in clockwise from the reference plane

→ put $\theta = -\theta$ in all formulae for calculation if θ is anti-clock wise from vertical reference plane.

Shear stress (τ_{xy})

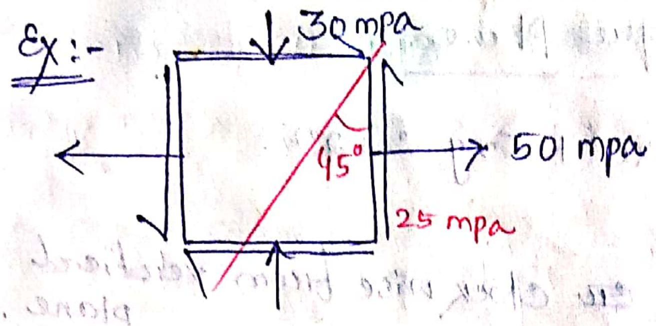


→ Take τ_{xy} as (+ve) if giving a clock wise rotation.



→ Take τ_{xy} as (−ve) if giving a anti-clock wise rotation.

Ex :-



$\sigma_x = 50$ mpa

$\sigma_y = -30$ mpa

$\tau_{xy} = -25$ mpa

$\theta = 45°$

Normal stress :- (σ_n or σ_θ) $\left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy} \sin 2\theta$.

Shear stress :- (τ or τ_θ) $-\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy} \cos 2\theta$.

Location of Principal plane

We Know that the plane on which the value of the shear stress is zero is called principal plane.

→ so put $\boxed{\tau = 0}$

$$\tau = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy} \cos 2\theta$$

$$0 = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta = \tau_{xy} \cos 2\theta$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

At $\theta = \theta_{P}$

$\theta_P = \theta_{P_1}$

$\theta_{P_2} = \theta_{P_1} + 90°$

$$\boxed{\theta_P = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y}\right)}$$

Location of principal plane.

Location of plane of Maximum Shear Stress

$\Rightarrow \dfrac{d\tau}{d\theta} = 0$

$\Rightarrow -\left(\dfrac{\sigma_x - \sigma_y}{2}\right) 2\cos 2\theta + \left\{-2\tau_{xy} \sin 2\theta\right\} = 0$

$\left(\dfrac{\sigma_x - \sigma_y}{2}\right) 2\cos 2\theta = -2\tau_{xy} \sin 2\theta$

$\dfrac{\sin 2\theta}{\cos 2\theta} = \dfrac{\sigma_x - \sigma_y}{-2\tau_{xy}}$

$\tan 2\theta = \dfrac{\sigma_x - \sigma_y}{-2\tau_{xy}}$

$$\boxed{\theta = \frac{1}{3}\tan^{-1}\left(-\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right)}$$

At this value of 'θ' τ is maximum.

we represent this as ϕ

ϕ is plane of maximum shear stress.

$$\boxed{\phi = \frac{1}{2}\tan^{-1}\left(-\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right)}$$

Principal Stresses

$$\sigma_{1,2} = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

σ_1 = Major principal stress (max^m value of Normal stress)

σ_2 = Minor principal stress (Min^m value of Normal stress).

→ consider magnitude

Ex:- $\sigma_{1,2} = 75 \pm 13$

$= 75 + 13$, $75 - 13$

Major → $\boxed{\sigma_1 = 88}$; $\boxed{62 = \sigma_2}$ ← minor

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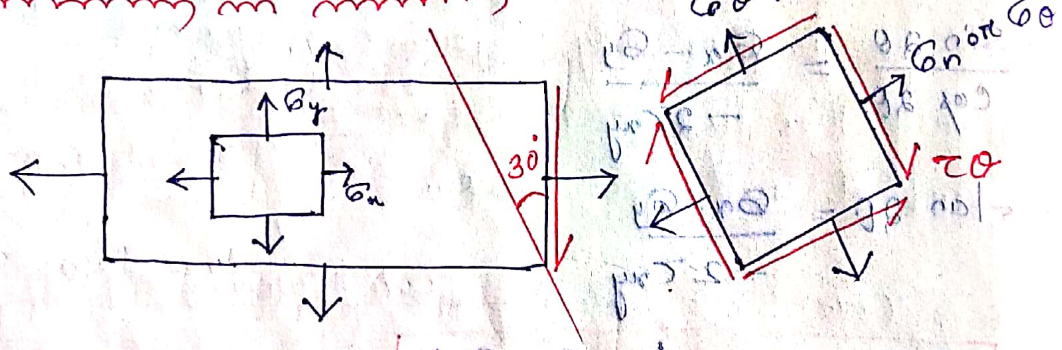
$\sigma_{1,2} = -75 \pm 13$

~~Major $\sigma_1 = -62$~~ ~~$\sigma_2 = -88$ minor~~

$\sigma_1 = -88$ $\sigma_2 = -62$

Major   minor

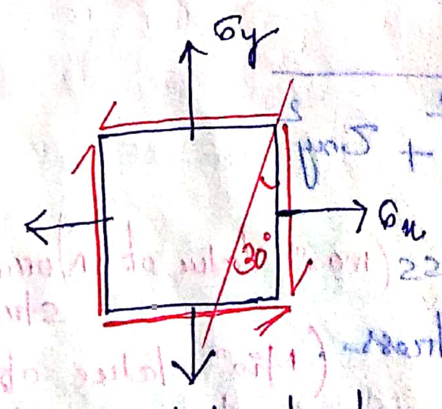
Transformation of stresses



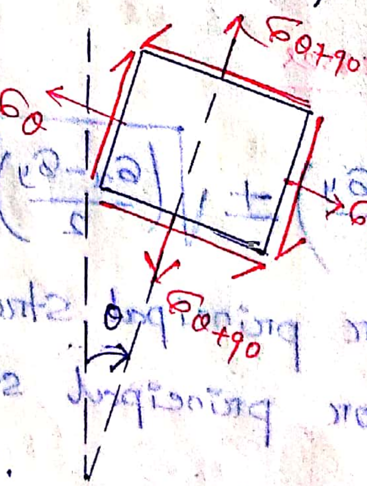
$$\sigma_x = \left(\frac{\sigma_\theta + \sigma_{\theta+90}}{2}\right) + \left(\frac{\sigma_\theta - \sigma_{\theta+90}}{2}\right)\cos(2\times30°) + \tau\sin(2\times30°)$$

$$\sigma_y = \frac{\sigma_\theta + \sigma_{\theta+90}}{2} + \left(\frac{\sigma_\theta - \sigma_{\theta+90}}{2}\right)\cos(2\times120°) + \tau\sin(2\times120°)$$

$$\tau_{xy} = -\left(\frac{\sigma_\theta - \sigma_{\theta+90}}{2}\right)\sin(2\times30°) + \tau\cos(2\times30°)$$

* 

Given state of stress.



→ find out normal stresses on an oblique plane at $\theta°$ from 'A' (vertical plane, i.e the reference plane).

→ find out state of stress when element is rotated by θ degrees.

$$\sigma_\theta = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad \text{—— (1)}$$

$$\sigma_{\theta+90} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2(\theta+90^\circ) + \tau_{xy}\sin 2(\theta+90^\circ).$$

$$= \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos(180+2\theta) + \tau_{xy}\sin(180+2\theta).$$

$$= \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \quad \text{—— (2)}$$

eqn (1) + eqn (2)

$$\sigma_\theta + \sigma_{\theta+90} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x+\sigma_y}{2}$$

$$\boxed{\sigma_\theta + \sigma_{\theta+90} = \sigma_x+\sigma_y}$$

→ for a given state of stress the sum of normal stresses on ϕ two mutually perpendicular planes remains const

→ for principal plane and principal stresses.

$\theta_p = \sigma_1$

$\theta_{p+90^\circ} = \sigma_2$

$\theta_p + \theta_{p+90}$

↑ Two mutually perpendicular principal planes.

$$\boxed{\sigma_1+\sigma_2 = \sigma_x+\sigma_y}$$

$$\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}.$$

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By adding σ_1 & σ_2

$$\boxed{\sigma_1+\sigma_2=\sigma_x+\sigma_y}$$

* Maximum shear stress

$$\phi=\frac{1}{2}\tan^{-1}\left(+\frac{\sigma_x-\sigma_y}{2\tau_{xy}}\right)$$

$$\tau \text{ or } \tau_\theta=-\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta+\tau_{xy}\cos 2\theta$$

put $\boxed{\theta=\phi}$

we will get τ_{max}.

so; $$\boxed{\tau_{max}=\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}}$$

We know

$$\sigma_{1,2}=\left(\frac{\sigma_x+\sigma_y}{2}\right)\pm\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$\sigma_1=\frac{\sigma_x+\sigma_y}{2}+\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} \quad (1)$$

$$\sigma_2=\frac{\sigma_x+\sigma_y}{2}-\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} \quad (2)$$

eq^n 1 − eq^n 2

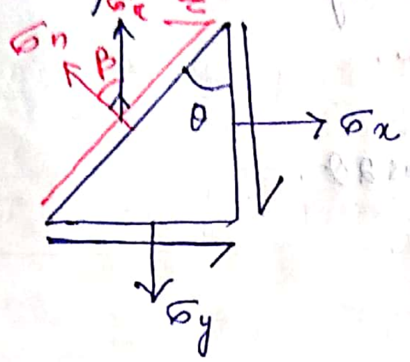
$$\sigma_1-\sigma_2=2\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$\frac{\sigma_1-\sigma_2}{2}=\tau_{max}$$

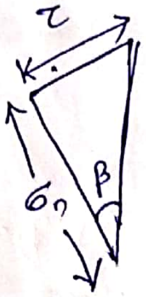
$$\Rightarrow \boxed{\tau_{max}=\frac{\sigma_1-\sigma_2}{2}}$$

Resultant Stress

σ_r is calculated on any oblique plane.



$$\boxed{\sigma_r = \sqrt{\sigma_n^2 + \tau^2}}$$

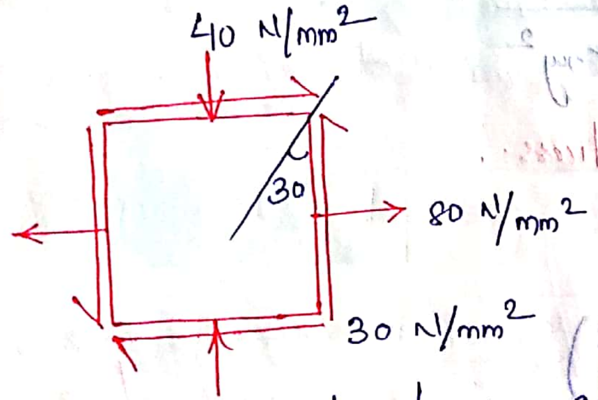


$$\tan \beta = \frac{\tau}{\sigma_n}$$

$$\boxed{\beta = \tan^{-1}\left(\frac{\tau}{\sigma_n}\right)}$$

22

Q.



for the given state of stress ; calculate

(i) Normal and shear stresses on oblique plane

(ii) Principal stress and it's location.

