Dt:- 23.03.2022

LAPLACE TRANSFORM:—

$$f(t) \underset{I.L.T(L^{-1})}{\overset{L.T(L)}{\rightleftharpoons}} F(s)$$

$f(t)$ → Time domain function

$F(s)$ → Frequency domain Function
→ Complex Quantity

$S = R + I_m$

S → Complex number
R → Real Part
I_m → Imaginary part

OR $S = \sigma + j\omega$

σ → Real Part
$j\omega$ → Imaginary Part.

ω = Angular Frequency
OR = Angular Velocity

Differencial Equation is under Time domain function

$L f(t) = F(s)$

$$\boxed{L f(t) = \int_0^\infty f(t)\, e^{-st} \cdot dt}$$ → Main Formula

Function

* $1 = t^0$

$$L f(t) = L(1) = \int_0^\infty 1 e^{-st} \cdot dt = \frac{1}{-s}\left[e^{-st}\right]_0^\infty$$

$$= -\frac{1}{s}\left[e^{-s\times\infty} - e^{-s\times 0}\right]$$

$$= -\frac{1}{s}\left[e^{-\infty} - e^{-0}\right]$$

$$= -\frac{1}{s}\left[\frac{1}{e^{\infty}} - \frac{1}{e^{0}}\right]$$

$$= -\frac{1}{s}\left[\frac{1}{\infty} - \frac{1}{1}\right]$$

$$= -\frac{1}{s}\left[0-1\right] \quad = -\frac{1}{s} \times -1 \quad = \frac{1}{s}$$

$$\boxed{L(1) = \frac{1}{s}}$$

* $f(t) = t$

then, $L(t) = \int_0^{\infty} t\, e^{-st} \cdot dt$ ['t' is algebraic function]

$$= \int_0^{\infty} t\, e^{-st}\, dt - \int_0^{\infty}\left[\frac{d}{dt} t \int_0^{\infty} e^{-st}\, dt\right] \cdot dt$$

$$= \left[\frac{t}{-s} e^{-st}\right]_0^{\infty} - \frac{-1}{s}\int_0^{\infty} e^{-st} \cdot dt$$

$$= \left[-\frac{t}{s} e^{-st}\right]_0^{\infty} + \frac{1}{s} \times \frac{1}{s}$$

$$= 0 + \frac{1}{s} \times \frac{1}{s}$$

$$= \frac{1}{s^2}$$

$$\boxed{L(t) = \frac{1}{s^2}}$$

ALGEBRIC FUNCTION

$$L(t) = \frac{1}{s^2}$$

$$L(f(t)) = L(t^2) = \frac{2!}{s^3}$$

$$L(t^3) = \frac{3!}{s^4}$$

$$\vdots$$

$$L(t^n) = \frac{n!}{s^{n+1}}$$

Formula

EXPONENTIAL FUNCTION :—

$$f(t) = e^t$$

$$L(e^t) = \int_0^\infty e^t \cdot e^{-st} \cdot dt$$

$$= \int_0^\infty e^{-(s-1)t} \cdot dt$$

$$= \frac{1}{s-1}$$

$$\boxed{L(e^t) = \frac{1}{s-1}}$$

$$L(e^{at}) = \int_0^\infty e^{at} \cdot e^{-st} \cdot dt$$

$$= \int_0^\infty e^{-(s-a)t} \cdot dt$$

$$= \frac{1}{s-a}$$

$$\boxed{L(e^{at}) = \frac{1}{s-a}}$$

* $F(t) = e^{-at}$

$$L(e^{-at}) = \frac{1}{s-(-a)} = \frac{1}{s+a}$$

$$\boxed{L(e^{-at}) = \frac{1}{s+a}}$$

$$L(e^{t}) = \frac{1}{s-1}$$

$$L(e^{at}) = \frac{1}{s-a} \quad \rightarrow \text{Formula}$$

$$L(e^{-at}) = \frac{1}{s+a}$$

PRODUCT OF ALGEBRIC & EXPONENTIAL :—

$$L(te^{at}) = \int_0^\infty te^{at} \cdot e^{-st} \cdot dt$$

$$= \frac{1}{(s-a)^2}$$

$$L(te^{at}) = \frac{1}{(s-a)^2}$$

$$L(te^{-at}) = \frac{1}{(s+a)^2}$$

$$L(t^2e^{at}) = \frac{2!}{(s-a)^3} \quad \longrightarrow \text{Formula}$$

$$L(t^2e^{-at}) = \frac{2!}{(s+a)^3}$$

$\vdots$

$$L(t^n e^{at}) = \frac{n!}{(s-a)^{n+1}}$$

$$L(t^n e^{-at}) = \frac{n!}{(s+a)^{n+1}}$$

* TRIGONOMETRIC FUNCTION :—

$\theta = \omega t$

$\sin\theta = \sin\omega t$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j} \quad \& \quad \sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2j} \quad \& \quad \cos\omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2j}$$

$$\boxed{j = \text{Imaginary part}}$$

$$\cos\theta = \cos\omega t = \frac{e^{j\omega t}}{2} + \frac{e^{-j\omega t}}{2}$$

$$= \frac{1}{2}$$

$$L(\cos\omega t) = L\left(\frac{e^{j\omega t}}{2}\right) + L\left(\frac{e^{-j\omega t}}{2}\right)$$

$$= \frac{1}{2} L\left(e^{j\omega t}\right) + \frac{1}{2} L\left(e^{-j\omega t}\right)$$

$$= \frac{1}{2}\left(\frac{1}{s - j\omega t} + \frac{1}{s + j\omega t}\right)$$

$$= \frac{1}{2}\left(\frac{s + j\omega + s - j\omega}{(s - j\omega)(s + j\omega)}\right)$$

$$= \frac{1}{2} \frac{2s}{s^2 - (j\omega)^2} = \frac{s}{s^2 - j^2\omega^2}$$

$$= \frac{s}{s^2 - (-1)^2\omega^2} \quad [\because j^2 = -1]$$

$$= \frac{s}{s^2 + 1\omega^2} = \frac{s}{s^2 + \omega^2}$$

$$\boxed{L(\cos \omega t) = \frac{s}{s^2+\omega^2}}$$

* $$L(\sin \omega t) = L\left(\frac{e^{j\omega t} - e^{-j\omega t}}{2j}\right)$$

$$= \frac{1}{2j}\left[L(e^{j\omega t}) - L(e^{-j\omega t})\right]$$

$$= \frac{1}{2j}\left[\frac{1}{(s-j\omega)} - \frac{1}{(s+j\omega)}\right]$$

$$= \frac{1}{2j}\left[\frac{(s+j\omega)-(s-j\omega)}{(s-j\omega)(s+j\omega)}\right]$$

$$= \frac{1}{2j}\,\frac{(s+j\omega - s + j\omega)}{s^2+\omega^2}$$

$$= \frac{1}{\cancel{2j}}\,\frac{\cancel{2j}\omega}{s^2+\omega^2}$$

$$= \frac{\omega}{s^2+\omega^2}$$

$$\boxed{L(\sin \omega t) = \frac{\omega}{s^2+\omega^2}}$$

* PRODUCT OF TRIGONOMETRIC & EXPONENTIAL FUNCTION:

$$L(e^{at}\cos\omega t) = \frac{(s-a)}{(s-a)^2+\omega^2}$$

$$L(e^{-at}\cos\omega t) = \frac{(s+a)}{(s+a)^2+\omega^2}$$

$$L(e^{at}\sin\omega t) = \frac{\omega}{(s-a)^2+\omega^2}$$

$$L(e^{-at}\sin\omega t) = \frac{\omega}{(s+a)^2+\omega^2}$$

→ Formula

* Initial Value Theorem :—

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

* Final Value Theorem :—

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

Dt:– 25.03.2022

* Laplace Transform Of Derivative Of a Time Domain Function :—

$$f(t),\ Lf(t) = F(s)$$

$$\boxed{L\left[\frac{d}{dt}(f(t))\right] = sF(s) - f(0^+)}$$

* $2\frac{dx}{dt}(t) + 8x(t) = 10 \qquad x(0^+) = 2$

Soln Using Laplace Transform on both side,

$$2L\left(\frac{dx}{dt}(t)\right) + 8Lx(t) = 10L(1)$$

$$\Rightarrow 2\left[sX(s) - x(0^+)\right] + 8X(s) = \frac{10}{s}$$

$$\Rightarrow 2\left[sXs - 2\right] + 8X(s) = \frac{10}{s}$$

$$\Rightarrow 2sX(s) - 4 + 8X(s) = \frac{10}{s}$$

$$\Rightarrow 2s^2X(s) - 4s + 8sX(s) = 10$$

$$\Rightarrow X(s)(2s^2 + 8s) = 10 + 4s$$

$$\Rightarrow X(s) = \frac{4s+10}{2s^2+8s} = \frac{2s+5}{s^2+4s}$$

$$\Rightarrow X(s) = \frac{2s+5}{s^2+4s} = \boxed{\cancel{\frac{2s}{s^2+4}} + \cancel{\frac{5}{s^2+4s}}}$$

$$\Rightarrow X(s) = \frac{2s+5}{s(s+4)}$$

$$\Rightarrow \frac{2s+5}{s(s+4)} = \frac{A}{s} + \frac{B}{s+4}$$

$$\Rightarrow \frac{2s+5}{s(s+4)} = \frac{A(s+4) + Bs}{s(s+4)}$$

$$\Rightarrow 2s+5 = A(s+4) + Bs$$

$$\Rightarrow 2s+5 = As + 4A + Bs$$

$$\Rightarrow 2s+5 = (A+B)s + 4A$$

$A + B = 2$

$B = 2 - A$

$= 2 - 1.25$

$= 0.75$

$5 = 4A$

$\Rightarrow A = \frac{5}{4} = 1.25$

$$X(s) = \frac{1.25}{s} + \frac{0.75}{s+4}$$

$$L^{-1} X(s) = 1.25\, L^{-1}\left(\frac{1}{s}\right) + 0.75\, L^{-1}\left(\frac{1}{s+4}\right)$$

$$\boxed{x(t) = 1.25 \times 1 + 0.75\, e^{-4t}}$$

* $$F(s) = \frac{1}{s^2+4s+8}$$

$$L^{-1}\left(\frac{1}{s^2+4s+8}\right)$$

$$= L^{-1}\left(\frac{1}{s^2+4s+4+4}\right)$$

$$= L^{-1}\left(\frac{1}{(s+2)^2+4}\right)$$

$$= L^{-1}\left(\frac{1}{(s+2)^2+(2)^2}\right)$$

$$= \frac{1}{2} L^{-1}\left(\frac{2}{(s+2)^2+(2)^2}\right)$$

$$= \frac{1}{2}\left[e^{-2t} \cdot \sin 2t\right]$$

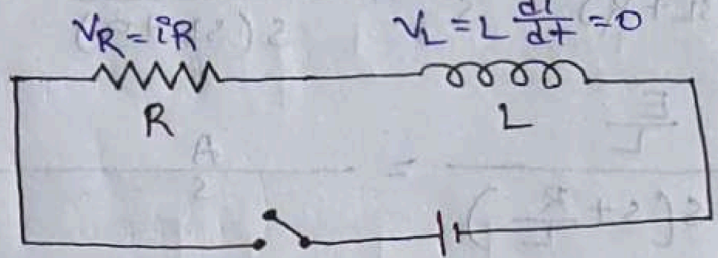
* $$F(s) = \frac{5}{s(s^2+4s+5)}$$

$$L^{-1}\left(\frac{5}{s(s^2+4s+5)}\right)$$

Dt:- 28.03.2022

Q:- A circuit having inductance 'L' & resistance 'R' is connected in series is excited by a DC source 'E'. Find out the steady state current using L.T ?

The initial value of inductor is 0.



$\frac{di}{dt}$ = current is changing w.r.t Time.

$$\frac{d(i = \text{constant})}{dt} = 0$$

$$L\frac{di}{dt} + iR = E$$

$\lim_{t \to \infty} i(t) = \lim_{s \to 0} sI(s)$ = steady state value.

$$L\frac{di}{dt} + iR = E$$

using Laplace Transform (L.T)

$$L\left[sI(s) - i(0^+)\right] + RI(s) = \frac{E}{s}$$

$$\Rightarrow I(s)\left[sL + R\right] = \frac{E}{s}$$

$$\Rightarrow I(s) = \frac{E}{s(sL+R)}$$

To get steady state current, $t \to \infty$

$$\lim_{s\to 0} sI(s) = \lim_{s\to 0} \cancel{s}\frac{E}{\cancel{s}(sL+R)}$$

$$= \frac{E}{R}$$

$$I(s) = \frac{E}{s(sL+R)} = \frac{A}{s} + \frac{B}{s(sL+R)}$$

$$\Rightarrow \frac{\frac{E}{L}}{s\left(s+\frac{R}{L}\right)} = \frac{A}{s} + \frac{B}{s\left(s+\frac{R}{L}\right)}$$

$$\Rightarrow \frac{E/L}{s\left(s+\frac{R}{L}\right)} = \frac{A\left(s+\frac{R}{L}\right)+Bs}{s\left(s+R/L\right)}$$

$$\Rightarrow \frac{E}{L} = A\left(s+\frac{R}{L}\right) + Bs$$

$$\Rightarrow 0\cdot s + \left(\frac{E}{L}\right) = (A+B)s + A\frac{R}{L}$$

$$A+B=0$$

$$\frac{E}{\cancel{L}} = \frac{AR}{\cancel{L}}$$

$$\Rightarrow A = \frac{E}{R}$$

$$A+B=0$$

$$\Rightarrow \frac{E}{R}+B=0$$

$$\Rightarrow B = -\frac{E}{R}$$

$$\Rightarrow I(s) = \frac{E}{R}\left(\frac{1}{s} - \frac{1}{s+\frac{R}{L}}\right)$$

Taking I.L.T

$$i(t) = \frac{E}{R}\left(1 - e^{-R/L\, t}\right)$$

$$\lim_{t\to\infty} i(t) = \frac{E}{R} \lim_{t\to\infty} \left(1 - e^{-\frac{R}{L}t}\right)$$

$$= \frac{E}{R} \lim_{t\to\infty} \left(1 - e^{-\frac{R}{L}\times\infty}\right)$$

$$= \frac{E}{R} \lim_{t\to\infty} \left(1 - \frac{1}{e^{\infty}}\right)$$

$$= \frac{E}{R} \lim_{t\to\infty} \left(1 - \infty\right)$$

$$= \frac{E}{R} \lim_{t\to\infty} (1-0)$$

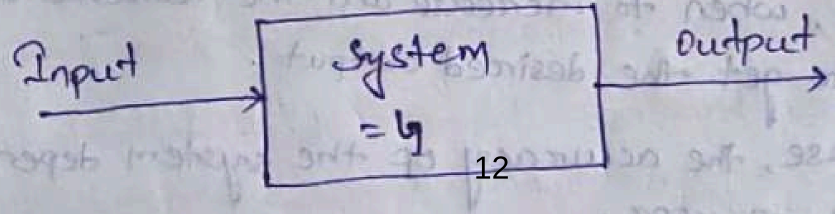
$$= \frac{E}{R} \times 1$$

$$\Rightarrow \boxed{i(t) = \frac{E}{R}}$$

* CONTROL SYSTEM :-

Def^n

Control system is a system which consist of no. of component together for a particular purpose or function and output is controlled by the input.



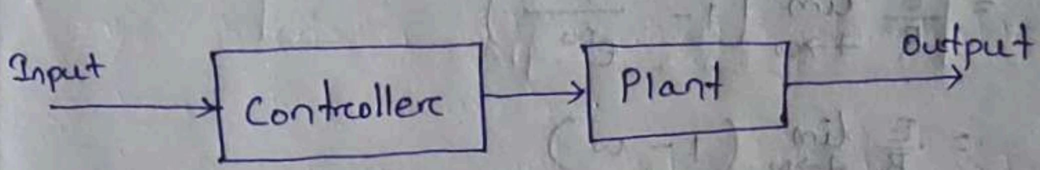
Dt:- 30.03.22

* Two types of control system,

(1)- Open Loop Control System

(2)- Closed Loop control system

(1)- Open Loop Control System :-



Plant :-

Plant is a portion of a system which convert or which transform input in to output.

Controller :-

- It is an external element to a system itself.
- It controlled the plant to get the desired output.

* An open loop control system is a system which the control action is totally independent of output of the system.
* This point is simple says that the input to the system is totally independent of the output, it means that we don't have a feedback signal that tells us how to varry the input, when to increase and the decrease input signal in order to get the desired output.
* In that case, the accuracy of the system depends on the experience of user.

Example :- (i)- Immersion Water Heater

(ii)- Toaster

Advantages :—

1- It is simple in construction and design because it doesn't have complex mechanisim.

2- It is economic because it doesn't have many elements present in it and the circuit is simple.

3- It is convenient to use when the output is difficult to measure.

4- Gain is High.

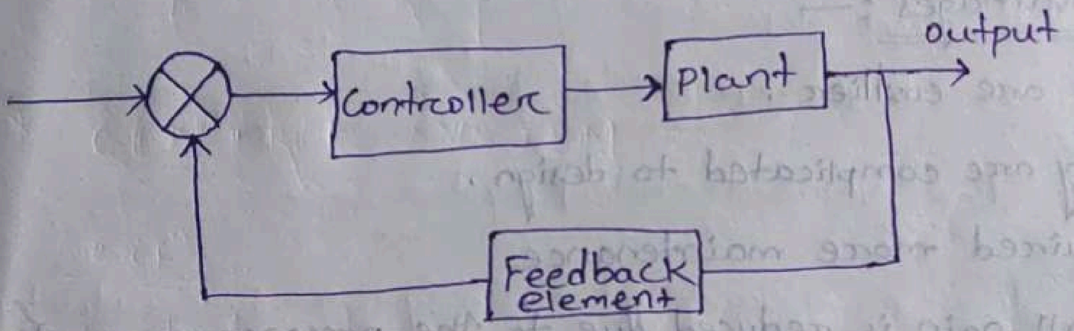
Disadvantages :—

1- The accuracy of open loop system is low.

2- The open loop system are not reliable

3- It is poorly equipped to handle disturbance.

(2)- Closed Loop control system :—



- In the closed loop system, the output is measured continuously and is feed back to the input, where the error w.r.t desired output is determine, we call the unit as the error detection unit and then after that the signal goes to the controller.
- The controller than controls amount of input, according to the desired response and then the controlled input goes to the process section and hence we get the desired output and this time desired output is mean and this way,

- The presence of feedback compensates for the disturbances for the disturbance & improves the accuracy of the system.

Example :- (i)- Air Conditioner (AC)

(ii)- Geyser

Advantages :-

1- The accuracy of closed loop system is high because there able to handle disturbance.

2- The closed loop systems are more reliable because there more accurate.

3- The sensitivity of the system may be made small to make the system more stable.

4- This system is less affected by noise.

5- Facilitates automation.

6- The bandwidth range is large.

Disadvantages :-

1- They are costlier.

2- They are complicated to design.

3- Required more maintenance.

4- Overall gain is reduced due to the presence of feedback.

5- Feedback leads to an oscillatory response.

6- Stability is the major problem and more care is needed to design a stable closed loop system.

NOTE

The feedback which is the distinguish factor between the open loop systems & closed loop systems and is abscent in the open loop and is present in the close loop system.

* TRANSFER FUNCTION :—

Def^n :—

$$\underset{\text{Excitation}}{r(t)} \xrightarrow{\;i/p\;} \boxed{\text{Control system}} \xrightarrow{\;o/p\;} \underset{\text{Response}}{c(t)}$$

- It is the ratio of L·T of output to L·T of input taking all initial condition is zero.
- This defination is only valid, when system is linear time invariant (L·T·I).
- It is denoted by symbol T.F or T.

$$T.F = T = \frac{L(c(t))}{L(r(t))}\Bigg|_{\substack{\text{initial}\\ \text{condition}}=0} \qquad \left[\because L[r(t)] = R(s) \;\&\; L[c(t)] = C(s)\right]$$

$$= \frac{C(s)}{R(s)}\Bigg|_{\text{initial condition}=0}$$

$$\boxed{C(S) = T \cdot R(S)}$$

* TYPES OF CONTROL SYSTEM :—

(1)- **Linear Control System** :—

This applies to systems made of devices which obey the superposition principle, which means roughly that the output is proportional to the input.

- The principle of superposition theorem includes two the important properties are,

(i)- Homogeneity.

(ii)- Additivity.

(2)- Non Linear Control System :—

Non-linear control system defined as a control system which does not follow the principle of homogeneity.

- In real life, all control systems are non-linear systems.

(3)- Time Invariant Control System :—

A time-invariant system has a time-dependent system function that is not a direct function of time.

- Such systems are regarded as a class of systems in the field of system analysis.
- The time-dependent system function is a function of the time-dependent input function.

(4)- Time Variant control system :—

A time-variant system is a system whose output response depends on moment of observation as well as moment of input signal application.

- In other words, a time delay or time advance of input not only shifts the output signal in time but also changes other parameters & behavior.

(5)- Linear Time Invariant Control system :-

Linear time invariant control system are a class of systems used in signals and systems that are both linear and time-invariant.

- Linear systems are systems whose outputs for a linear combination of inputs are the same as a linear combination of individual responses to those inputs.

(6)- Linear Time Variant Control system :-

Linear time variant systems are the ones whose parameters vary with time according to previously specified laws.

- Mathematically, there is a well defined dependence of the system over time and over the input parameters that change over time.

(7)- Non Linear Time Invariant control system :-

If a nonlinearity does not under-go any changes with time, the system is nonlinear time-invariant.

- In control systems, nonlinearity is typically required of the certain shape and can even be synthesized.

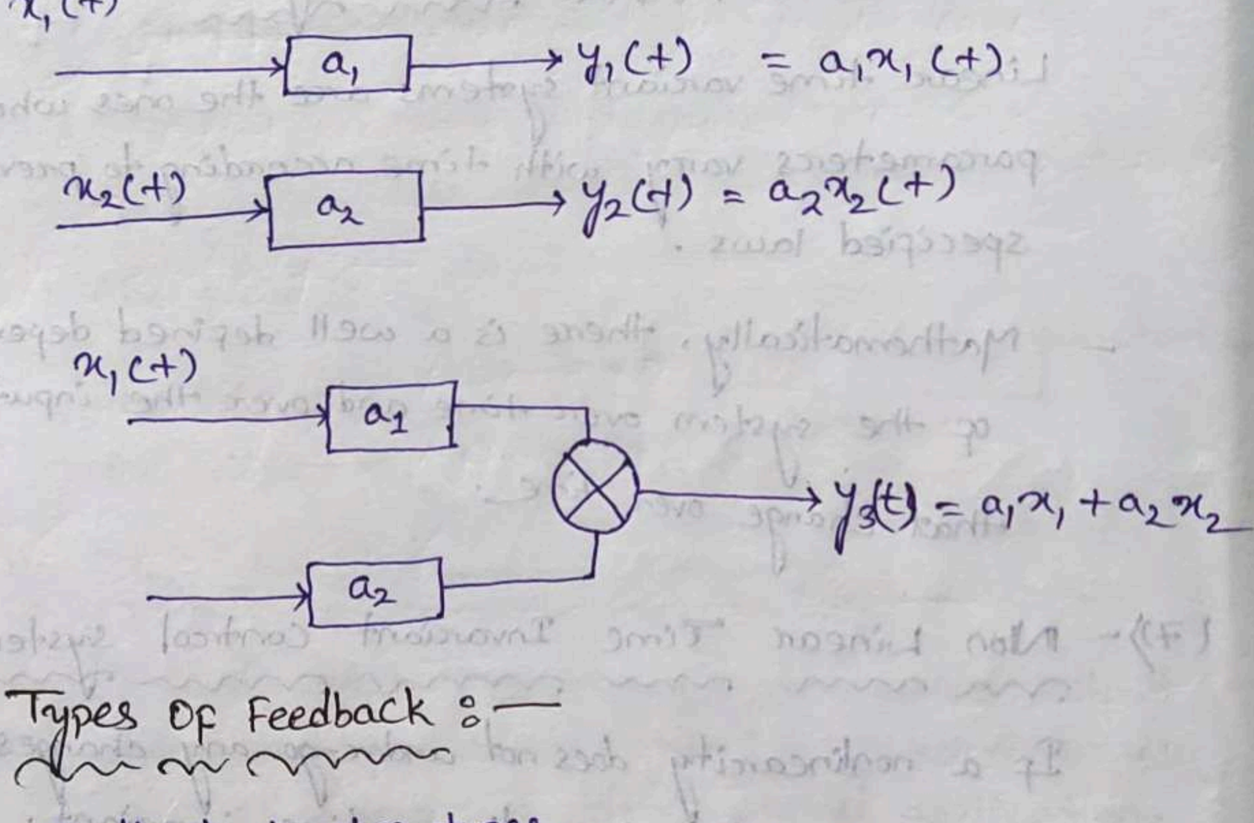
(8)〉- Non-Linear Time Variant Control system :—

If a non-linearity changes with time, the system is non-linear time variant system.

Dt:- 05.04.22

* Linear Time Invariant Control System :—

It is the control system is both linear and time invariant, then it is called linear time invariant control system.



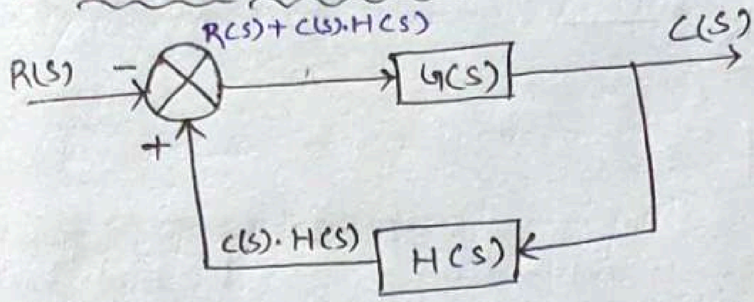
* Types Of Feedback :—

Feedback is two types,

(i)〉- Positive Feedback

(ii)〉- Negative Feedback

* Positive Feedback :—



Adding feedback element to the reference input

$$C(S) = \left[R(S) + C(S)\cdot H(S)\right]\cdot G(S)$$

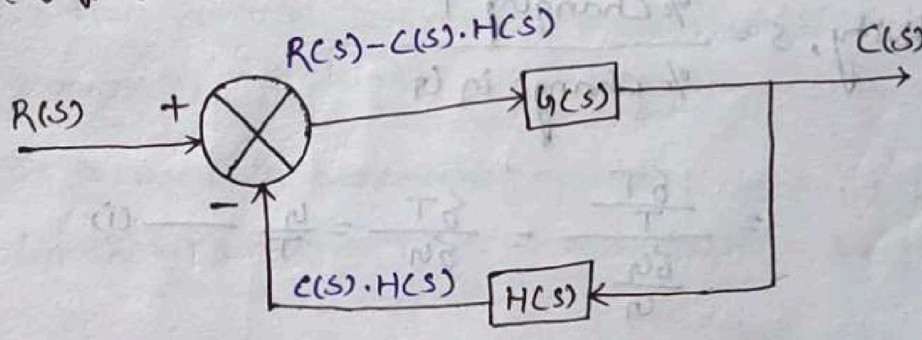
$$\Rightarrow C(S) = R(S)\cdot G(S) + C(S)\cdot H(S)\cdot G(S)$$

$$\Rightarrow C(S) - C(S)\cdot H(S)\cdot G(S) = R(S)\cdot G(S)$$

$$\Rightarrow C(S)\left[1 - G(S)\cdot H(S)\right] = R(S)\cdot G(S)$$

$$\Rightarrow \boxed{T = \frac{C(S)}{R(S)} = \frac{G(S)}{1 - G(S)\cdot H(S)}}$$

* Negative Feedback :—



$$C(S) = \left[R(S) - C(S)\cdot H(S)\right]\cdot G(S)$$

$$\Rightarrow C(S) = R(S)\cdot G(S) - C(S)\cdot H(S)\cdot G(S)$$

$$\Rightarrow C(S) + C(S)\cdot H(S)\cdot G(S) = R(S)\cdot G(S)$$

$$\Rightarrow C(S)\left[1 + G(S)\cdot H(S)\right] = R(S)\cdot G(S)$$

$$\Rightarrow \boxed{T = \frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)\cdot H(S)}}$$

* Effect Of Feedback :—

(i) - On Gain

(ii) - On sensitivity

(iii) - On stability

(i) - On Gain :—

$$T = \frac{G(s)}{1+GH} \quad (-\text{ve Feedback})$$

If, $GH > 1$ then,

$1+GH$ ~~1+(GH)~~ increase and gain is decrease.

& $1+GH$ decrease and gain increase.

(ii) - Sensitivity :—

$$\text{Sensitivity}, S = \frac{\%\text{ Changing } T}{\%\text{ Change in } G}$$

$$= \frac{\frac{\delta T}{T}}{\frac{\delta G}{G}} = \frac{\delta T}{\delta G} \cdot \frac{G}{T} \quad \text{—— (i)}$$

$$\frac{\delta T}{\delta G} = \frac{\delta}{\delta G}\left[\frac{G}{1+GH}\right] = \frac{(1+GH) - GH}{(1+GH)^2}$$

$$= \frac{1+GH-GH}{(1+GH)^2} = \frac{1}{(1+GH)^2}$$

$$S = \frac{1}{(1+GH)^2} \frac{G}{T} \quad (\text{putting value of } \frac{\delta T}{\delta G} \text{ in eqn})$$

$$= \frac{1}{(1+GH)^2} \times \frac{G}{\frac{G}{1+GH}} = \frac{1}{1+GH}$$

$$S_G^T = \frac{1}{1+GH}$$

(iii) - Stability :-

Output is more controllable the system is stable.