(iii) maximum shear stress and it's plane.

(iv) Obliquity and resultant stress.

Ans. To find

1. σ_n, σ_θ & τ or τ_θ
2. σ_1 & σ_2 & θ_p
3. τ_{max} & ϕ
4. β , σ_r

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formulae

$$\sigma_n \text{ or } \sigma_\theta = \left(\frac{\sigma_x+\sigma_y}{2}\right)+\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

Normal stress on any oblique plane.

$$\tau \text{ or } \tau_\theta = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta .$$

Shear stress at an oblique plane.

$$\sigma_{1,2} = \left(\frac{\sigma_x+\sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

principal stresses.

$$\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x-\sigma_y}\right)$$

Location of principal plane.

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

maximum shear stress.

$$\tau_{max} = \frac{\sigma_1-\sigma_2}{2}$$

$$\phi = \frac{1}{2}\tan^{-1}\left(-\frac{\sigma_x-\sigma_y}{2\tau_{xy}}\right)$$

Location of plane of maximum shear stress.

$$\sigma_r = \sqrt{\sigma_n^2+\tau^2}$$

obliquity & Resultant stress

$$\sigma_r = \sqrt{\sigma_\theta^2+\tau_\theta^2}$$

$$\beta = \tan^{-1}\left(\frac{\tau_\theta}{\sigma_\theta}\right) \text{ or } \tan^{-1}\left(\frac{\tau}{\sigma_n}\right)$$

Given data

$\sigma_x = 80\ N/mm^2$ $\qquad$ $\theta = 30^\circ$ (cw).

$\sigma_y = -40\ N/mm^2$

$\tau_{xy} = -80\ N/mm^2$

a plane y direction stress

1. Normal stress on oblique plane

$$\sigma_n = \left(\frac{\sigma_x+\sigma_y}{2}\right)+\left(\frac{\sigma_x-\sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$= \frac{80-40}{2} + \frac{80-(-40)}{2}\cos(2\times 30^\circ) + (-80)\sin(2\times 30^\circ)$$

$$\boxed{\sigma_n = -19.28 \ N/mm^2}$$

2. Shear stress on oblique plane

$$\tau \text{ or } \tau_\theta = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{80-(-40)}{2}\right)\sin(60) + (-80)\cos(60)$$

$\boxed{\tau = -91.96}$ N/mm^2 {here −ve means τ_{xy} anti-clockwise

3. Principal stresses

$$\sigma_{1,2} = \left(\frac{\sigma_x+\sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \left(\frac{80-40}{2}\right) \pm \sqrt{\left(\frac{80+40}{2}\right)^2 + (-80)^2}$$

$$= 20 \pm 100$$

for (+ve) $20+100 = 120 \ N/mm^2$
for (−ve) $20-100 = -80 \ N/mm^2$
} only compare magnitude not the sign.

$\sigma_1 = 120 \ N/mm^2 \rightarrow$ major principal stress

$\sigma_2 = -80 \ N/mm^2 \rightarrow$ minor principal stress.

4. Location of principal plane.

$$\theta_p = \left(\frac{1}{2}\right)\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_x-\sigma_y}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{2\times(-80)}{80+40}\right) = -26.565^\circ$$

principal plane is at 26.56° from vertical plane in anti-clockwise direction

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5.

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$= \sqrt{\left(\frac{80+40}{2}\right)^2+(-80^2)}$$

$$= 100 \ N/mm^2.$$

6. plane of maximum shear stress

$$\phi = \frac{1}{2}\tan^{-1}\left(-\frac{\sigma_x-\sigma_y}{2\tau_{xy}}\right)$$

$$= \frac{1}{2}\tan^{-1}\left(-\frac{80+40}{2\times(-80)}\right)$$

$$= 18.435°$$

plane of max^m shear stress is at 18.435° from vertical plane in clock-wise direction.

* $\sigma_n = -19.28 \ N/mm^2$

$\tau = -91.96 \ N/mm^2$

Resultant stress $= \sqrt{\sigma_n^2+\tau^2}$ or $\sqrt{\sigma_\theta^2+\tau_\theta^2}$

$$= \sqrt{(-19.28)^2+(-91.96)^2}$$

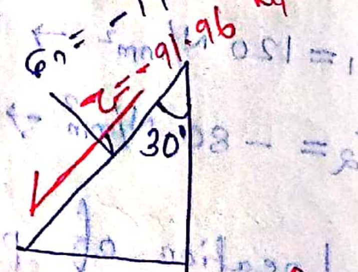
$$= 93.96 \ N/mm^2$$

Obliquity $(\beta) = \tan^{-1}\left(\frac{\tau}{\sigma_n}\right)$ or $\tan^{-1}\left(\frac{\tau_\theta}{\sigma_\theta}\right)$

$$= \tan^{-1}\left(\frac{-91.96}{-19.28}\right)$$

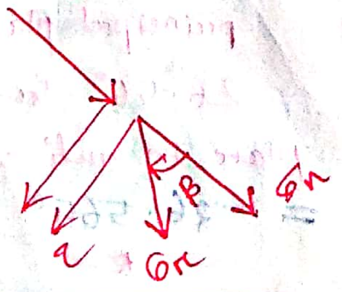
$$= 78.16 \ N/mm^2.$$

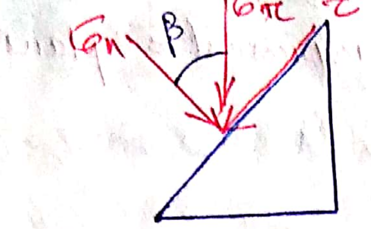
Always measured w.r.t Normal stress or direction of Normal stress



LR
(↑) +ve
(↓) -ve

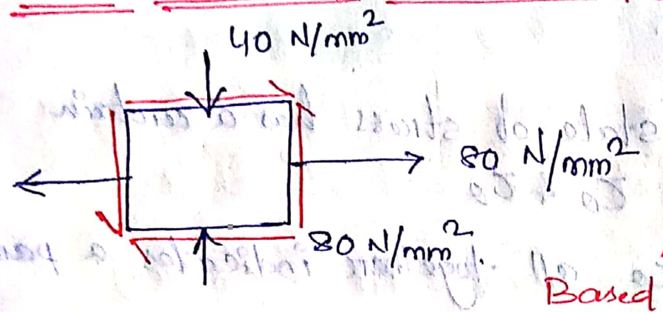
RL
(↓) +ve
(↑) -ve





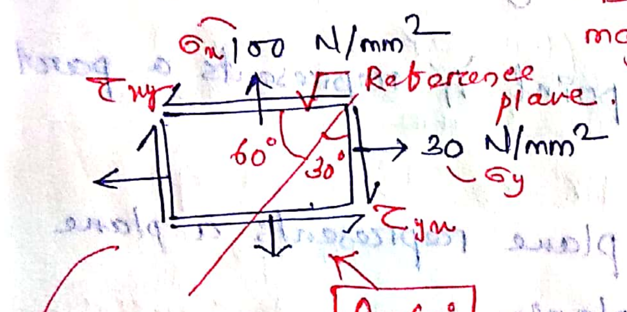
σ_n is towards the surface
So it is compressive.

* How to select reference plane



The plane which carries $\boxed{\text{maximum stress}}$ in a given problem should be taken as reference plane.

Based on magnitude not



$\boxed{\theta = 60^\circ}$

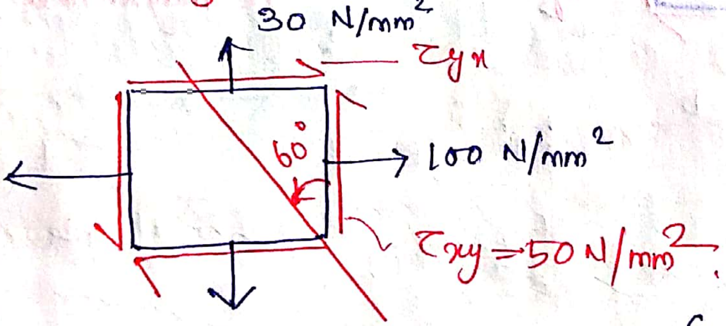
$\boxed{\sigma_x > \sigma_y}$

The highest stress value should be taken as σ_x

τ_{xy}

⟂ shear stress on x-plane and y-direction.

Simply rotate 90° if this type of figure is given.



$\theta = 60^\circ$ {anti-clock wise}

(23)

Mohr's Circle Method.

for τ

CW ⇒ +ve

ACW ⇒ −ve

$\tau \to$ shear stress

ordinate.

Abscissa

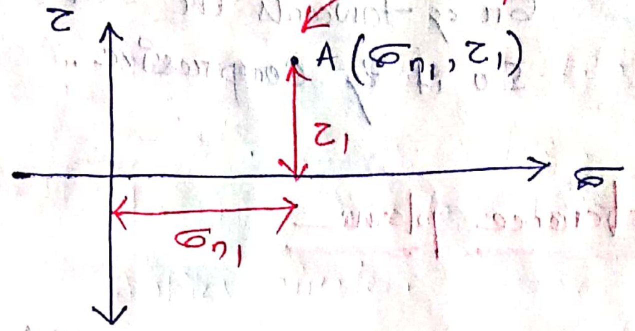
$-\sigma_n$ (Compressive)

0,0

σ_n (Tensile)

└ Normal Stress

→ This point represent a plane.



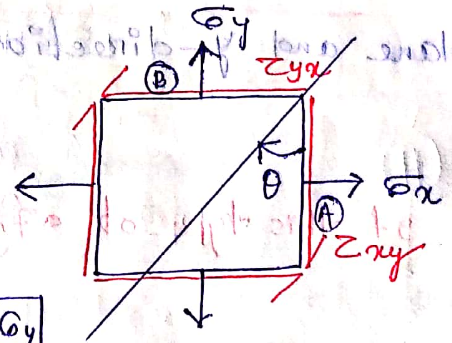
→ Each plane on a given state of stress has a certain values of σ_n & τ or τ_θ & τ_θ

→ σ_n and τ or σ_θ and τ_θ all together indicates a particular oblique plane.

→ In the above figure the point 'A' represents a particular oblique plane.

→ A point drawn on $(\sigma_n - \tau)$ plane represents a plane or state of stress of a plane.