* SERVOMECHANISM :-

- Automatic control of any physical quantity (position, velocity, displacement) is called servomechanism.
- The word servo means controlling mechanical position or derivatives of position like velocity and acceleration.
- It is an automatic device that uses the error sensing negative feedback to correction of performance of mechanisim.
- A servo drive is a special electric amplifier used to power electric servomechanisims.
- Servomechanism uses negative feedback to control mechanical position.
- Position control servomechanism used in hydraulic and pneumatic machines to control the position.
- It is used in automatic machine tools, statellite and tracking antenna, Air craft system & navigation system.
- A servomechanism primarily consists of 3-basic components,

    (1) - Feedback system

    (2) - Error Detector

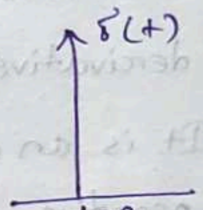
    (3) - Electric Motor

* TEST SIGNALS :—

(a)- Impulse signal :—

An unit implulse is defined as a signal which has zero value every where except at $t=0$, where its magnitude in infinite.

- It is generally called δ-function.



$$\delta(t) = \begin{cases} 1 & t=0 \\ 0 & t \neq 0 \end{cases}$$

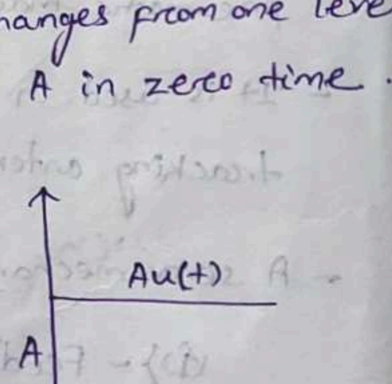
$$\delta(t) = 0, \ t \neq 0, \quad \int_{-t}^{+t} \delta(t) \cdot dt = 1 \quad \epsilon \to 0$$

$$\delta(t) = i(t) = \frac{d}{dt} u(t)$$

$$\mathcal{L}\delta(t) = (1) = R(s)$$

(b)- Step Signals :—

The step is a signal whose value changes from one level (usually zero) to another level A in zero time.



$$r(t) = Au(t)$$

$$u(t) = 1, \ t \geq 0$$

$$= 0 \quad t < 0$$

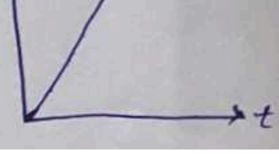
$$Lr(t) = A/s = R(s)$$

(c)- Ramp Signal :—

The ramp is a signal which starts at a value of zero & increases linearly with time.

$$r(t) = At \ ; \ t \geq 0$$

$$= 0 \ , \ t < 0$$



$L\, r(t) = A/s^2 = R(s)$

(d) :- Parabolic signal in the integral of ramp signal.

Relation

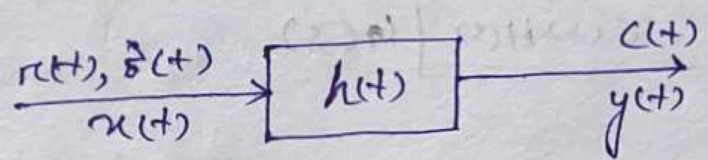
$\int \delta(t) = u(t) \Rightarrow \delta(t) = \frac{d}{dt} u(t)$

$\int u(t) = r(t) \Rightarrow u(t) = \frac{d}{dt} r(t)$

$\int r(t) = x(t) \Rightarrow r(t) = \frac{d}{dt} x(t)$

* Impulse Response of a system :—

The response of the system for an impulse is called the impulse response of the system.



$y(t) = x(t) \cdot h(t)$

$Y(W) = X(W) \cdot H(W)$

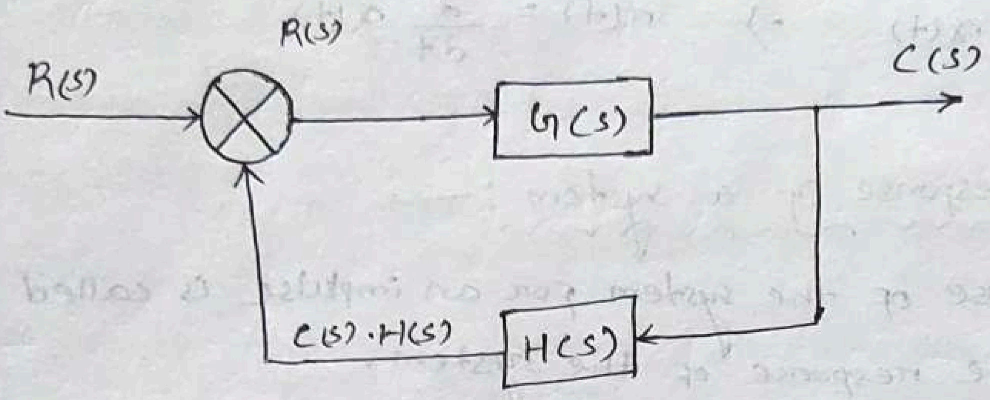
$\Rightarrow H(W) = \frac{Y(W)}{X(W)}$ (Fourier Series)

Transfer function $= H(s) = \frac{Y(s)}{X(s)}$ (Laplace Transfer)

* TRANSFER FUNCTION :-

Transfer function of a control system is the ratio of Laplace transform of output to laplace transform of input.

i.e, $\text{Transfer Function} = \left.\dfrac{\text{L.T. of Output}}{\text{L.T. of input}}\right|_{\text{initial condition}=0}$



$$= \left[R(s) - C(s).H(s) \right] G(s)$$

$$T.F = \frac{C(s)}{R(s)}$$

$$C(s) = \left[R(s) - C(s).H(s) \right] G(s)$$

$$\Rightarrow C(s) = R(s).G(s) - C(s).H(s).G(s)$$

$$\Rightarrow C(s) + C(s).H(s).G(s) = R(s).G(s)$$

$$\Rightarrow C(s) \left[1 + H(s).G(s) \right] = R(s).G(s)$$

$$\Rightarrow \boxed{T.F = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s).H(s)}}$$

* Impulse Response of a System :—

The response of the system for an impulse is called as impulse response of the system.

- Generally this can be represented with $h(t)$ or $h(n)$.

$$x(t) \longrightarrow \boxed{h(t)} \xrightarrow{y(t)}$$

$$y(t) = x(t) \cdot h(t)$$

$$Y(W) = X(W) \cdot H(W)$$

$$H(W) = \frac{Y(W)}{X(W)}$$

$$F^{-1}(H(W)) = h(t)$$

$$R(s) \longrightarrow \boxed{H(s)} \xrightarrow{C(s)}$$

$$C(s) = R(s) \cdot H(s)$$

$$\Rightarrow H(s) = \frac{C(s)}{R(s)}$$

$$\Rightarrow L^{-1} H(s) = h(t)$$

Problem :- $H(s) = \dfrac{s^2+2s-4}{s^2-4} = 1 + \dfrac{2s}{s^2-4}$

$$\delta(t) \xrightarrow{L.T} 1$$

$$L^{-1}(1) = \delta(t)$$

$$H(s) = 1 + \frac{2s}{(s+2)(s-2)}$$

$$= 1 + \frac{s+s+2-2}{(s+2)(s-2)} = 1 + \frac{(s+2)+(s-2)}{(s+2)(s-2)}$$

$$H(s) = 1 + \frac{1}{s+2} + \frac{1}{s-2}$$

$$L^{-1}H(s) = L^{-1}(1) + L^{-1}\left(\frac{1}{s+2}\right) + L^{-1}\left(\frac{1}{s-2}\right)$$

$$h(t) = \delta(t) + \left[e^{-2t} + e^{2t}\right] u(t)$$

* Uses of Transfer Function :—

Transfer functions are used,

(i)- Analysis of SISO filters in the fixed of signal processing.

(ii)- Communication Theory

(iii)- Control Theory

(iv) Used exclusively LTI system.

* Advantages of Transfer Function :—

(a)- If transfer function of a system is known, the response of the system to any input can be determined easily.

(b)- A transfer function is the mathematical model & give gain of the system.

(c)- Since it involves L.T. the terms are simple algebric expression and no differential terms are present.

(d)- Poles & zeros of the system can be determined from the knowledge of Transfer Function.

* Disadvantage of Transfer Function :—

(a)- T.F doesn't take onto accounts the initial condition of the system.

(b)- T.F can be defined only for linear system.

(c)- No inform'n can be drawn ~~absent~~ about the physical structure of the system.

(d)- It is applicable for SISO system.

(e)- To find frequency response, we need to shift the system into fourier domain.

* Properties Of Transfer Function :—

(1)- Mathematical model expressing the differential equation that relates the output & the input of the system.

(2)- Independent of the magnitude nature of input.

(3)- Doesnot provide any information about the physicals structure of the system.

(4)- Transfer function of physically different systems can be identical.

(5)- If the transfer function is known the output response can be studied for various input to understand the nature of the system.

* Poles & Zeros Of A Transfer Function :—

$$G(s) = \frac{Y(s)}{X(s)} = \frac{C(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_n}$$

Transfer function of a control system can be written in above form,

$$G(s) = \frac{(s-z_0)(s-z_1)\cdots(s-z_m)}{(s-P_0)(s-P_1)\cdots(s-P_n)}$$

Roots of numerator polynomial → zeros

$Z_0, Z_1, Z_2, \ldots\ldots Z_m$; $Z_i ; i = 0, 1, 2, \ldots m$

Roots of denominator polynomial → Poles

$P_0, P_1, P_2 \ldots\ldots P_n \quad P_j , j = 0, 1, 2, \ldots\ldots n$.

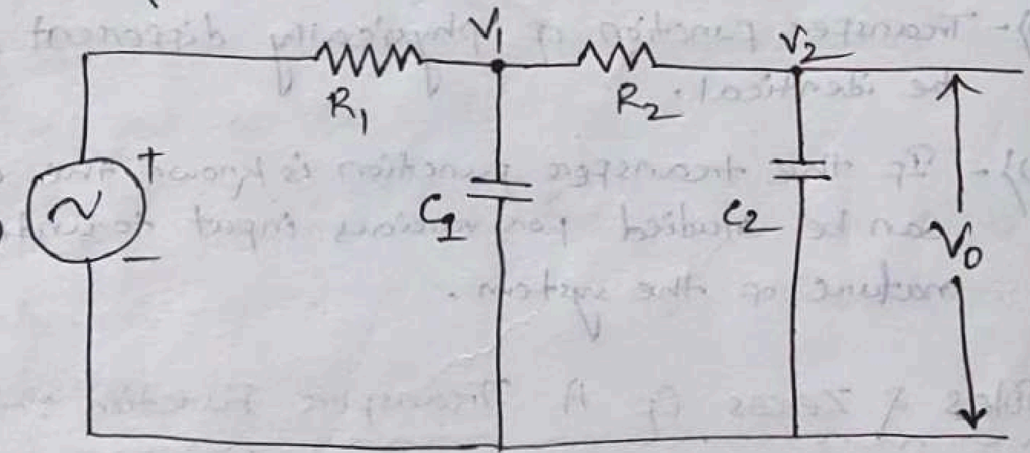
* Characteresticks Eqⁿ of Transfer Function :—

$$G(s) = \frac{C(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \ldots\ldots + b_m}{a_0 s^n + a_1 s^{n-1} + \ldots\ldots + a_n}$$

If equate denominator polynomial to zero of transfer function, it is called the characteristics eqⁿ of the transfer function.

Problem

(1)- Obtain the transfer function of the elecric network shown below,



Applying nodal analysis node-1,

$$\frac{V_1(t) - e(t)}{R_1} + C_1 \frac{dV_1(t)}{dt} + \frac{V_1(t) - V_2(t)}{R_2} = 0$$

Taking Laplace Transform on both sides,

$$\frac{V_1(S)-E(S)}{R_1}+C_1SV_1(S)+\frac{V_1(S)-V_2(S)}{R_2}=0$$

$$\Rightarrow V_1(S)\left(\frac{1}{R_1}+C_1S+\frac{1}{R_2}\right)=\frac{E(S)}{R_1}+\frac{V_2(S)}{R_2} \quad \text{——(1)}$$

Applying nodal analysis for node-2

$$\frac{V_2(t)-V_1(t)}{R_2}+C_2\frac{dV_2(t)}{dt}=0$$

Taking laplace Transform both side,

$$\frac{V_2(S)-V_1(S)}{R_2}+SC_2V_2(S)=0$$

$$\Rightarrow \frac{V_1(S)}{R_2}=\frac{V_2(S)}{R_2}+SC_2V_2(S)$$

$$\Rightarrow \frac{V_1(S)}{R_2}=\left(\frac{1}{R_2}+SC_2\right)V_2(S)$$

$$\Rightarrow V_1(S)=\left(\frac{R_2}{R_2}+SR_2C_2\right)V_2(S)$$

$$V_1(S)=(1+SC_2R_2)V_2(S) \quad \text{——(2)}$$

Putting value of $V_1(S)$ from eqn (2) to eqn (1),

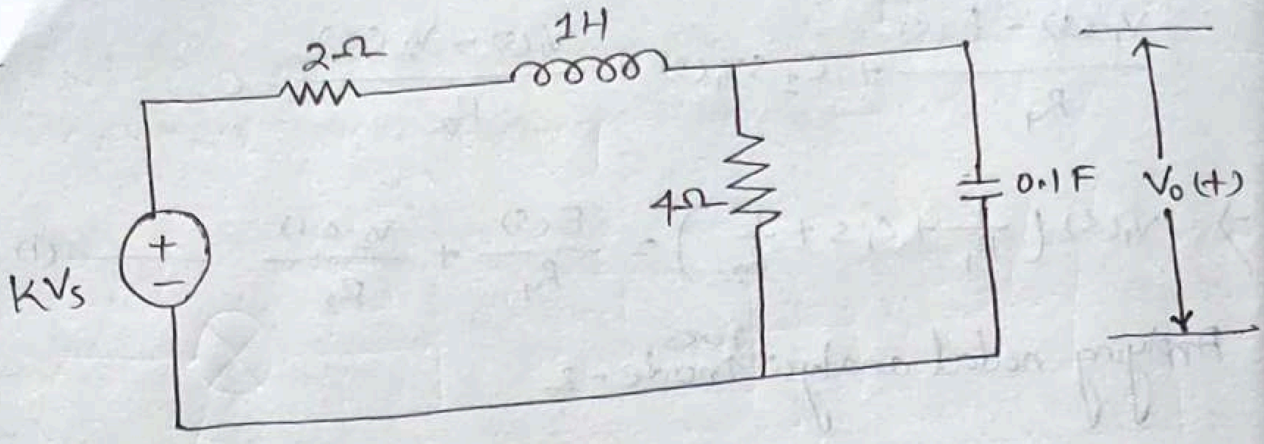
$$(1+R_2C_2S)V_2(S)\left(\frac{1}{R_1}+C_1S+\frac{1}{R_2}\right)=\frac{E(S)}{R_1}+\frac{V_2(S)}{R_2}$$

$$\Rightarrow V_2(S)\left[(1+C_2R_2S)\left(\frac{1}{R_1}+C_1S+\frac{1}{R_2}\right)-\frac{1}{R_2}\right]=\frac{E(S)}{R_1}$$