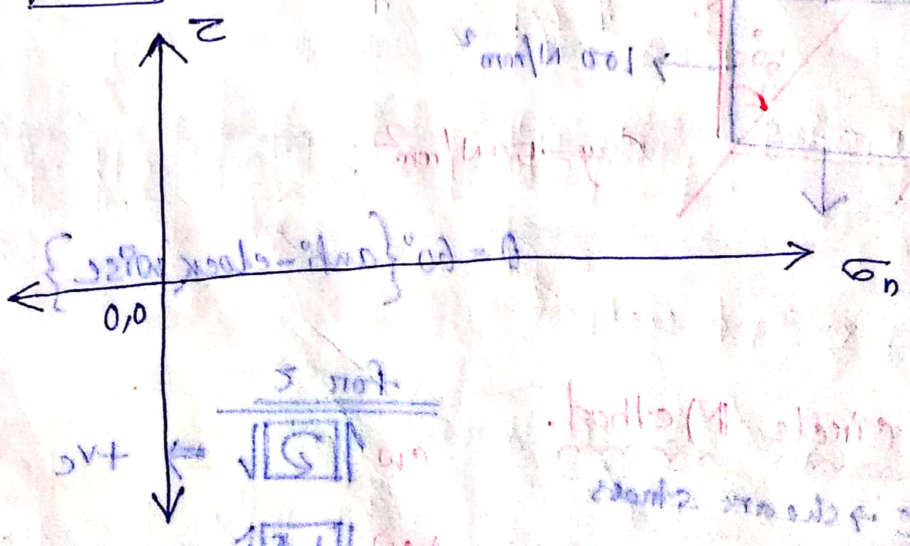
✳



$\boxed{\tau_{xy} = \tau_{yx}}$

In magnitude.

Assume $\boxed{\sigma_x > \sigma_y}$

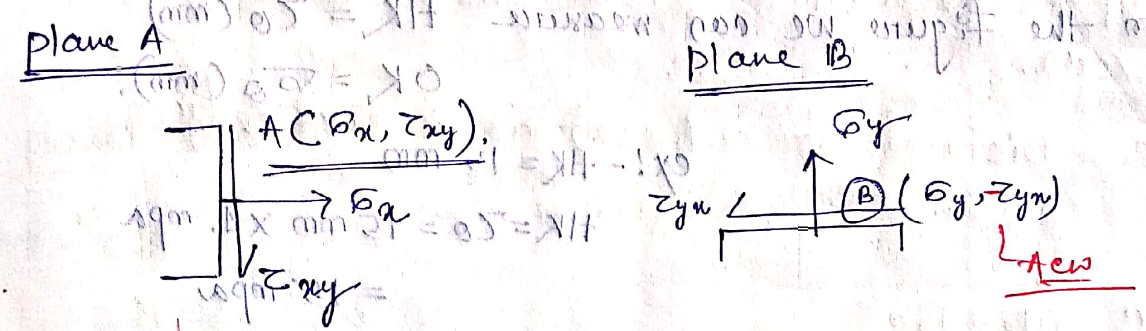


Sign Convention

$\sigma \rightarrow$ Tensile (+ve)
Compressive (−ve)

$\tau \rightarrow$ CW in plane (+ve)
ACW in plane (−ve)

$\theta \rightarrow$ CW from reference (+ve)
ACW from reference (+ve)

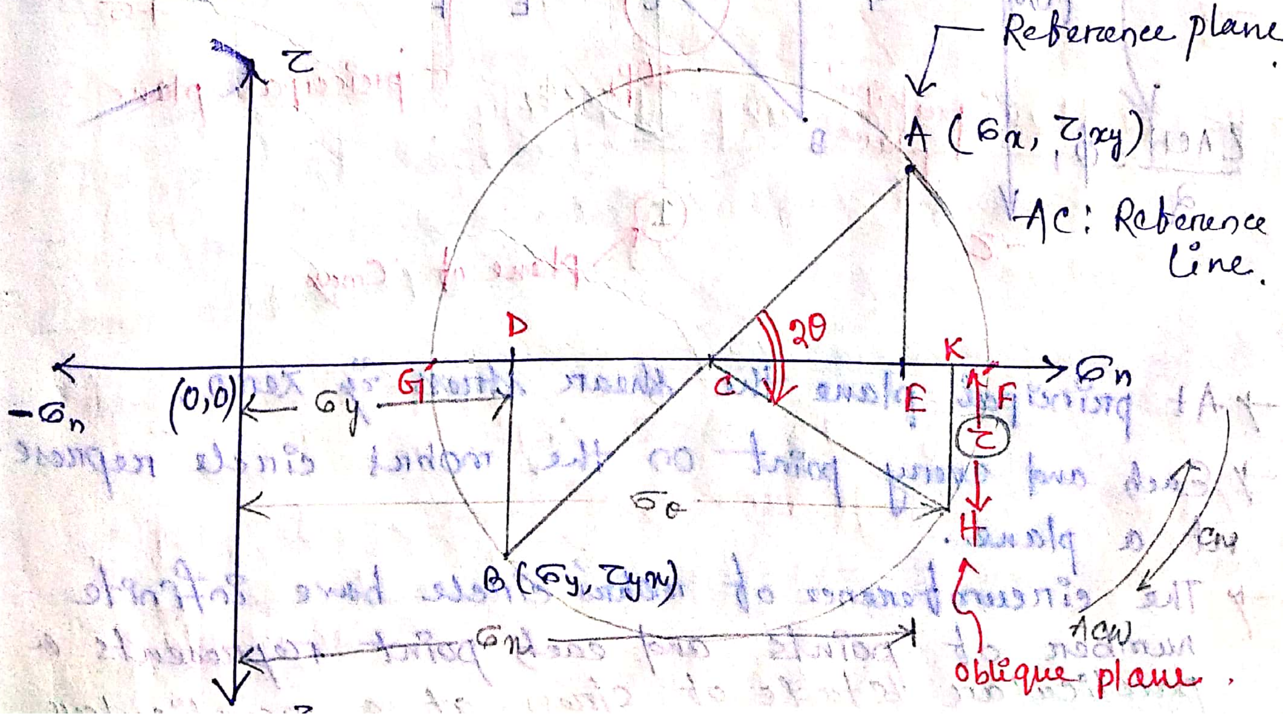


Step-1 Locate point 'A' & 'B' on σ_n-τ plane.
for given state of stress take suitable scale → To convert stress in terms of length.

ex = 100 mpa ⟶ 10 cm

$1 \text{ mpa} \longrightarrow \frac{10}{100} \text{ cm} = 0.1 \text{ cm} = 1 \text{ mm}$

50 mpa ⟶ 5 cm



Step-2 : Join 'A' & 'B'

Step-3 : The intersection of line 'AB' with σ_n axis is 'c'. C as centre, AC or CB as radius draw a circle. This circle is called as mohr's circle.

Note:- The angles between any two planes on mohr's circle is twice that of the actual angle.

from the figure we can measure $HK = \tau_\theta\ (mm)$

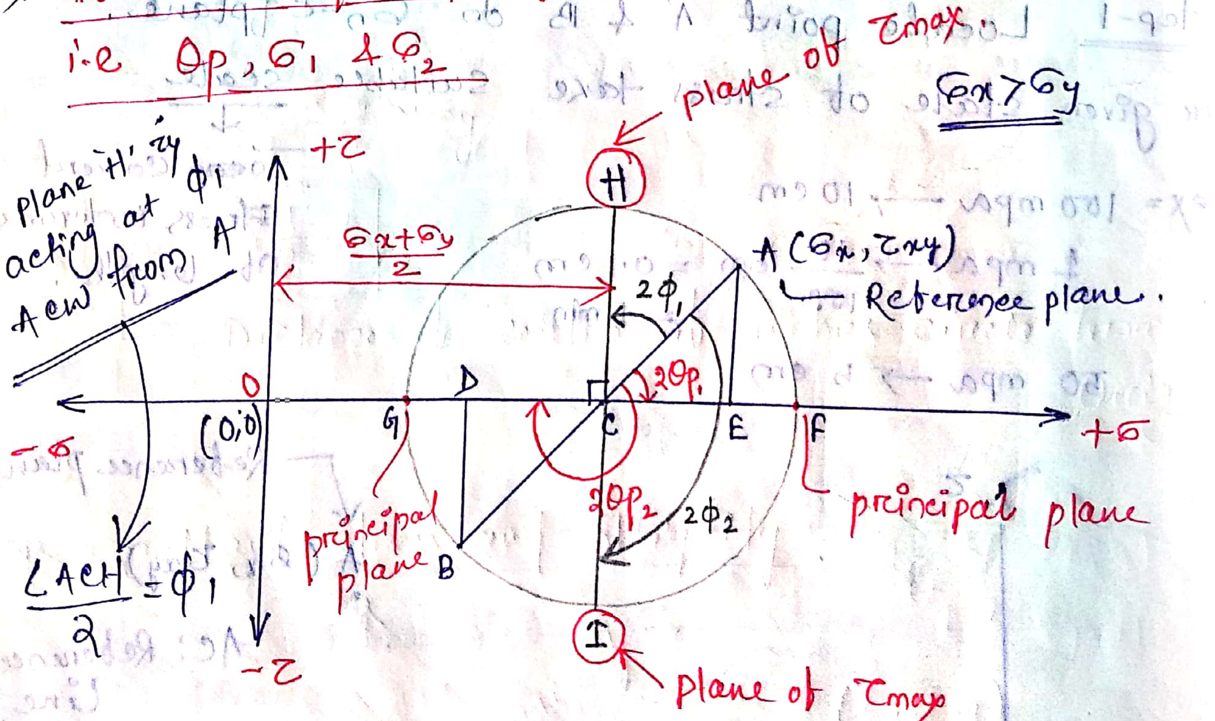
$OK = \sigma_\theta\ (mm)$.

ex:- $HK = 15\ mm$

$HK = \tau_\theta = 15\ mm \times 1\ mpa$

$= 15\ mpa$

* Location of principal plane and principal stresses i.e $\theta_p, \sigma_1 \ \& \ \sigma_2$



→ At principal plane the shear stress is zero.

→ Each and every point on the mohr's circle represent a plane.

→ The circumference of mohr's circle have infinite number of points and each point represents a particular state of stress at a particular plane.

→ Location of principal plane is $\angle ACF = 2\theta_p$

$2\theta_p = \angle ACF$

$\theta_p = \frac{\angle ACF}{2}$ (clock wise)

→ from the mohr's circle we can see that, At point 'G' and 'F' the value of 'τ' is zero so 'G' & 'F' point are the principal planes.

→ 'F' is the major principal plane coz value of normal stress is highest and the 'G' is the minor principal plane coz the value of normal stress is lowest.

→ Location of minor principal plane.

$2\theta p_2 = \angle ACG$

$\theta p_2 = \frac{\angle ACG}{2}$ (clock wise)

→ principal stress

major principal stress is $OF = \sigma_1$ (in mm)

minor principal stress is $OG = \sigma_2$ (in mm).

→ The point where the mohr's circle intersect the σ_n axis is called as principal plane.

* Maximum shear stress and it's plane

→ The point on the mohr's circle where the value of τ is maximum that point is known as maximum shear stress.

→ Draw a perpendicular line from σ_n axis from point 'c' i.e centre of mohr's circle

Now $CH = \tau_{max}$ (+ve) clock wise

maximum shear stress is equal to the radius of the mohr's circle.

→ GF = Diameter of mohr's circle

τ_{max} i.e radious of mohr's circle $= \frac{GF}{2}$

→ Diameter of mohr's circle i.e $GF = \sigma_1 - \sigma_2$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{GF}{2}$$

$CI = -\tau_{max}$ (-ve) i.e Anticlock wise

→ In plane of maximum shear stress normal stress is not zero.

→ OC = Distance of centre of mohr's circle from origin.

$$OC = OD + DC$$

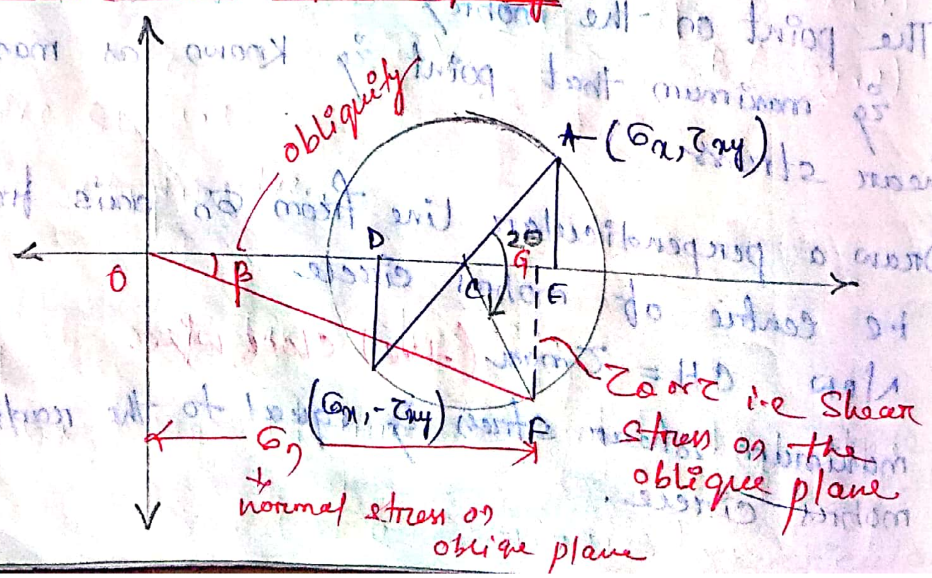
$$= \sigma_y + \frac{\sigma_x - \sigma_y}{2}$$

$$= \frac{\sigma_x}{2} - \frac{\sigma_y}{2} + \sigma_y$$

$$\boxed{OC = \frac{\sigma_x + \sigma_y}{2}}$$

Distance of centre of mohr's circle from Origin.

* Resultant stress and obliquity



Length of OF = $\underline{\sigma_R}$

$\angle GOF = \beta$

$OG = \sigma_n$

$GF = \tau$

$OF = \sigma_R$

$$\left\{ \sigma_R = \sqrt{\sigma_n^2 + \tau^2} \right.$$

← According to analytical method.

(24)

→ plane state of stress means 2D state of stress.

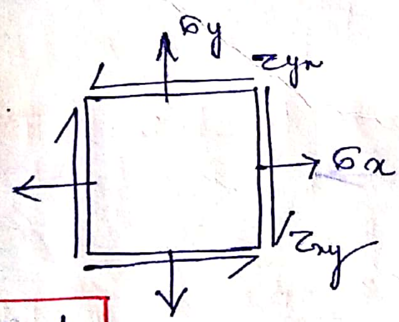
where $\boxed{\sigma_z = 0}$ for x-y plane.

$$\left.\begin{matrix}\tau_{zx} \\ \tau_{zy} \\ \tau_{yz} \\ \tau_{xz}\end{matrix}\right\} = 0$$

stress tensor for 2-D is

$$[\tau] = \begin{bmatrix} \tau_{xx} & \tau_{xy} \\ \tau_{yx} & \tau_{yy} \end{bmatrix}$$

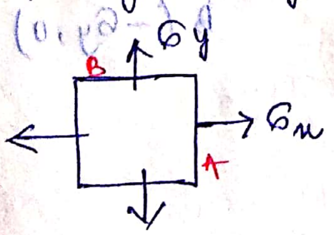
$$= \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix}$$

* 

⇐ Complex stress.

Case -1

(i) If τ_{yx} & τ_{xy} becomes zero.



This is like stress

∴ Nature of two stress is same.

σ_x & σ_y both tensile in nature.

→ What is principal plane in this case.

for principal plane $\tau = 0$.

So here plane 'A' & 'B' are principal plane

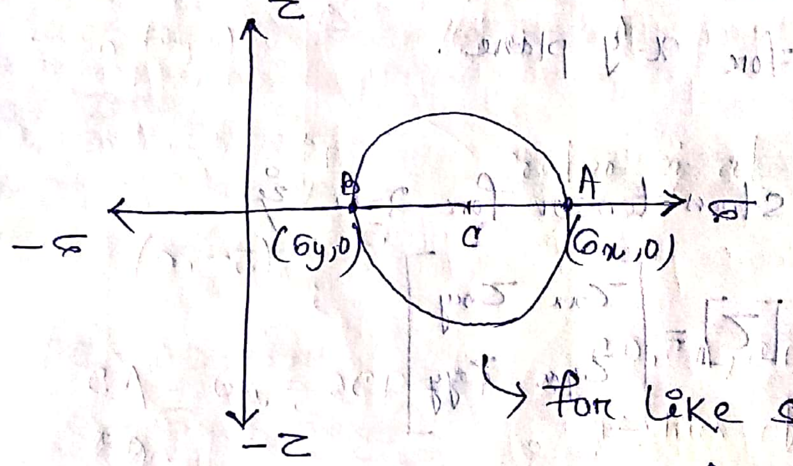
→ What is principal stress in this case.

we know that the stress acting on the principal plan is called as principal stress. So σ_x & σ_y are the principal stresses.

If $\sigma_x > \sigma_y$ then $\sigma_1 = \sigma_x$

$\sigma_2 = \sigma_y$

→ Mohr's circle for above case.

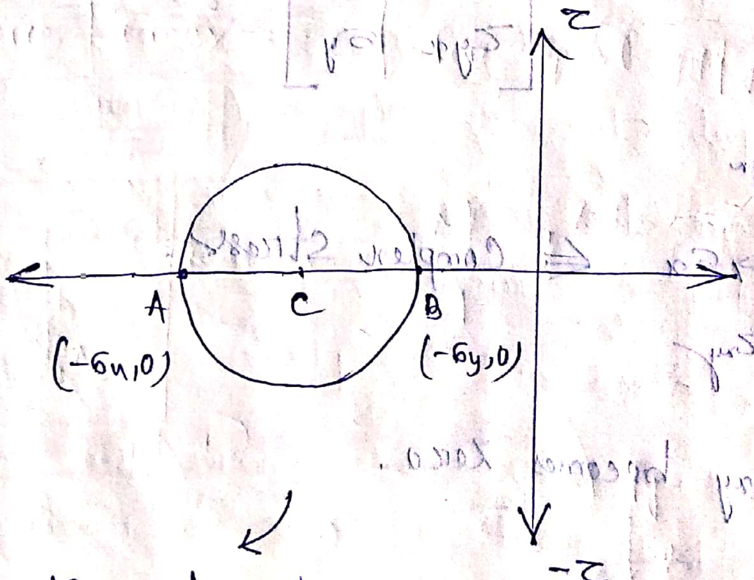


A (σ_x, 0)

B (σ_y, 0)

↳ for like stresses.

for tensile



A($-\sigma_x$, 0)

B ($-\sigma_y$, 0).

for like stresses

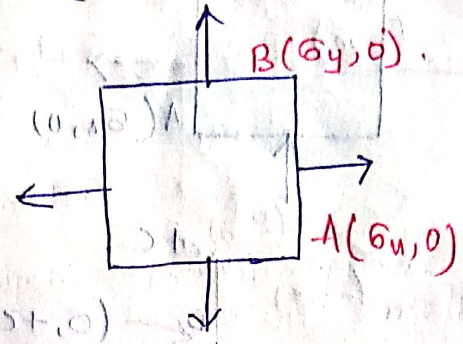
for compressive

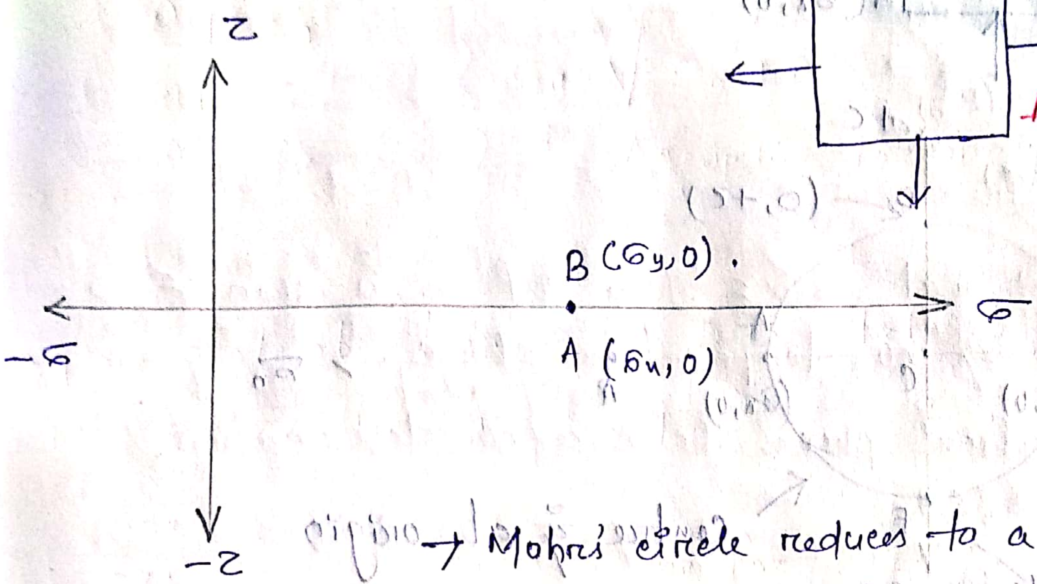
→ for equal and like stresses.

i.e σ_x & σ_y Nature is same, but the
(tensile, compressive)

magnitude of σ_x & σ_y is same ($\sigma_x = \sigma_y$).

let σ_x & σ_y both are tensile.





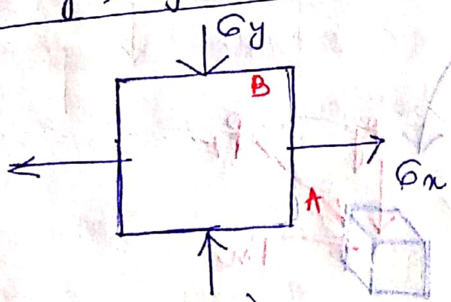
→ Mohr's circle reduces to a point.
radious of mohr's circle is the maximum shear stress.