$$\Rightarrow \frac{V_2(S)}{E(S)}=\frac{1}{R_1\left[(1+C_2R_2S)\left(\frac{1}{R_1}+C_1S+\frac{1}{R_2}\right)-\frac{1}{R_2}\right]}$$

$$=\frac{R_2}{\left[(1+C_2R_2S)(R_2+C_1SR_1R_2+R_1)-R_1\right]}$$

(2) -



$L \to sL = s$

$C \to \frac{1}{sC} = \frac{1}{\frac{s}{10}} = \frac{10}{s}$

Applying nodal analysis to node,

$$\frac{KV_s - V_0}{2+s} = \frac{V_0}{4} + \frac{V_0}{\frac{10}{s}}$$

$$\Rightarrow \frac{K}{2+s} V_s = V_0 \left(\frac{1}{4} + \frac{s}{10} + \frac{1}{2+s} \right)$$

$$\Rightarrow \frac{K}{2+s} V_s = V_0 \left[\frac{5(s+2) + 2s(s+2) + 20}{20(2+s)} \right]$$

$$\Rightarrow \frac{K}{(2+s)} V_s = V_0 \left[\frac{5s + 10 + 2s^2 + 4s + 20}{20(2+s)} \right]$$

$$\Rightarrow \frac{V_0}{V_s} = \frac{2s^2 + 9s + 30}{20K}$$

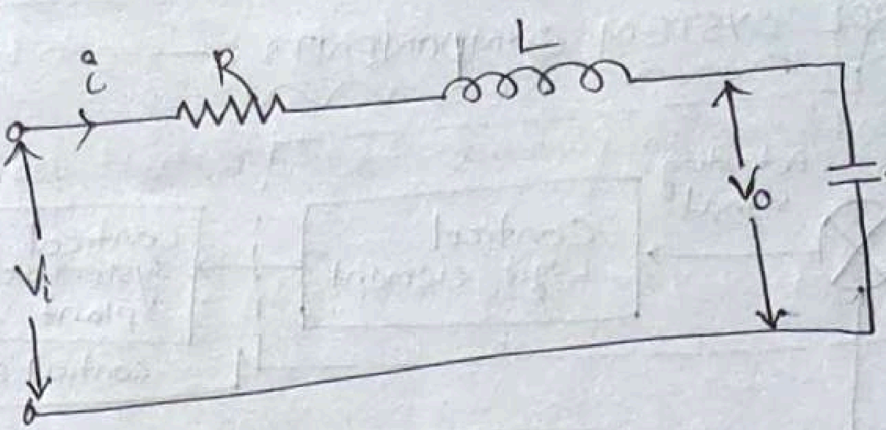
(3) - Mathematical Model of control system :-

(a) - Differential Eq^n Model.

(b) - Transfer function Model.

(c) - State space Model.

Taking an example,



$$V_i = Ri + L\frac{di}{dt} + V_0$$

But $i = C\frac{dV_0}{dt}$

$$V_i = RC\frac{dV_0}{dt} + L\frac{d^2V_0}{dt^2} + V_0$$

$$V_i = L\frac{d^2V_0}{dt^2} + RC\frac{dV_0}{dt} + V_0 \quad \text{——— (i)}$$

(Differential Eq^n Model.

Transfer Function Model

$$V_i = L\frac{d^2V_0(t)}{dt^2} + RC\frac{dV_0}{dt} + V_0$$

Taking L.T on both sides,

$$V_i(s) = L s^2 V_0(s) + RC s V_0(s) + V_0(s)$$

$$\Rightarrow V_i(s) = \left(Ls^2 + RCs + 1\right) V_0(s)$$

$$\Rightarrow \frac{V_0(s)}{V_i(s)} = \left(\frac{1}{Ls^2 + RCs + 1}\right)$$

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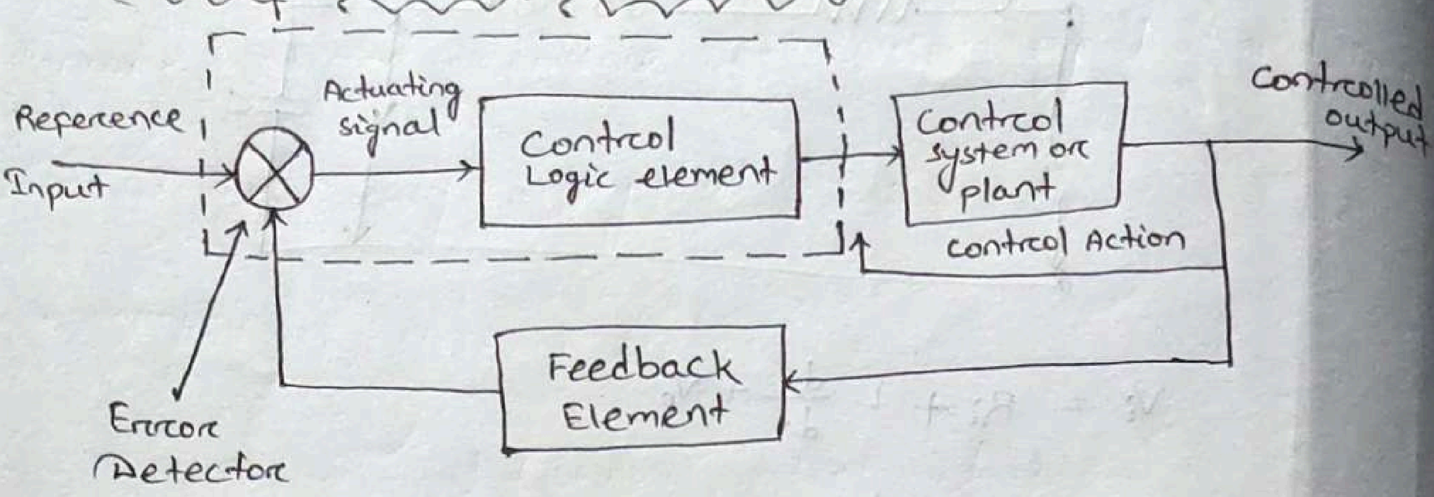
* CONTROL SYSTEM COMPONENTS :—



Block diagram representation of a closed loop control system

Basic Elements

(a)- Feedback Element

(b)- Controller

(c)- Control system

(a)- Feedback Elements :—

The feedback element is used to ~~fe~~ feedback the output signal to the error detector for comparison with the input.

(b)- Controller :—

It consist of the error detector and the control logic elements.

* Error Detector :—

Receives the measured signal (feedback/output) and compare it with reference input & determines the error signal also known as actuating signal.

- Actuating signal → Low power level → Not sufficient to operate the plant.
- Need for an intermediate device bet^n the error dector & plant.
- It can manipulate the actuating signal as desired.
- Manipulation in the form of amplification or generation of desired function.
- Control system components → Manipulation is done by them (components).

* CONTROL SYSTEM COMPONENTS :—

- Employed or introduced in a system to perform a specific function or purpose in the system.
- Components can be mechanical, electrical, hydraulic, pneumatic, thermal or any other type.
- Modern control systems uses sensors and encoders as control system components.

Following devices - (1) - Potentiometers.
Sensors
(2) - AC servomotors.
(3) - D.C servomotors.
(4) - stepper motors.
(5) - Tacho Generators.

* POTENTIOMETERS :—

A potentiometer is an electromechanical transducer which converts the mechanical energy (displacement) (either linear or rotational) into electrical energy (voltage).

- It is also called as error detecting device, because it is used as an error detector in control system.

Error - Find the difference betⁿ output signal & input signal.

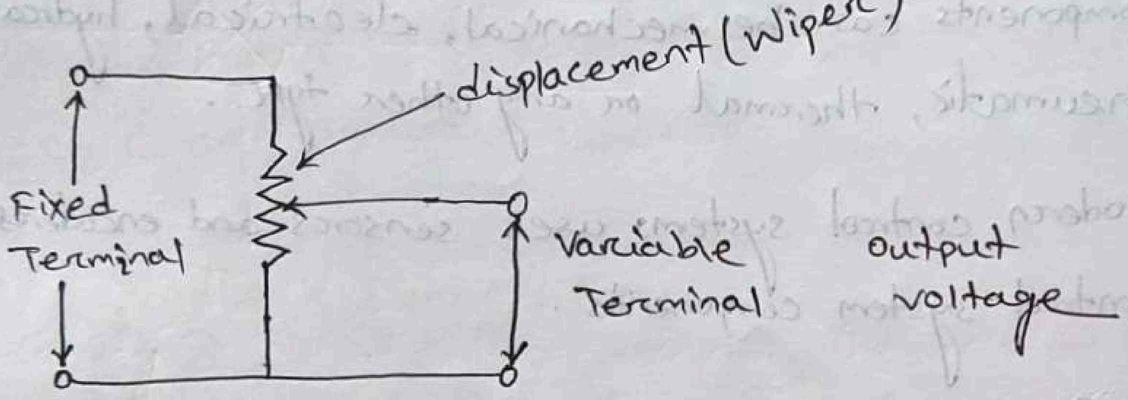
→ Inexpensive & easy to apply and use.

* TYPES OF POTENTIOMETERS :—

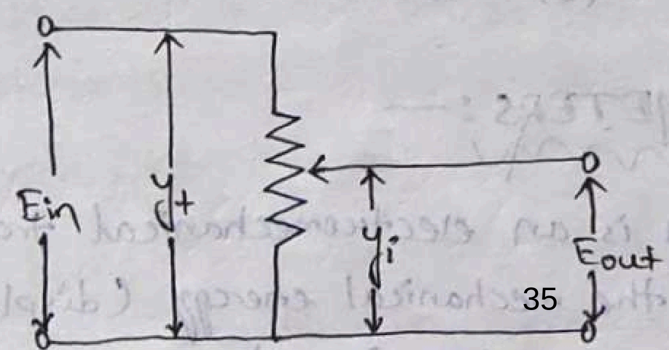
(1) - Translational (linear) potentiometers

(2) - Rotational Potentiometers

(1) - Translational Potentiometers :—



When voltages is applied across the fixed terminals of the potentiometer, the output voltage which is measured across the variable terminal is proportional to input displacement.

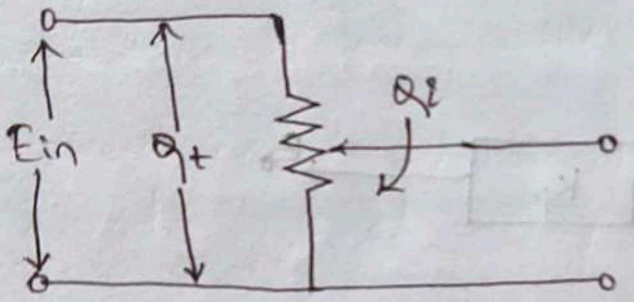


- Under ideal conditions the ratio between output voltage and input voltage is given,

$$\boxed{\frac{E_{out}}{E_{in}} = \frac{y_i}{y_t}}$$

y_i = Displacement from zero position.
y_t = Total length of the translational potentiometer.

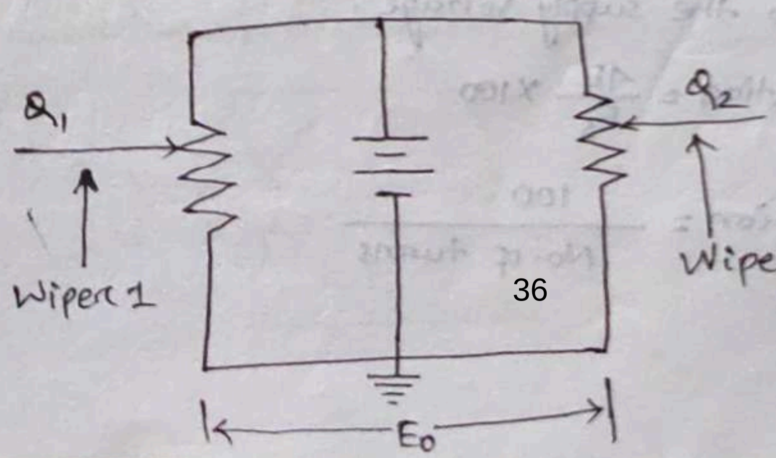
(2)- Rotational Potentiometer :—



$$\boxed{\frac{E_{out}}{E_{in}} = \frac{Q_i}{Q_t}}$$

Q_i = Input angular displacement
Q_t = Total length of the wiper

→ Potentiometer can be used as a error detector to compare the position of two remotely located shafts.



Double ended Arrangement

Circuit for potentiometer as an error detector.

- The output voltage E_0 is given by,

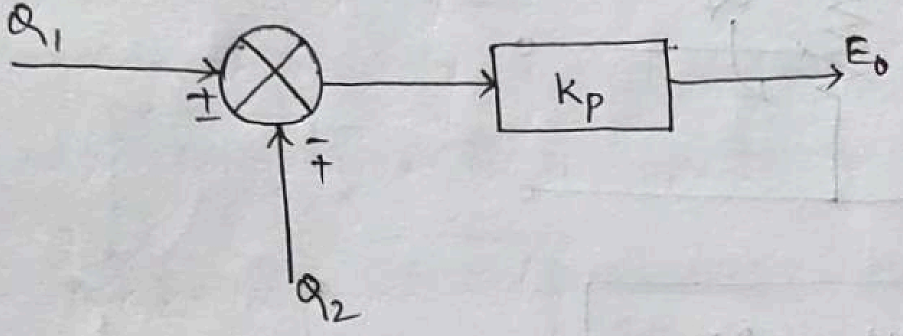
$$E_0 = K_p(\theta_1 - \theta_2)$$

normally D.C. voltage

$$K_p = \frac{\text{The ratio of the input of excitation}}{\text{Total angle of rotation}}$$

$$= \frac{E_i}{\theta_T}$$

$$E_0 = \frac{E_i}{\theta_T}(\theta_1 - \theta_2)$$



- Polarity of output voltage describe the relation position of the shaft.

- In case of A.C, the phase distance will find the relative positions of the shafts.

Resolution of Potentiometers :-

It is defined as the ratio of change in the output voltage in one step to the supply voltage.

$$\%\ \text{Resolution} = \frac{\Delta E}{E_s} \times 100$$

$$\Rightarrow \%\ \text{Resolution} = \frac{100}{\text{No. of turns}}$$

* TACHOGENERATORS :—

→ It is an electromechanical device which produces an output voltage that is proportional to its shaft speed.

Mechanical signal. → Electrical signal.

→ It works on the principle of induction motor.

→ Two types of tachogenerators,

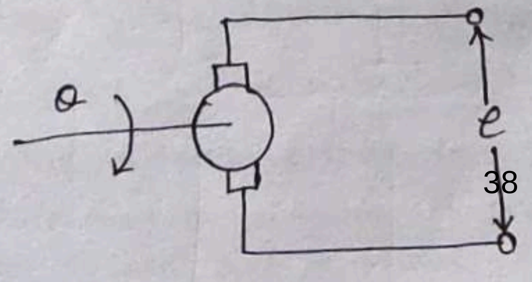
(a) – A.C. Tachogenerator.

(b) – D.C. Tachogenerator.