→ here shear stress is zero

→ In case of equal and like stresses each and every oblique plane in a given state of stress is a principal plane.

* For Unlike stresses

$\boxed{\tau_{xy}, \tau_{yx} = 0}$



→ Plane 'A' & 'B' are principal plane.

→ σ_x & σ_y are principal stresses

* Equal & Unlike Stresses

→ magnitude is same i.e $\boxed{\sigma_x = \sigma_y}$ But Nature is Different.

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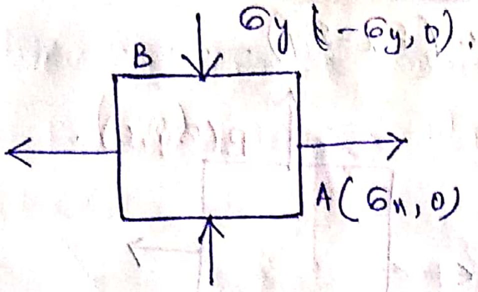
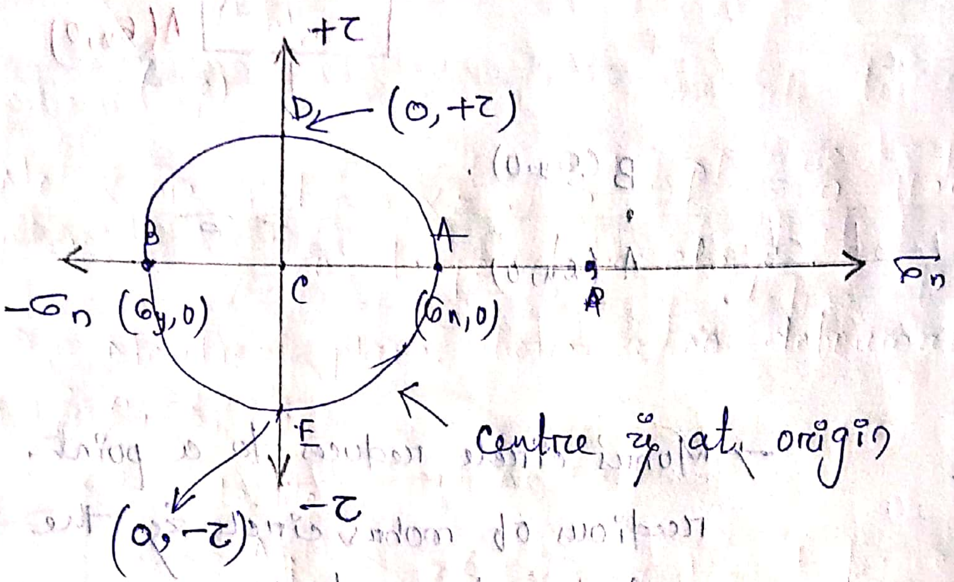
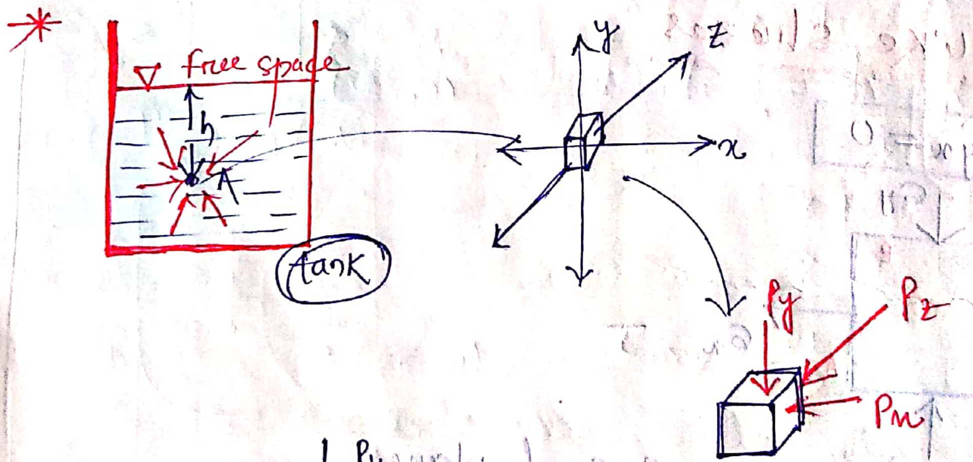


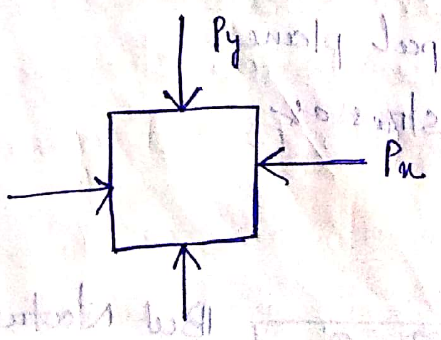


$CA = \sigma_x = CD = CE = CB$ } CD, CA, CE, CB are the radius of mohr's circle.

here $\boxed{\tau_{max} = \sigma_x}$ for equal and unlike stress.

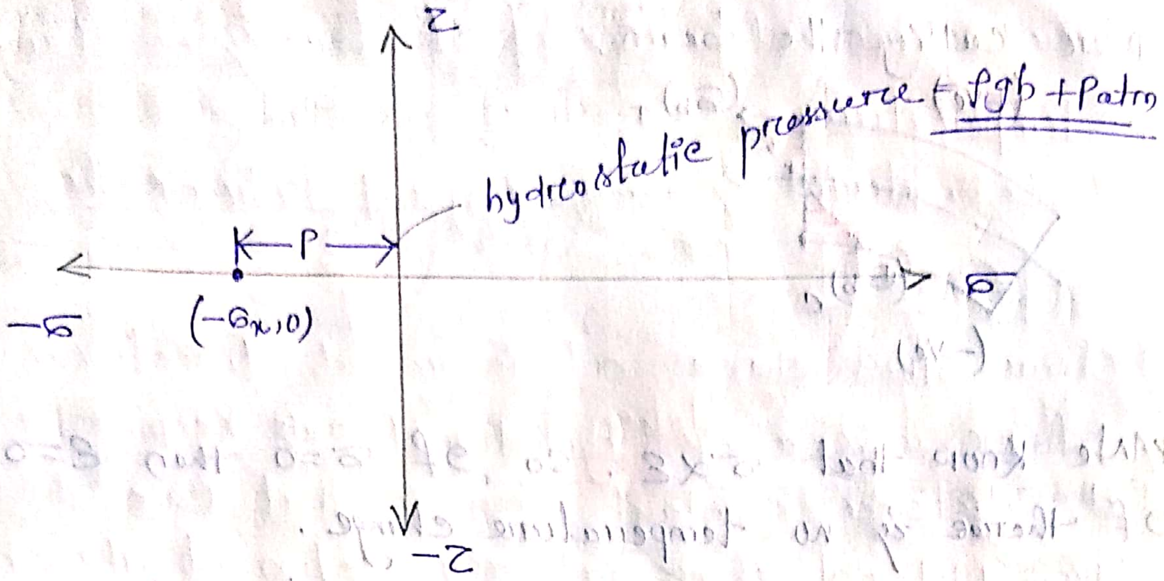
*





$\boxed{P_x = P_y}$

Equal & Like stress → ~~[illegible]~~ Increase of hydrostatic pressure in a fluid element



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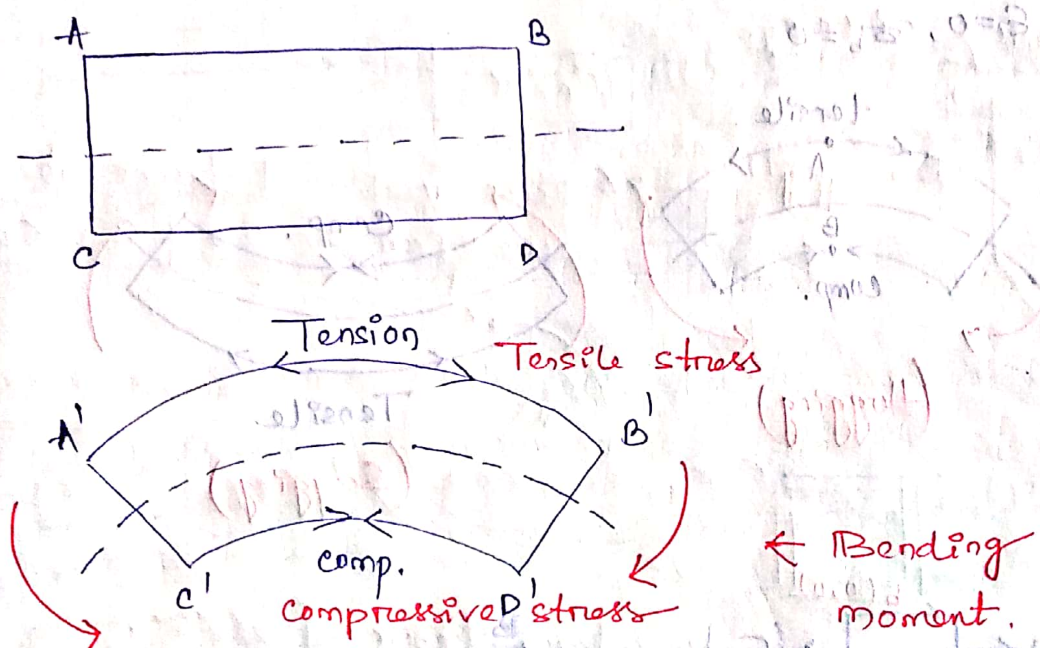
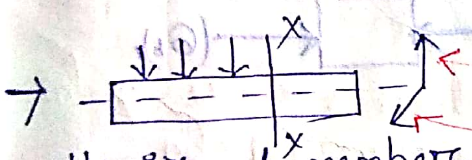




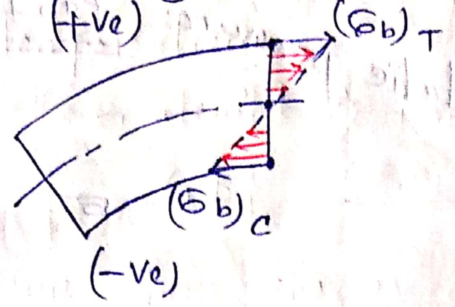
Horizontal member is called beam.
Beam carry vertical load in transverse direction.

$M = f \cdot d.$

$\Sigma M_x \rightarrow$ Summation of moment of forces upto section 'XX'

→ Due to application of Bending moment the tensile & Compressive stress are setup in a in the beam these tensile and compressive stress are called Indirect stress.