* D.C. Tachogenerator :—

The D.C. tachogenerator resembles a small motor in that it compaires of a stator with a permanent magnetic field a rotating armature circuit & a commutator & brush assembly.

- The rotor is connected to the shaft.
- The output voltage is proportional to the angular velocity of the shaft.
- Polarcity of the output voltage depends on the direction of the rotation of the shaft.



- Dynamics of D.C tachogenerator can be represented by the equation,

$$e(t) = K_t \frac{d\theta(t)}{dt} = K_t \dot{\theta}$$

e = Output voltage (volts)

θ = Rotor Displacement (radians)

K_t = Sensitivity of the tachogenerators. (volts per rad/sec)

Problem :-

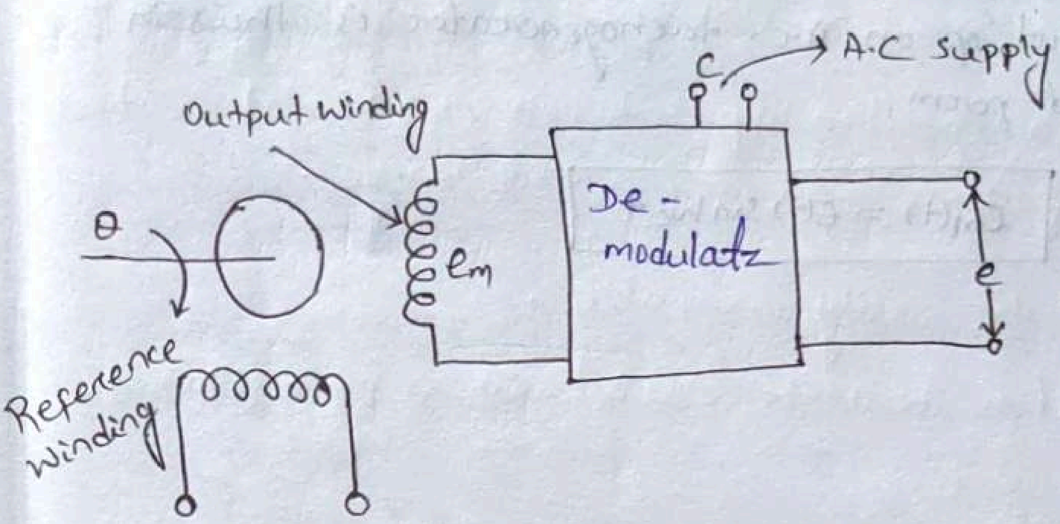
(a) High-frequency ripple generated by the commutator-brush assembly.

(b) - Maintenance is difficult.

* A.C. Tachogenerator :-

- Resembles two phase induction motor.
- Comprises of
  - (a) - two stator windings (Arranged in space quadrature)
  - (b) - Rotor is not conductively connected to an external circuit.
- A sinusoidal voltage is applied to the excitation winding (reference).

$e_r = E_r \sin \omega_c t$



- When the rotor is stationary ($\theta = 0$), no emf is induced in the output winding and therefore the output voltage is zero.
- When the motor rotates, a voltage at the reference frequency W_c is induced.
- The magnitude of the output voltage is proportional to the rotational speed.

$$e \propto W$$

$$e = K_1 W$$

$$e = K_1 \frac{d\theta}{dt}$$

Taking laplace transform we get,

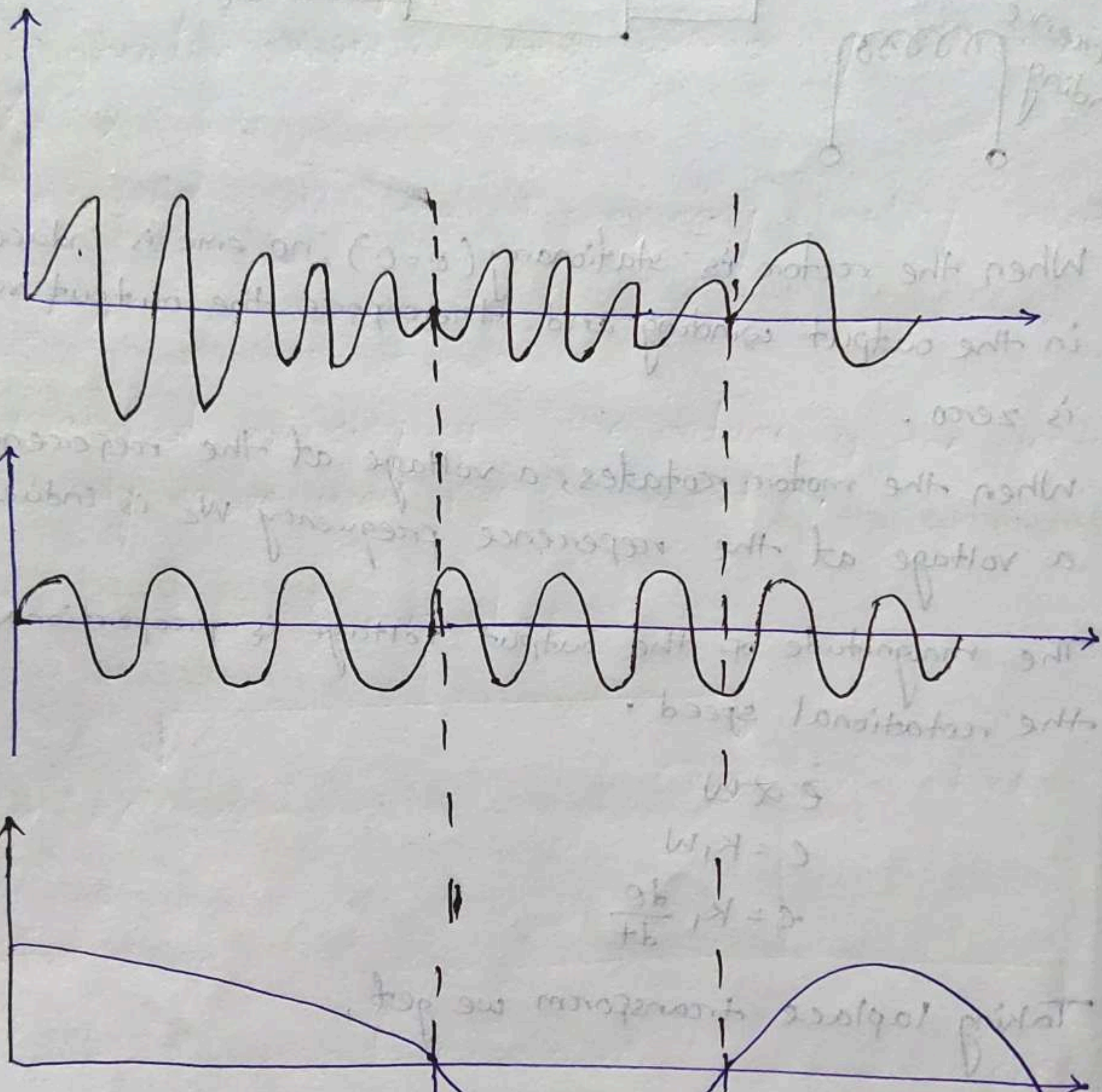
$$\underbrace{E(s)}_{\text{output}} = K_1 \underbrace{s\theta(s)}_{\text{Input}}$$

$$\boxed{\frac{E(s)}{\theta(s)} = sK_1 = T.F}$$ Transfer Function

- A change in the direction of the shaft rotation causes a 180° phase shift is the output voltage.
- When the output voltage is in phase with the reference voltage, the direction of rotation is said to be positive & when the output voltage is 180° out of phase with the reference voltage the direction of rotation is said to be negative.

The output of an A.C. tachogenerator is thus in modulated form,

$$\boxed{e_m(t) = e(t)\sin W_c t}$$



* SERVOMOTORS :—

Servomechanism

↓

Servo + mechanism

↓

Servant
(slave)

- Servomechanism is defined as a closed loop control system in which a small input power controls a larger output power in a strictly proportinate manner.
- The controlled variable (output variable) is some mechanical variable like position, velocity or acceleration.
- Servo systems are used in automatic control system which works on the error signals.
- The error signals are used to drive the motor used in servo systems.
- Motor used in servo systems are called servomotors.
- Servomotors usually drive a final control element. These motors are coupled to the output shaft i.e. load through gear train for power matching.
- These motors are used to convert electrical signal applied into the angular velocity or movement of shaft.

Requirment of a Good Servomotor :—

- Inertial of the rotor should be as low as possible.
- Its responsible of the servomotor should be as fast as possible.
- For quickly changing error signal if must react with good response. (This is achieved by keeping the torque weight high).

- It should have linear torque-speed characteristics.
- ~~Lea~~ Linear relationship between electrical control signal & rotor speed over a wide range.
- It should be easily reversible.
- It operation should be stable within any oscillation in overshapt.
- The motor should withstand frequency starting operation.

* TYPES OF SERVOMOTORS :—

Classified depending upon the nature of the electric supply to be used for its operation.

- Servomotors
  - A.C. Servomotors
  - D.C Servomotors
    - Armature controlled D.C. Servomotors
    - Field controlled D.C servomotors.

* D.C. Servomotor :—

- More or less same as normal DC motor.
- D.C servomotor behaves like a mechanical transducer which convert D.C voltage into mechanical signal i.e, angular displacement.
- All D.C servomotors are essentially separately excited type. This entire • torque-speed characteristics.

- The control of d.c servomotor can be from field side and armature side.

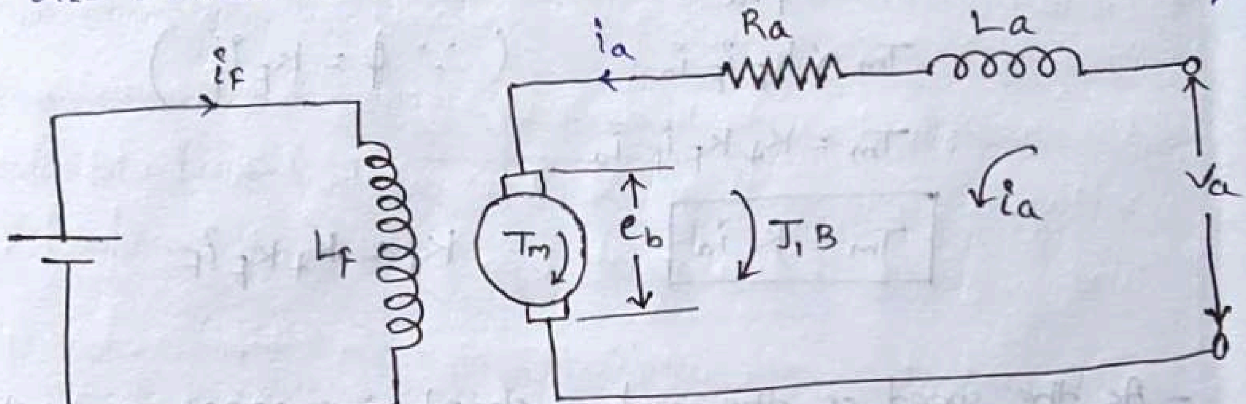
- Depending upon this D.C servomotor are classified,

(a) - Field controlled D.C servomotor.

(b) - Armature controlled D.C servomotor.

* Armature controlled D.C servomotor :-

- In this motor, the field current is held constant and armature current is varied to control the torque.



- Circuit diagram for armature controlled D.C servomotor.

Let, R_a = Armature resistance

L_a = Armature inductance

i_a = Armature current

V_a = Armature voltage

W_m = Angular velocity.

e_b = Back emf

J = Moment of inertia

i_f = Field current

L_f = Field inductance

Now, air flux ϕ is proportional to field current,

$$\phi \propto i_f$$
$$\phi = K_f i_f$$

i_f = constant armature current i_a produces the torque T_m (due to application of V_a) which in turn produces angular shaft in the motor shaft.

- Produced torque T_m is proportional to flux ϕ & armature current I_a.

$$T_m \propto \phi i_a$$
$$T_m \propto K_f i_f i_a \qquad (\because \phi = K_f i_f)$$
$$T_m = K_t K_f i_f i_a$$
$$\boxed{T_m = K_1 i_a} \qquad K_1 = K_t K_f i_f$$

- As the speed of the motor shaft increases a back emf (e_b) is induced in the armature circuit.

- The back emf (e_b) is proportional to the speed of the motor shaft & direction of the back emf is opposite to the armature input voltage V_a.

$$e_b \propto w$$
$$e_b = K_b \frac{d\theta}{dt} \quad \text{——— (1)}$$

Applying KVL in the armature circuit, we get

$$V_a = i_a R_c + L_a \frac{di_a}{dt} + e_b \quad \text{——— (2)}$$

The load-torque equation is given by

$$J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} = T_m = K_1 i_a \quad \text{——— (3)}$$

Taking laplace transform of eq^n (1), & (2) & (3)

$$E_b(S) = K_b S\theta(S) \quad \text{------} (4)$$

$$Va(s) = Ia(s)Ra + SLaIa(S) + E_b(S)$$

$$\Rightarrow Va(s) - E_b(s) = (Ra + SLa)\, Ia(S) \quad \text{------} (5)$$

$$S^2 J\theta(s) + SB\theta(S) = Tm(S) = K_1 Ia(S)$$

$$[JS^2 + BS]\,\theta(S) = Tm(S) = K_1 Ia(S) \quad \text{------} (6)$$

From eq^n (5), ~~in eq^n (6) we get~~

$$Ia(S) = \frac{Va(S) - E_b(S)}{Ra + SLa}$$

$$Ia(S) = \frac{Va(S) - K_b S\theta(S)}{Ra + SLa}$$

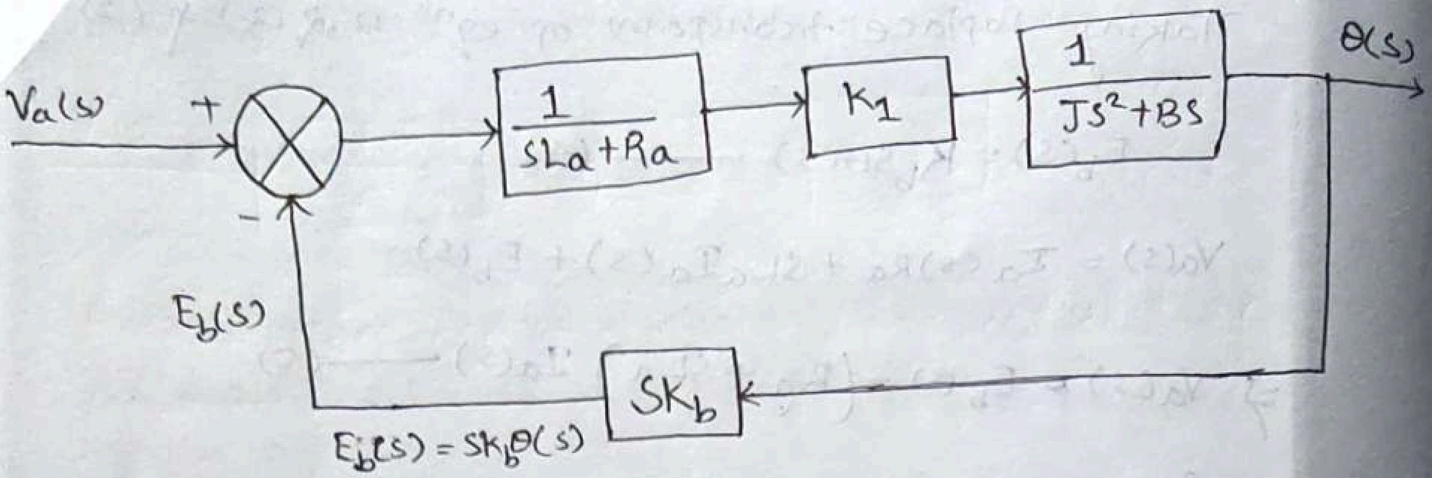
substitute, Ia(s) in eq^n (6) we get,

$$[JS^2 + BS]\,\theta(s) = K_1 \frac{[Va(s) - K_b S\theta(s)]}{SLa + Ra}$$

$$\Rightarrow \theta(s) \cdot \left[\frac{(JS^2 + BS)(SLa + Ra)}{K_1} + K_b S \right] = Va(S)$$

$$\Rightarrow \frac{\theta(S)}{Va(S)} = \frac{K_1}{(JS^2 + BS)(SLa + Ra) + K_1 K_b S}$$

$$\boxed{\frac{\theta(S)}{Va(S)} = \frac{K_1}{1 + \dfrac{(JS^2 + BS)(SLa + Ra) K_1 K_b S}{(JS^2 + BS)(SLa + Ra)}}}$$ Transfer function.