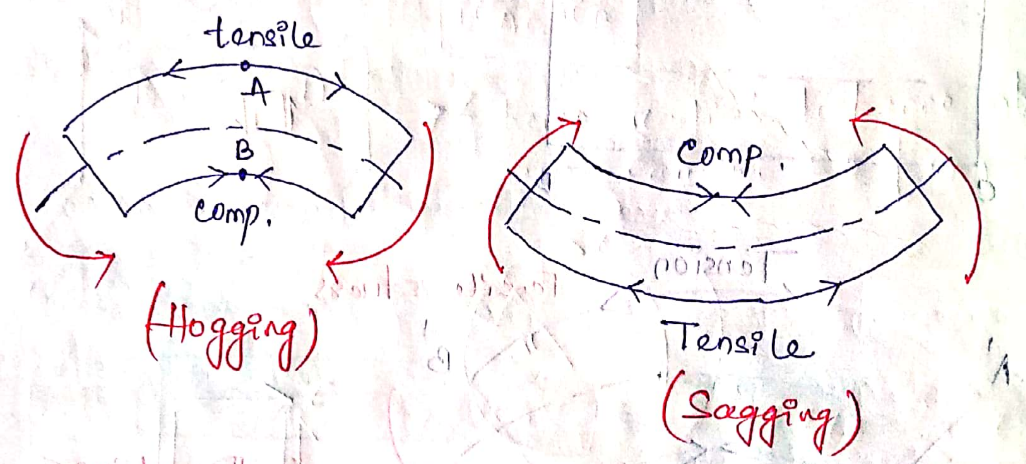
After cutting the beam,

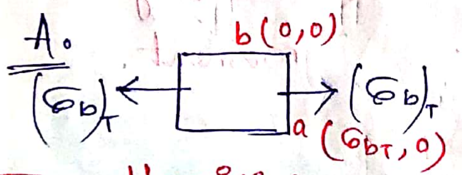


* We know that $\sigma \propto \varepsilon$. So if $\sigma = 0$ then $\varepsilon = 0$ if there is no temperature change.
* In Neutral plane there is no deformation.

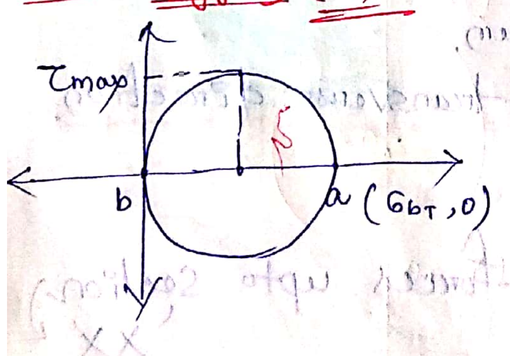
$\varepsilon = 0, \; \sigma_b = 0$

*





For Hogging (A)

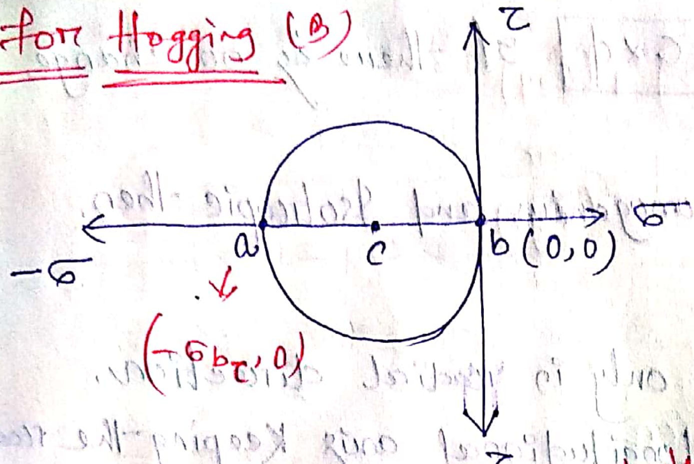


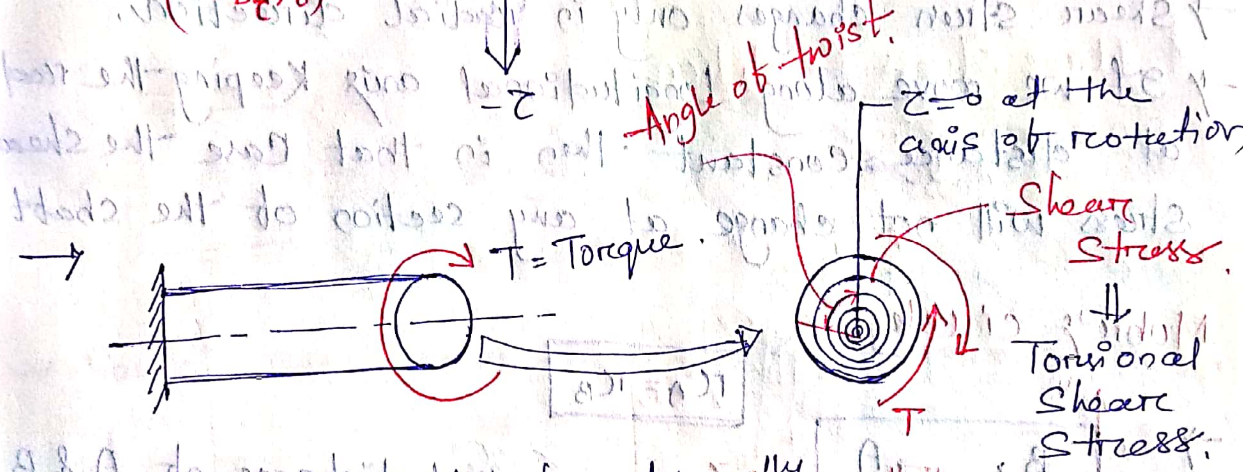
$$\tau_{max} = \frac{\sigma_{bT}}{2}$$

$(\tau_{max})_A = (\tau_{max})_B$ if $(\sigma_b)_A = (\sigma_b)_B$

→

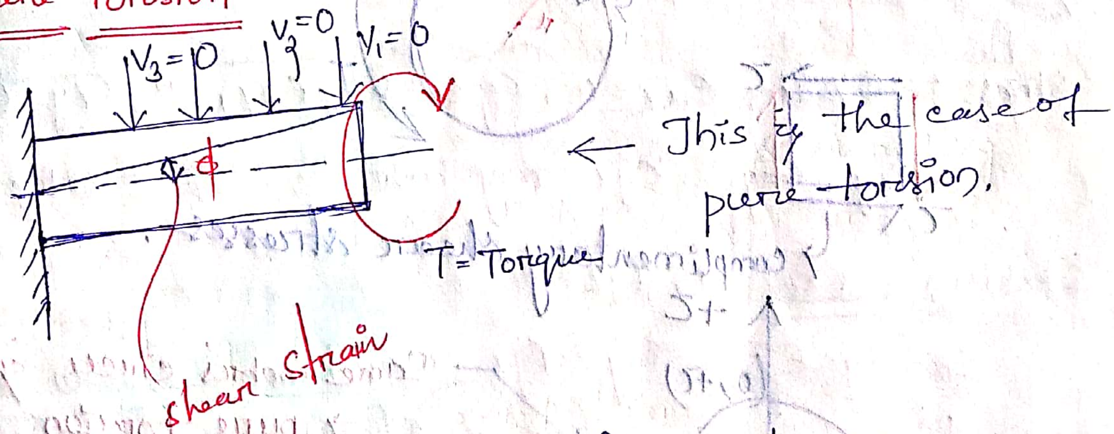
For Hogging (B)





Twist produced due to externally applied torque is known as pure Torsion.

Pure torsion

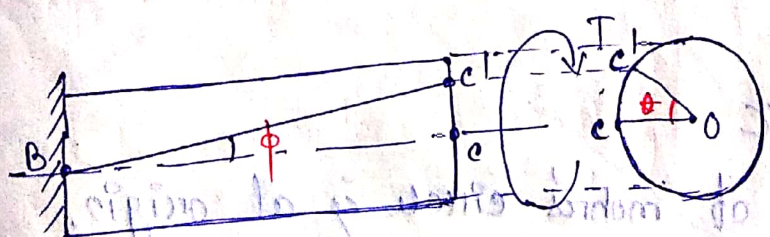


← This is the case of pure torsion.

→ Torsional shear stress is function of radius.

i.e $\tau = f(r)$

$\tau \propto$ radius.



θ = Angle of twist

ϕ = Shear strain.

• According hooks law $\boxed{\tau \propto \phi}$ If there is no change in temperature.

• If the material is homogenius and Isotropic then.

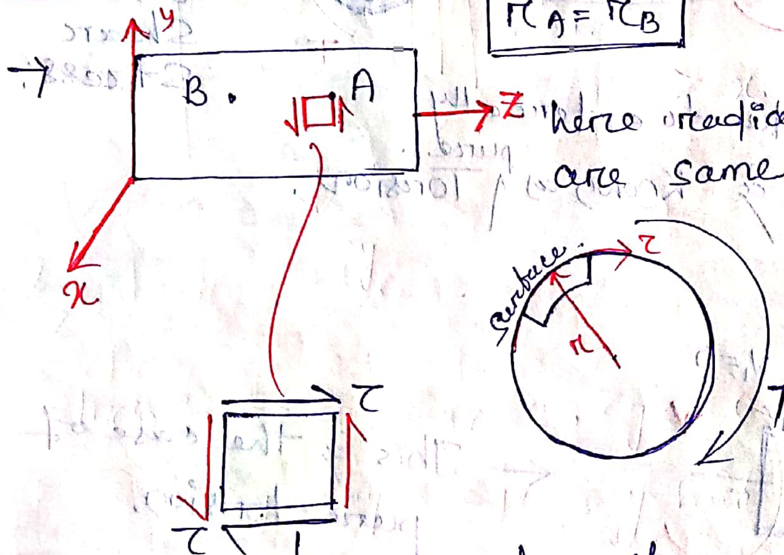
$$\boxed{\tau = G\phi}$$

→ Shear stress changes only in radial direction.

→ If we move along longitudional axis keeping the radial distance constant then in that case the shear stress will not change at any section of the shaft.