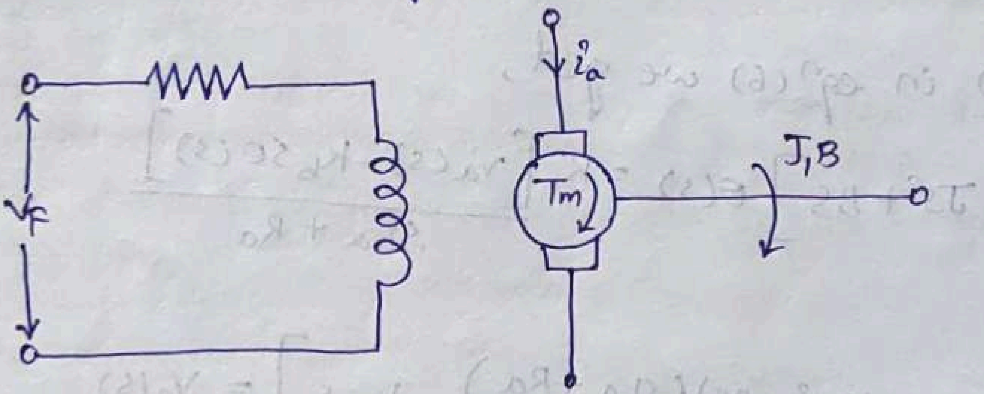


Block diagram Armature control D.C servomotor.

* Field Controlled D.C servomotor :—

In the field controlled D.C. servomotor variable input voltage (field voltage V_f) is applied to field winding and armature current (I_a) is kept constant.

- The output is the angular shift in the motor shaft.



Let, R_f = Field resistance

L_f = Field inductance

i_f = Field current

V_f = Variable field voltage

θ = Angular displacement of the motor shaft.

T_m = Torque developed by the motor.

B = co-efficient of viscous friction.

J = Moment of inertia.

I_a = Armature current is kept constant & the motor shaft is controlled by the input voltage V_f.

- As the input voltage is applied a current i_f flows which produces flux in the machine,

↓

Torque at the motor shaft.

↓

Angular shift in the motor shaft.

- $T_m \propto i_f$

$T_m = K_f i_f$ —— (1)

K_f = motor torque constant.

Field eq^n, $V_f = i_f R_f + L_f \frac{di_f}{dt}$ —— (2)

Torque eq^n,

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T_m \quad \text{—— (3)}$$

Taking laplace transform of eq^n, we get

$T_m(s) = K_f I_f(s)$

$(sL_f + R_f) I_f(s) = V_f(s)$ —— (4)

$(s^2 J + Bs)\,\theta(s) = T_m(s) = K_f I_f(s)$ —— (5)

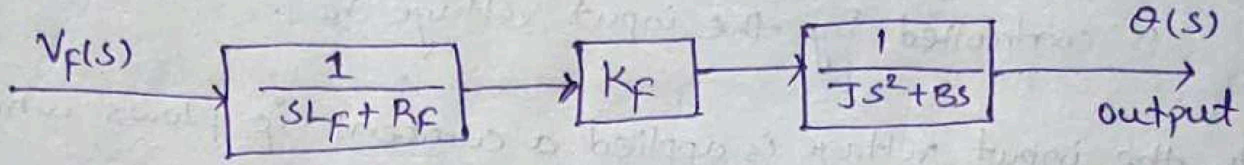
$(s^2 J + Bs)\,\theta(s) = K_f \frac{V_f(s)}{(sL_f + R_f)}$

$$\Rightarrow \boxed{\frac{\theta(s)}{V_f(s)} = \frac{K_f}{(sL_f + R_f)(s^2 J + Bs)}}$$

Transfer Function

$\theta(s)$ = Angular shaft

$V_f(s)$ = Field voltage



Block diagram representation of Field controlled D.C servomotor.

Armature Controlled D.C. servomotor	Field Controlled D.C. Servomotor
(1)- Better performance in expected due to closed loop.	(1)- Poor performance due to open loop structure.
(2)- The inductance of the armature circuit is small & hence Ta in negligible. This reduces the order of the system eqn also.	(2)- The inductance of the field circuit is not negligible, It offers significant T.F.
(3)- Speed of response of the motor to changing current is fast.	(3)- speed of response of the motor to changing current is slow.
(4)- The damping due to the armature resistance & the motor friction and extra damping in produced. Increased damping improves the transisent response of the system	(4)- Improve damping is not possible.

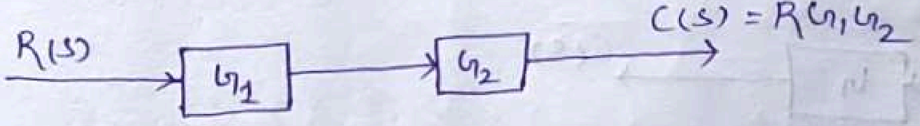
Dt:- 11.04.22

* RULES FOR BLOCK DIAGRAM REDUCTION :—

- Block diagram is the picturial representation of components of control engineering or control system.
- Complex system having more member of block diagram in complex form.
- To get the transfer function we need to simplify the block diagram of the control system.
- To reduce the block diagram - we should follow some rules mentioned below,

Rules :—

(1) Blocks are in series or cascade :—



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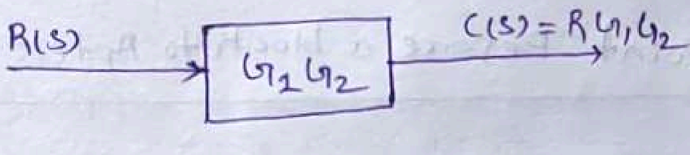
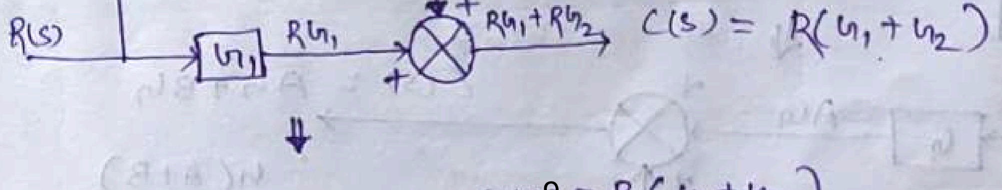


(2) - Blocks are in Parallel :—



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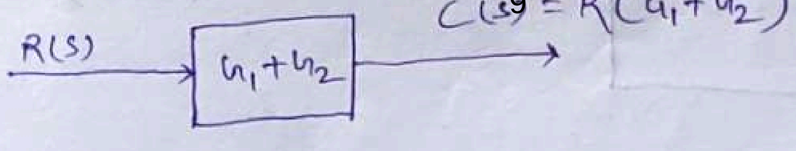
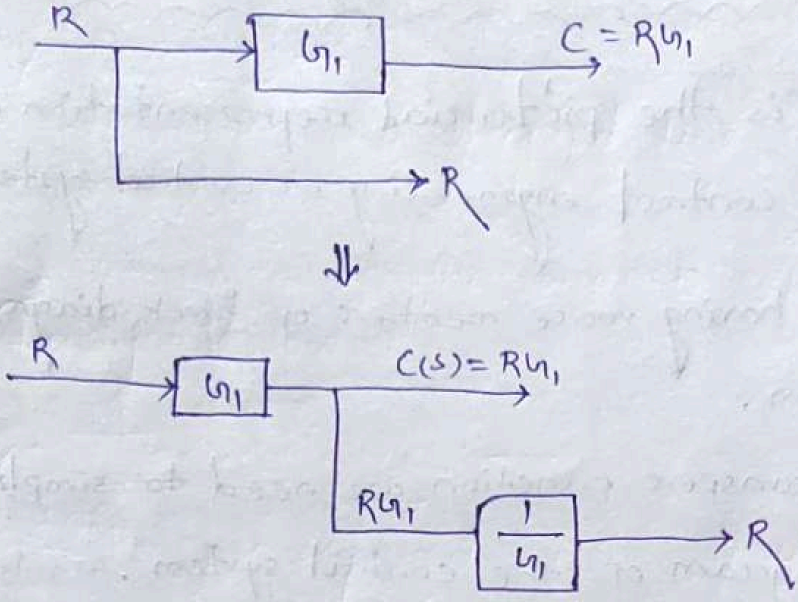
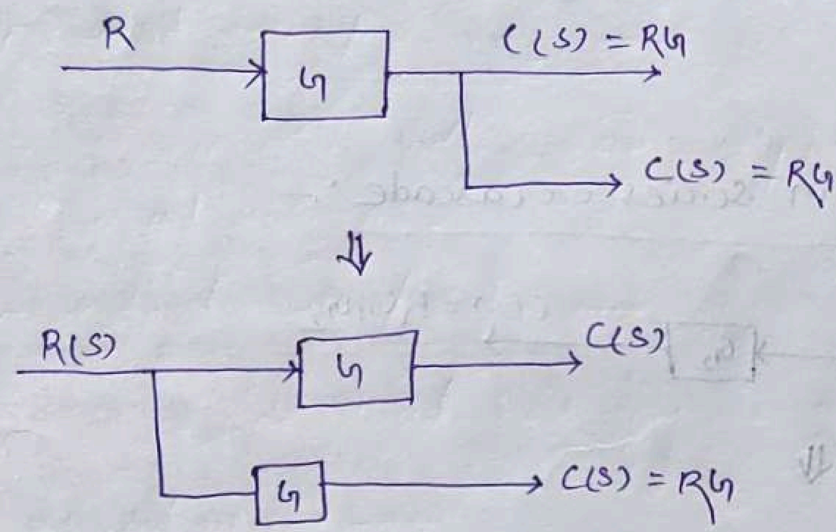


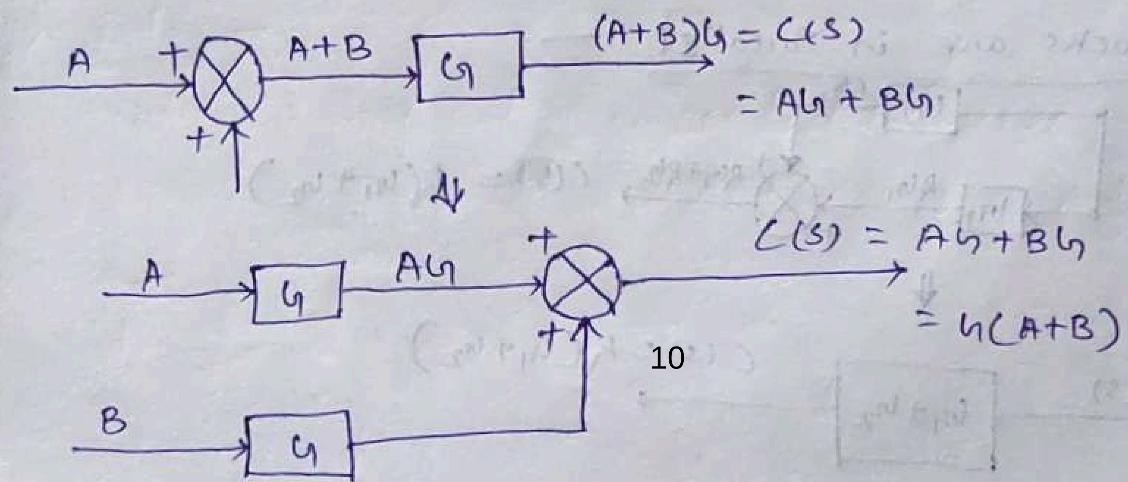
(3) - Moving Take Off Point Before a Block to After a Block :-



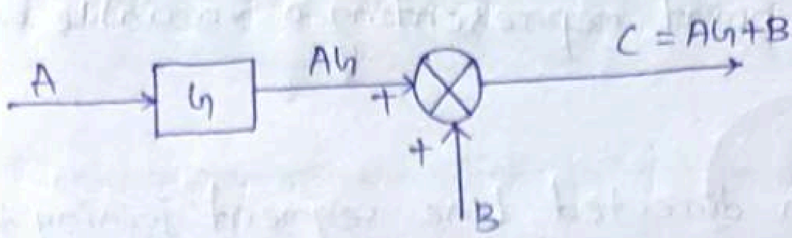
(4) - Moving take off Point After a block to Before a Block :-



(5) - Moving Summing Point Before a block to After a Block :-



(6)- Moving summing point After a Block to Before a Block:-



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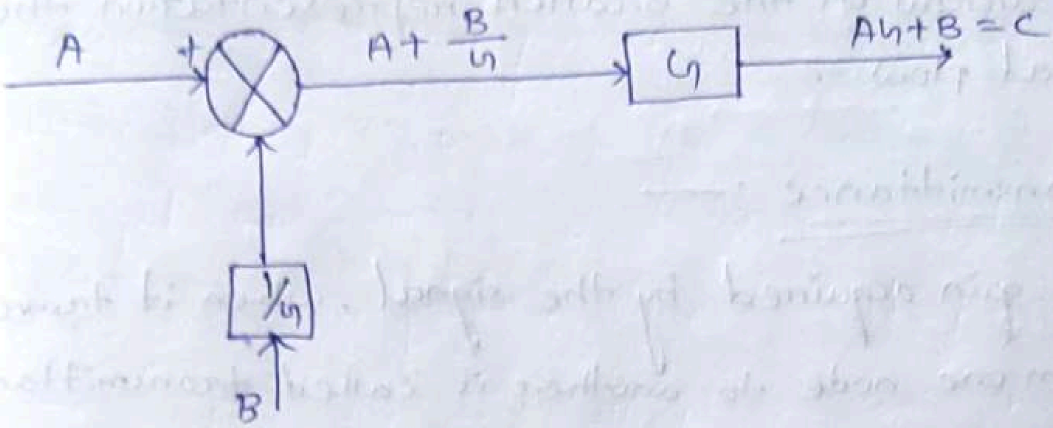
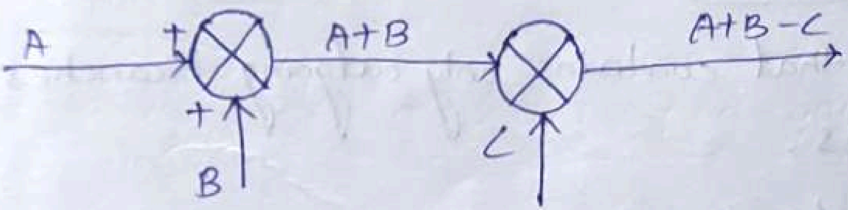
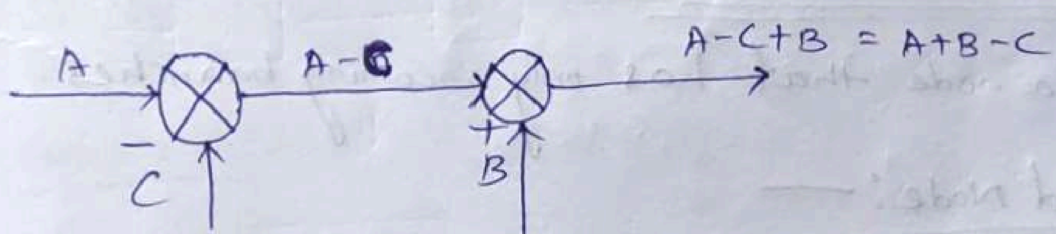


(7)- Interchange of Summing point :-





* SIGNAL FLOW GRAPH :-

Signal flow graph is a graphical representation of components of control system engg. or control system.