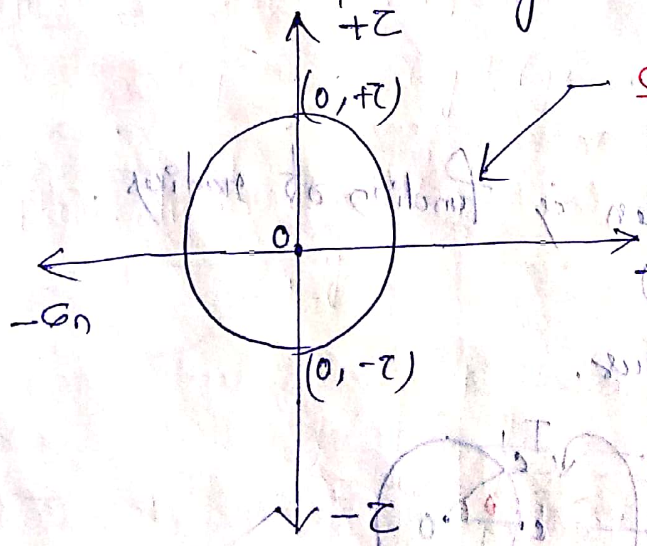
Mohr's circle

$\boxed{\tau_A = \tau_B}$



→ here radial distance of A & B are same. so $\boxed{\tau_A = \tau_B}$

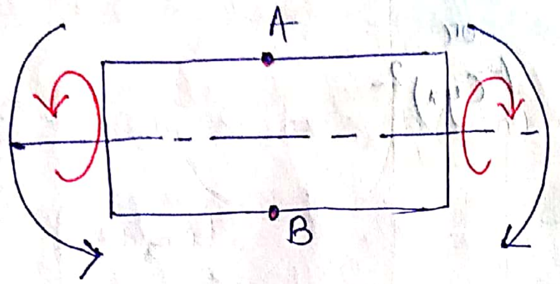
↳ complimentary shear stresses.

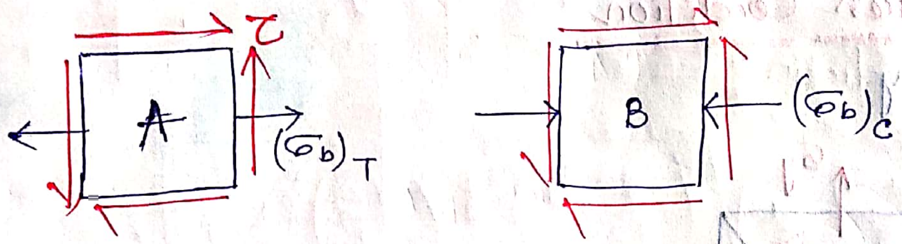


Same mohr's circle is for
→ pure torsion
→ equal & unlike stress

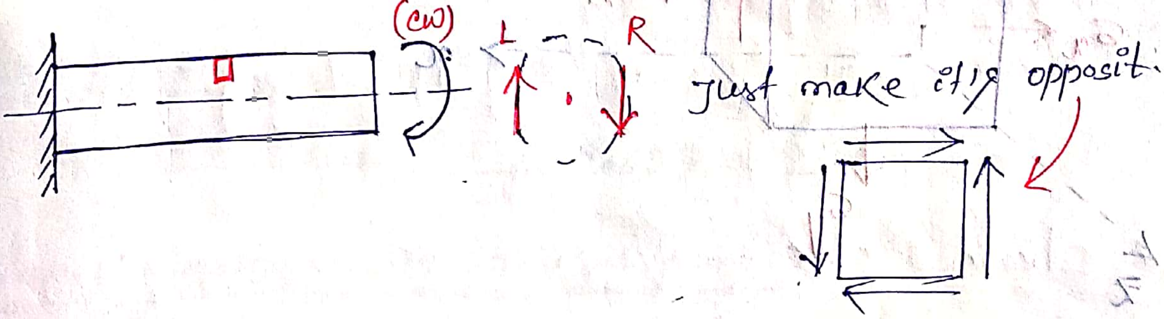
→ Here the centre of mohr's circle is at origin.

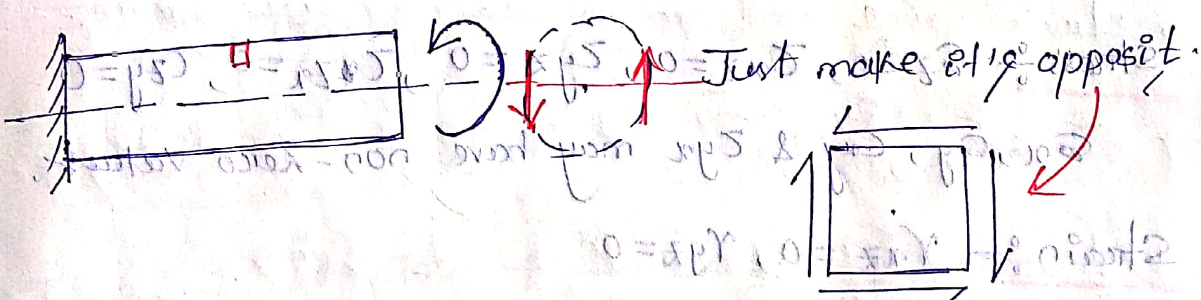
* Mohr's Circle for Combined Bending & Torsion.

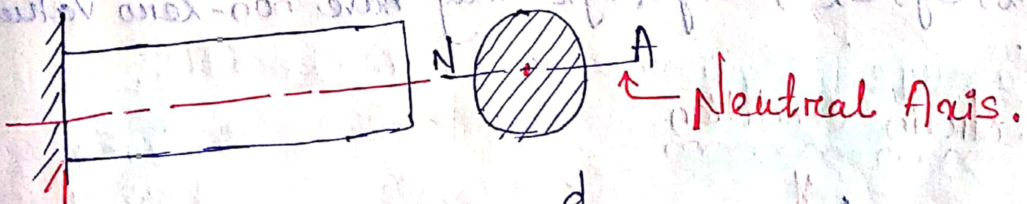




* How to place shear stress (trick)

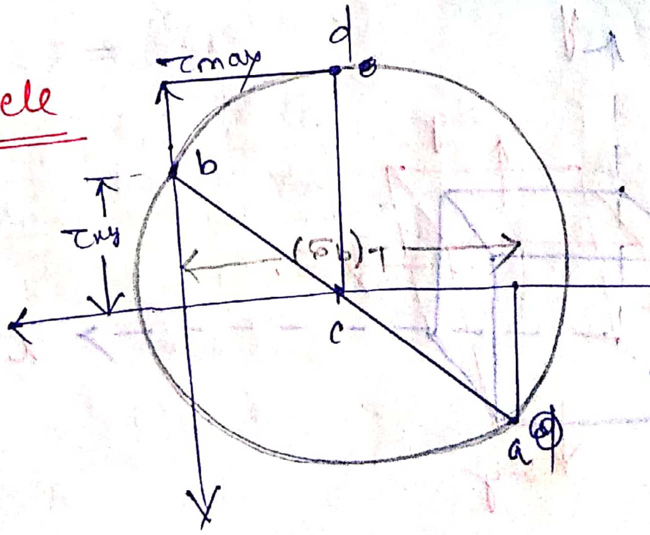






Mohr's circle

for Element (A)



Coordinates

A $(\sigma_{b_T}, -\tau_{xy})$

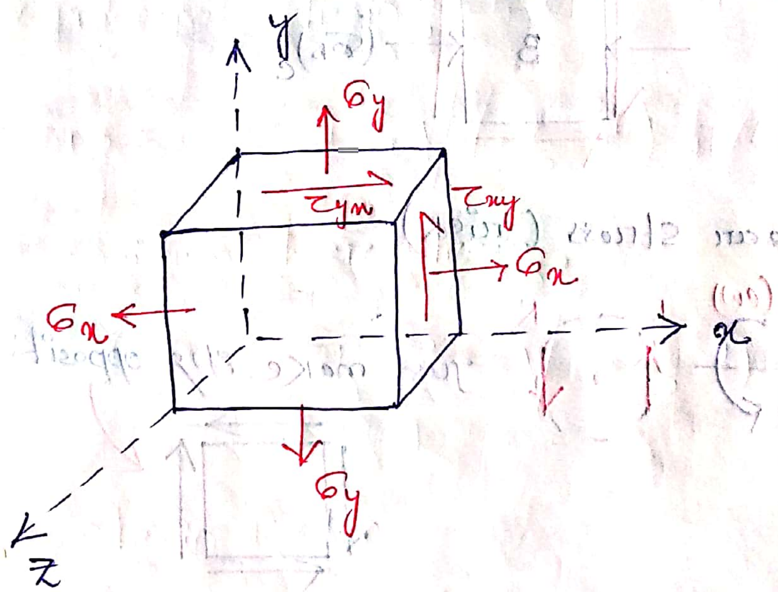
B $(0, +\tau_{xy})$.

→ Here $\tau_{max} = \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + (\tau_{xy})^2}$ or $(\tau_{yx})^2$

Radius of mohr's circle

(26)

* Plane stress Condition



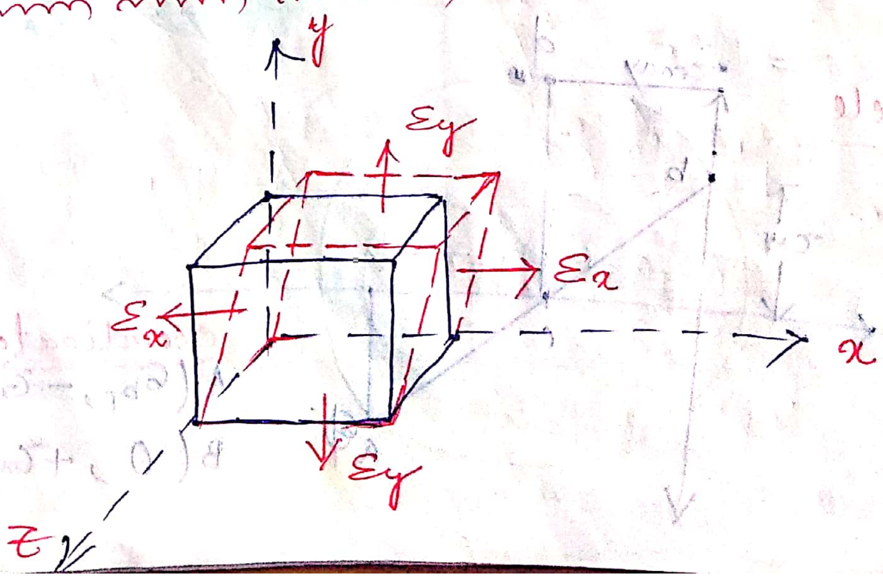
Stress :- $\sigma_z = 0$, $\tau_{xz} = 0$, $\tau_{yz} = 0$, $\tau_{zx} = 0$, $\tau_{zy} = 0$

$\sigma_x, \sigma_y, \tau_{xy}$ & τ_{yx} may have non-zero values.

Strain :- $\gamma_{xz} = 0$, $\gamma_{yz} = 0$

$\varepsilon_x, \varepsilon_y, \varepsilon_z$, γ_{xy} & γ_{yx} may have non-zero values.

* Plane strain condition



→ $\tau_{xz} = 0$, $\tau_{yz} = 0$

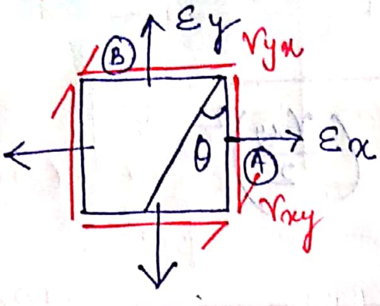
$\sigma_z, \sigma_x, \sigma_y$ and τ_{xy} & τ_{yz} may have non zero values.

→ $\varepsilon_z = 0$, $\gamma_{xz} = 0$, $\gamma_{yz} = 0$

$\varepsilon_x, \varepsilon_y, \gamma_{xy}$ & γ_{yz} may have zero values.

Transformation of strain :

→



Normal strain

$$\varepsilon_n = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta.$$

Shear strain on an oblique plane

$$\frac{\gamma}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta.$$

principal plane $(\gamma = 0)$

$$-\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta = 0$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

$$\tan 2\theta = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

$$\boxed{\theta_p = \frac{1}{2}\tan^{-1}\left(\frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}\right)}$$

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Location of plane of maximum shear strain:

$$\frac{d\gamma}{d\theta} = 0$$

$$\boxed{\phi = \frac{1}{2}\tan^{-1}\left(-\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}}\right)}$$

$\sigma_x \to \varepsilon_x$
$\sigma_y \to \varepsilon_y$
$\tau_{xy} \to \frac{\gamma_{xy}}{2}$

* Principal strain :-

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

* Maximum shear strain :-

$$\frac{\gamma_{max}}{2} = \frac{\varepsilon_1 - \varepsilon_2}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

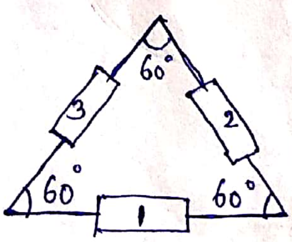
* Strain Rosette

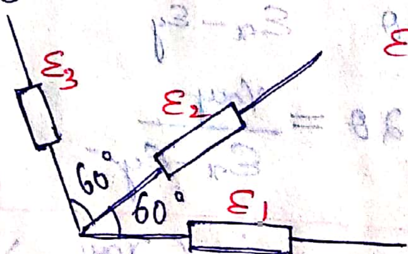
→ strain gauge used to ~~measure~~ measure strain.

→ $$\varepsilon_n = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta.$$

→ $$\frac{\gamma}{2} = -\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta.$$

(i) Delta type, Equi angular strain Rosette:





$\varepsilon_1, \varepsilon_2, \varepsilon_3$ was given

$\theta_1 = 0°$

$\theta_2 = 60°$

$\theta_3 = 120°$

put values of 'θ' in ε_n

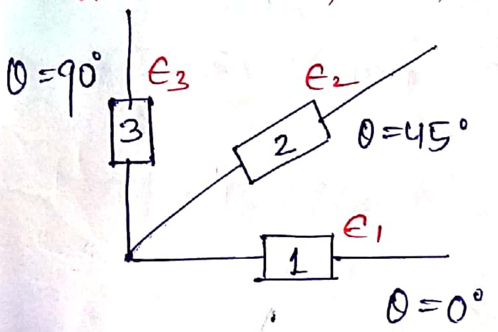
$$\varepsilon_n = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta .$$

then we get three equations.

$$\varepsilon_1 = \varepsilon_x, \varepsilon_y, \gamma_{xy} \text{ --- } (1)$$
$$\varepsilon_2 = \varepsilon_x, \varepsilon_y, \gamma_{xy} \text{ --- } (2)$$
$$\varepsilon_3 = \varepsilon_x, \varepsilon_y, \gamma_{xy} \text{ --- } (3)$$

here three equations and three unknowns present by solving we can get the value of $\varepsilon_x, \varepsilon_y, \gamma_{xy}$

2. Rectangular or Square strain rosette.



$\varepsilon_1, \varepsilon_2$ & ε_3 was given.

By putting values of 'θ' in ε_n we get 3 equations

$$\varepsilon_1 = \varepsilon_x, \varepsilon_y, \gamma_{xy} \text{ --- } (1)$$
$$\varepsilon_2 = \varepsilon_x, \varepsilon_y, \gamma_{xy} \text{ --- } (2)$$
$$\varepsilon_3 = \varepsilon_x, \varepsilon_y, \gamma_{xy} \text{ --- } (3)$$

By solving these three equations, we will get $\varepsilon_x, \varepsilon_y, \gamma_{xy}$.

* principal stresses through principal ~~plane~~ Strain.

$$\boxed{\sigma_1 = \frac{E(\varepsilon_1 + \mu\varepsilon_2)}{1-\mu^2}} \qquad \boxed{\sigma_2 = \frac{E(\mu\varepsilon_1 + \varepsilon_2)}{1-\mu^2}}$$

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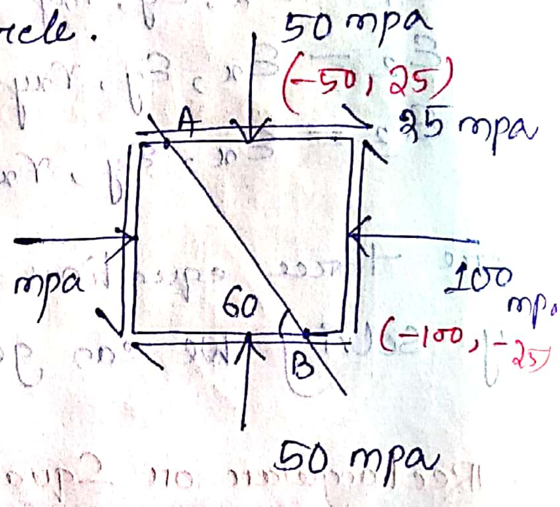
(27) A machine component is subjected to the stresses as shown in figure. Find the normal stress and shear stress on plane AB, inclined at an angle of 60° with X-axis. Also find resultant ~~Eo~~ stress on the section by Mohr's circle.



$\sigma_x = 100$ mpa (comp) $= -100$

$\sigma_y = 50$ mpa $= -50$

Vertical planes are ocer reference plane.

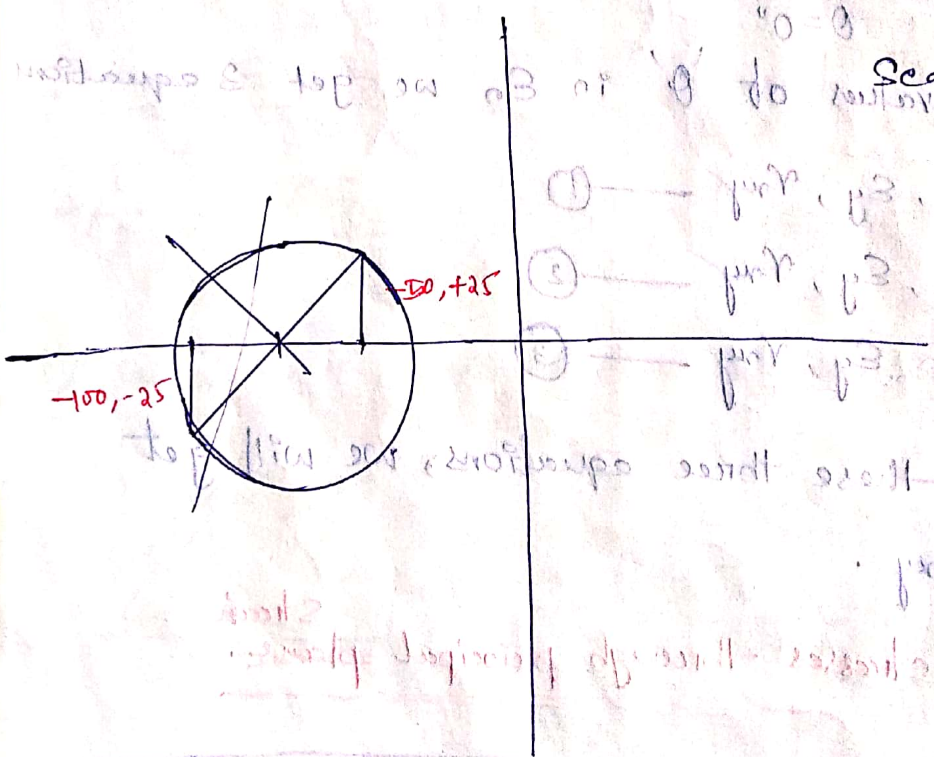
$\tau_{xy} = 25$ mpa (ccw)

$= -25$ mpa

$\theta = \cancel{60^\circ \text{ (ccw)}}$

$= 30^\circ$ (ccw)

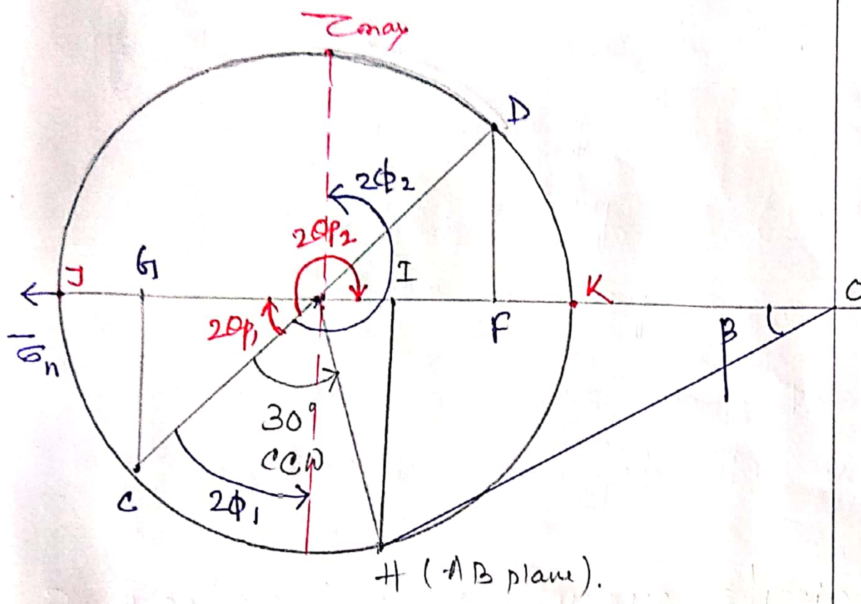
$= -30^\circ$



Scale = 1 cm = 20 mpa

100 mpa → 5 cm

1 mm = $\frac{20}{10}$



Scale -
10 cm = 100 mpa
$\boxed{1mm = 1 mpa}$

$OI = \sigma_n = 66$ mm $= 66$ mpa (Normal stress) } on
$IH = \tau = 34$ mm $= 34$ mpa (Shear stress) } oblique plane.
OH = Resultant stress $= 74$ mm
$= 74$ mpa

→ $\theta p = ?$, $\phi = ?$, σ_1 & $\sigma_2 = ?$, $\tau_{max} = ?$, $\beta = ?$

let us find these values.

$OJ = \sigma_1$ → major principal stress #
$OK = \sigma_2$ → minor principal stress.

$2\theta p_1$ → location of major principal plane.
$2\theta p_2$ → location of minor principal plane.

(28)