- In SFG there are so many elements they are,

(a) - Node :—

A node is a point representing a variable or signal.

(b) - Branch :—

A branch is a directed line segment joining two nodes.

- The arrow on the branch representation the direction signal flow.

(c) - Transmittance :—

The gain acquired by the signal, when it travels from one node to another is called transmittance.

- It is either real or complex.

(d) - Input Nodes / source :—

It is a node that contains only outgoing branches or called sources.

(e) - Output Node / sink :—

It a node that has only incoming branches.

(f) - Mixed Node :—

It is a node that has both incoming & outgoing branches.

(g) - Path :—

A path is a traversal of connected branches in the direction of arrows. The path shouldn't cross a node more than one.

path is two types,

(i) - Open path & (ii) - Closed Path.

(h)- Forward Path or Forward Path Gain :—

Path from input to output is called forward path.

- Product of all branch node is called forward path gain.

(i)- Loop Gain :—

> The product of all branch gain of path is called path gain

Product of all gains of loop is called loop gain.

(j)- Non-touching Loop :—

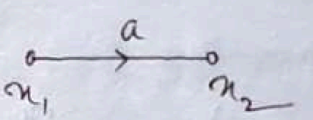
If loop don't have common node.

(k)- Individual Loop :—

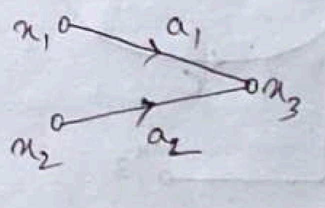
Starting from a node & after moving certain distance on the graph & come to the same node & not touching node more than one.

V.I.M.P * SIGNAL FLOW GRAPH ALGEBRA :—

Rule-1

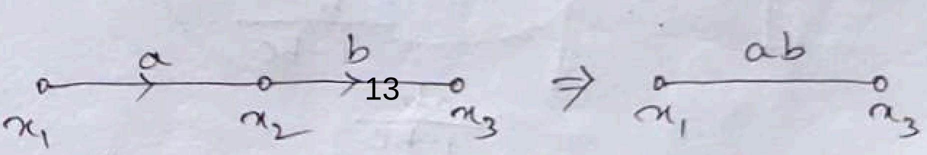
(i)- 

$x_2 = ax_1$

(ii)- 

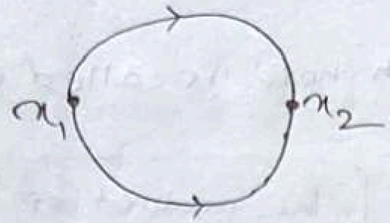
$x_3 = a_1x_1 + a_2x_2$

Rule-2

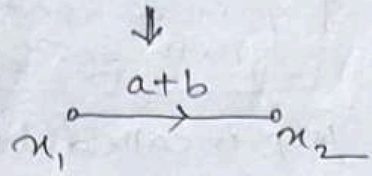


$x_2 = ax_1$, $x_3 = bx_2 = b(ax_1) = abx_1$

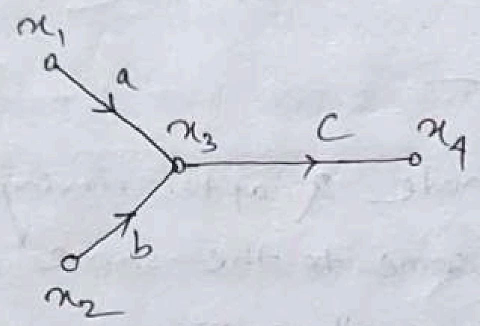
Rule-3

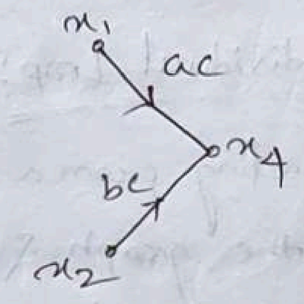


$$x_2 = ax_1 + bx_1$$
$$x_2 = (a+b)x_1$$



Rule-4

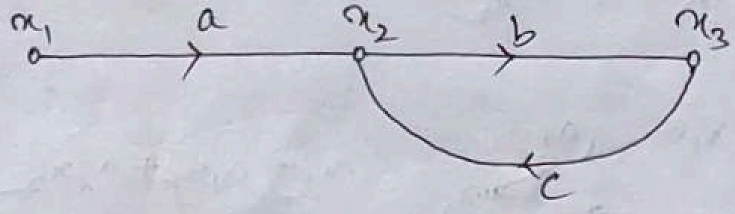


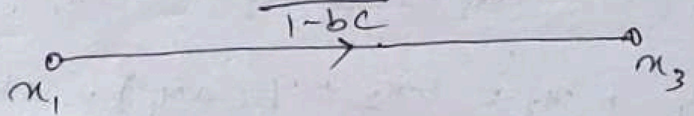


$= acx_1 + bcx_2$

$$x_3 = ax_1 + bx_2$$
$$x_4 = cx_3 = c(ax_1 + bx_2)$$
$$x_4 = acx_1 + bcx_2$$

Rule-5





* Procedure For Converting Block Diagram Into Signal Flow Graph :—

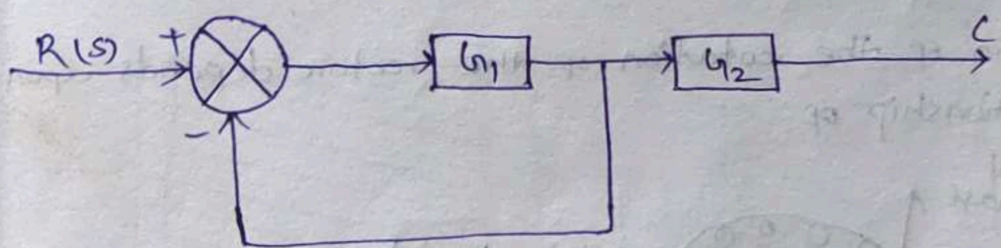
(1)- Assume node at the input, output, at every summing point at every branch point & in between cascaded blocks take off.

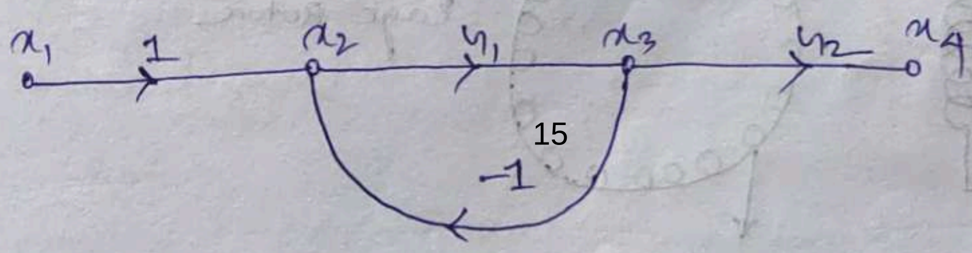
(2)- Draw the nodes separately as small circle & number the circle in the order 1, 2, 3 .

(3)- From the block diagram find the gain betn each node in main forward path & connect all the corresponding circle by straight lines & mark the ~~(4)~~ gain on the node.

(4)- Draw the field forward path ~~contro~~ (other than main forward path) betn various nodes & mark the gain of the field forward path along with sign.

(5)- Draw the field forward path betn various nodes & mark the gain of the feedback paths & sign.





Dt:- 13.04.22

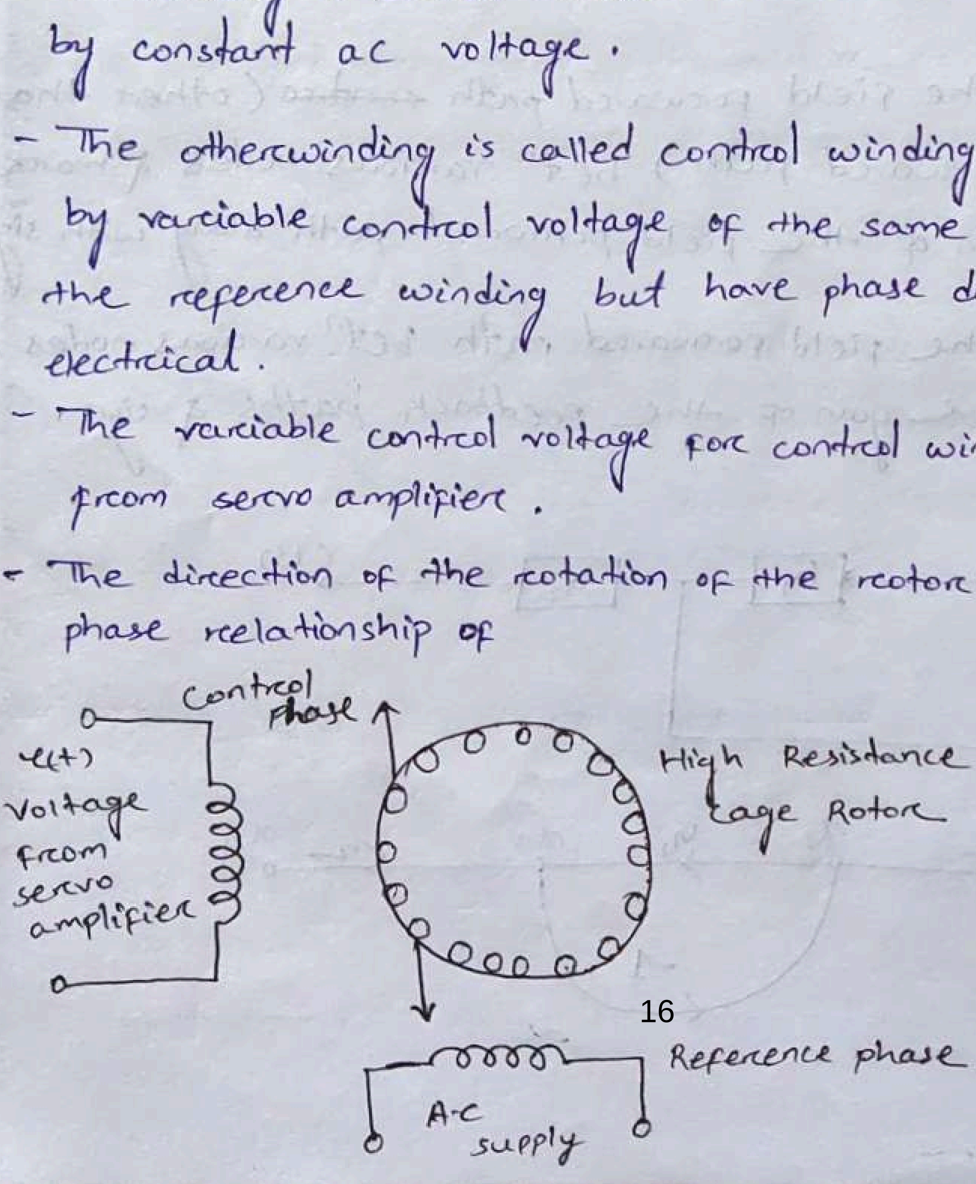
* A.C SERVOMOTOR :—

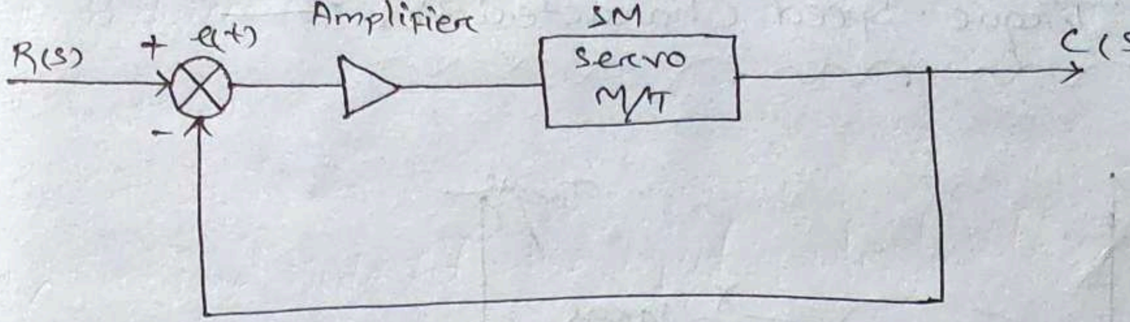
A.C. Servomotor

→ 2-ϕ A.C. servomotor

→ 3-ϕ A.C. Servomotor.

- These motor has two parts,
  (i) - stator (ii) Rotor
- These are two phase induction motor, stator has two distributed windings.
- These windings are displaced each other by 90° electrical.
- One winding is called main or reference winding and is excited by constant ac voltage.
- The other winding is called control winding and is excited by variable control voltage of the same frequency as the reference winding but have phase displacement 90° electrical.
- The variable control voltage for control winding is obtained from servo amplifier.
- The direction of the rotation of the rotor depends upon phase relationship of



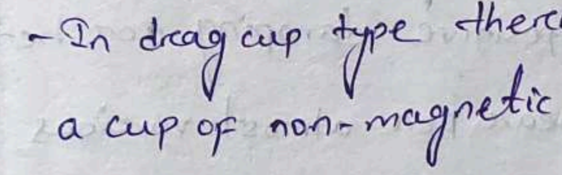


Two types of rotor,

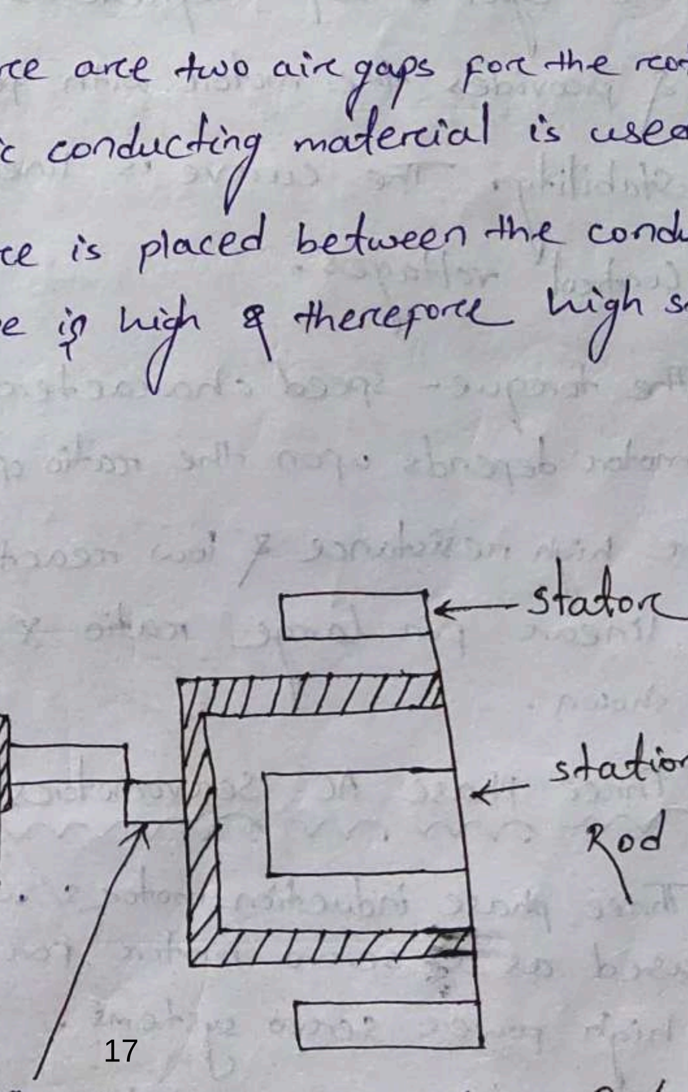
(a) - Squirrel Cage Rotor

(b) - Drag ~~cut~~ cup type rotor.

- The squirrel cage rotor have large length & small diameter. So its resistance is very high the air gap of squirrel cage is kept small.
- In drag cup type there are two air gaps for the rotor a cup of non-magnetic conducting material is used.
- A stationary iron core is placed between the conducting cup to drag cup type is high & therefore high starting torque.

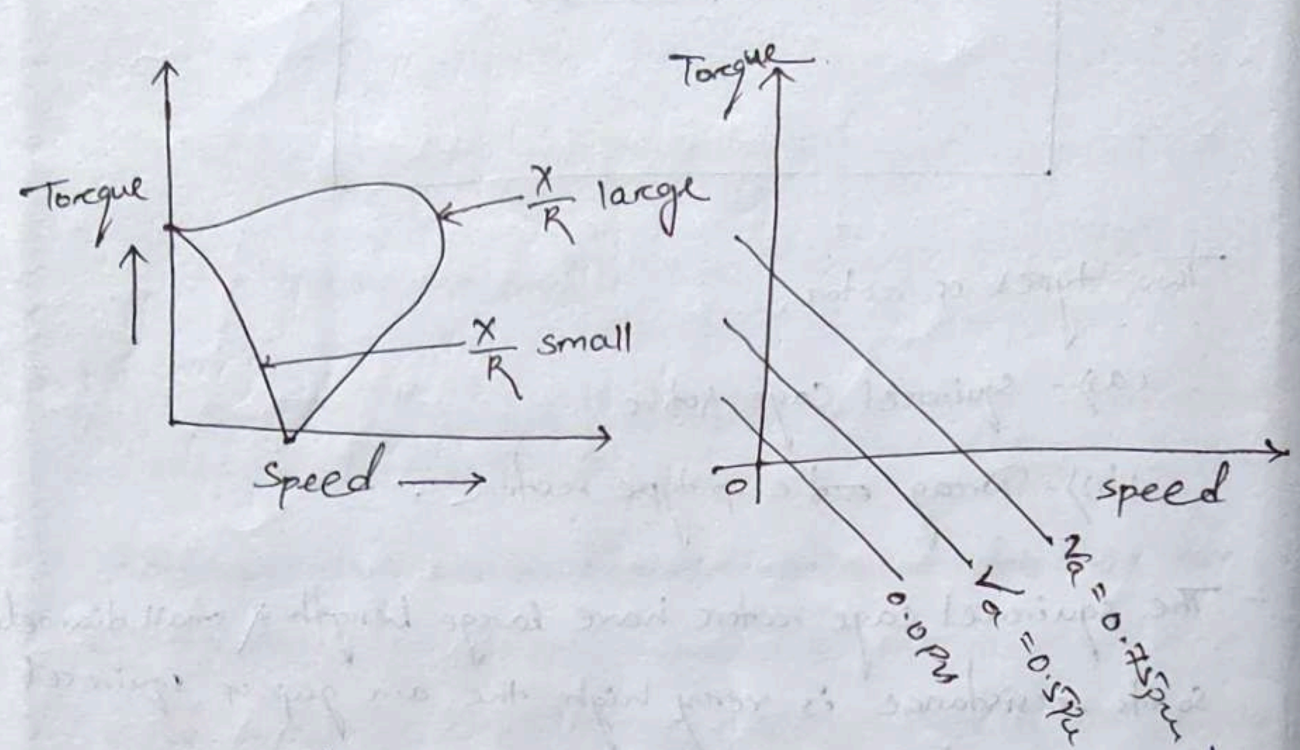


squirrel Cage Rotor



Drag cup type Rotor

* Torque - Speed Characteristics :—



- The negative slope represents a high rotor resistance & provides the motor with positive damping for better stability. The curve is linear for almost various control voltages.
- The torque-speed characteristics of two phase induction motor depends upon the ratio of reactance to resistance.
- For high resistance & low reactance, the characteristics is linear for large ratio X to R it becomes non-linear as shown.

* Three Phase AC Servomotors :—

Three phase induction motors with the voltage control are used as a servo motor for the applications in the high power servo systems.

- A 3-ϕ squirrel cage induction motor is a highly non-linear coupled ckt device. It is used as a linear decoupled machine by using a control method known as a vector control or field oriented control.

- The current in this type of machine is controlled in such a way that the torque & flux are decoupled. The decoupling result in high speed & high torque response.

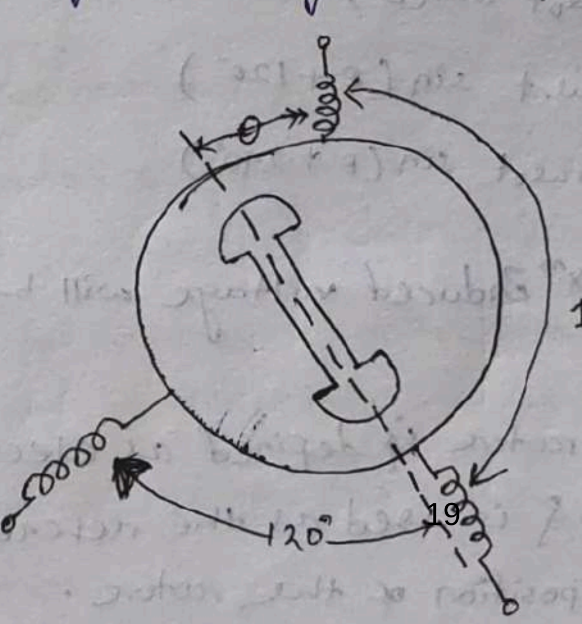
* SYNCHROS :—

A synchros is an electromagnetic transducer which converts the angular position of a shaft into electrical signal.

- synchros are used as detectors & encoders.

* Synchro Transmitter :—

- The construction is similar to 3-ϕ alternators.
- stator is made of laminated silicon steel & carries three phase star connected winding.
- Rotor is a rotating part, dump-bell shaped magnet with single winding.



- A single phase AC voltage is applied to rotor through slip ring.
- Let the voltage applied be,

$$e_r = E_r \sin\omega t$$

- Magnetizing current will flow in the rotor coil. It produces sinusoidal varying flux & distributed in air gap, bcoz of transformer action voltage get induced in all stator coil which is proportional to cosine of angle betⁿ stator & rotor coil axis.
- Now, consider rotor of synchro transmitter is at an angle θ, the voltage in each stator coil with respect to neutral are,

$$E_{an} = KE_r \sin\omega t \cos\theta$$

$$E_{bn} = KE_r \sin\omega t \cos(\theta + 120°)$$

$$E_{cn} = KE_r \sin\omega t \cos(\theta + 240°)$$

Magnitude of stator terminal voltages are,

$$E_{cb} = E_{cn} - E_{bn}$$

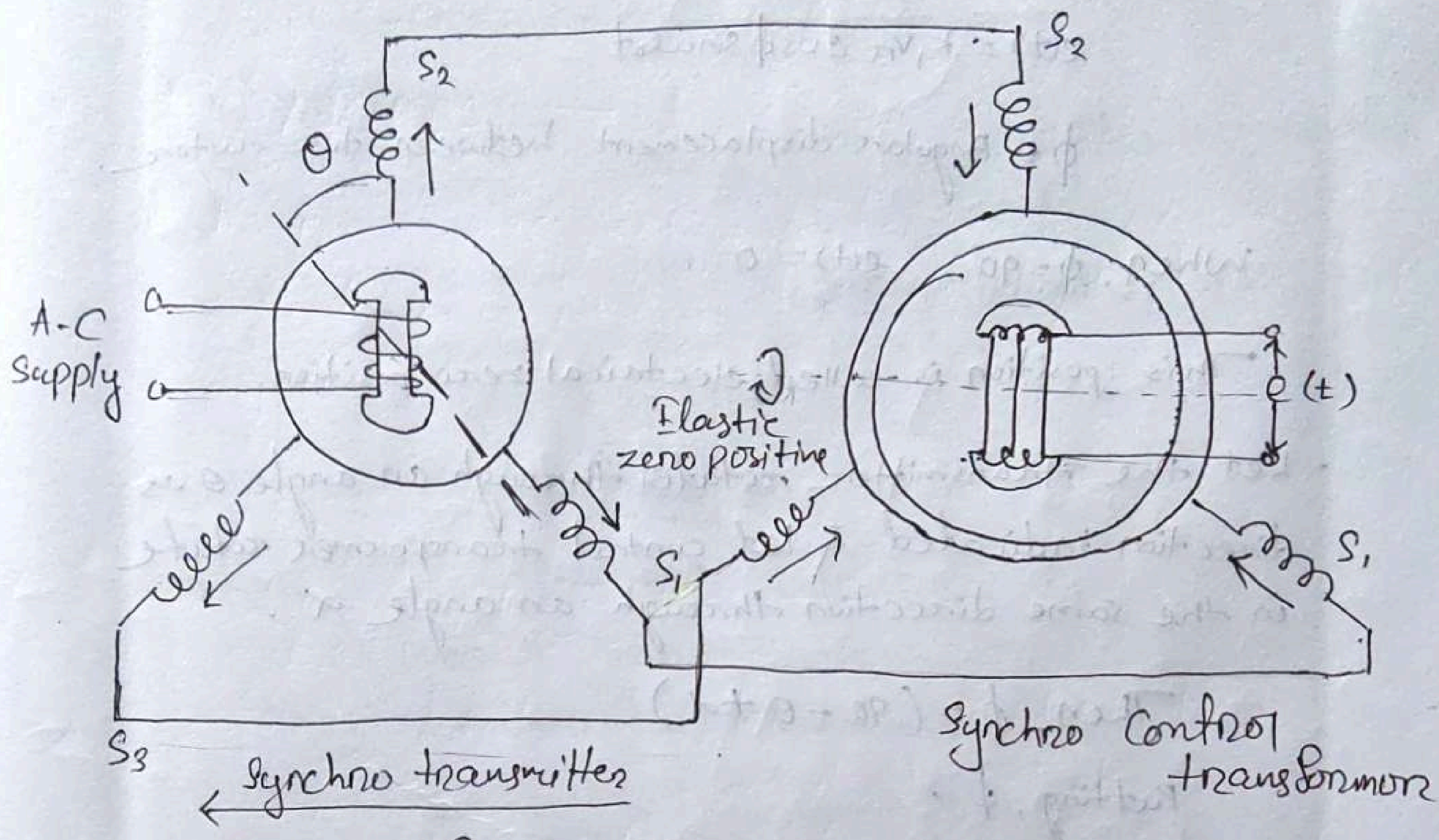
$$E_{cb} = \sqrt{3} KE_r \sin\omega t \sin(\theta)$$

$$E_{ac} = \sqrt{3} KE_r \sin\omega t \sin(\theta + 120°)$$

$$E_{ba} = \sqrt{3} KE_r \sin\omega t \sin(\theta + 240°)$$

- When $\theta = 0$ the maxᵐ induced voltage will be E_{an} & E_{cb} will be zero.
- This position of the rotor is defined as electrical zero of the transmitter & is used as the reference for indicating angular position of the rotor.

- Thus the input to the synchro transmitter is the angular position of the rotor shaft & the output are the three single phase voltage which are the function of the shaft position.



Synchro Error ~~Dedec~~ Detector.

- Principle of operation of synchro control transformer is same as that of synchro transmitter.
- Rotor of synchro control transformer is cylindrical type.
- The combination of synchro transmitter & synchro control transformer is used as error detector.
- The function of error detector is to convert the difference of two shaft position into electrical signal.
- The output of synchro transmitter is input to synchro control transformer.

- Same current will flow in the stator winding of synchro control transformer but in opposite direction.

- The voltage across the rotor terminals of control transformer is,

$$e(t) = K_1 V_r \cos\phi \sin\omega_0 t$$

ϕ = Angular displacement between two rotor.

When, $\phi = 90^\circ$, $e(t) = 0$.

This position is called electrical zero position.

- Let the transmitter rotate through an angle θ as direction indicated & let control transformer rotate in the same direction through an angle α.

Then, $\phi = (90 - \theta + \alpha)$

Putting, ϕ.

$$e(t) = K_1 V_r \sin(\theta - \alpha) \sin\omega_0 t$$

- We see that when the two rotor shaft are not in alignment, the rotor voltage of control transformer is approximately a sine function of the difference betn the two shaft angle.

Dt:- 18.04.22

* Mason's Gain Formula :—

- A technique to reduce a signal-flow graph to a signal transfer function requires the application of one formula.

- The transfer function, $\frac{C(s)}{R(s)}$ of a system represented by a signal flow graph is,

$$\boxed{T = \sum_{k=1}^{K} \frac{P_K \Delta_k}{\Delta}}$$

Where,

T = Overall Transmittance

Δ = Determinant of Transfer function

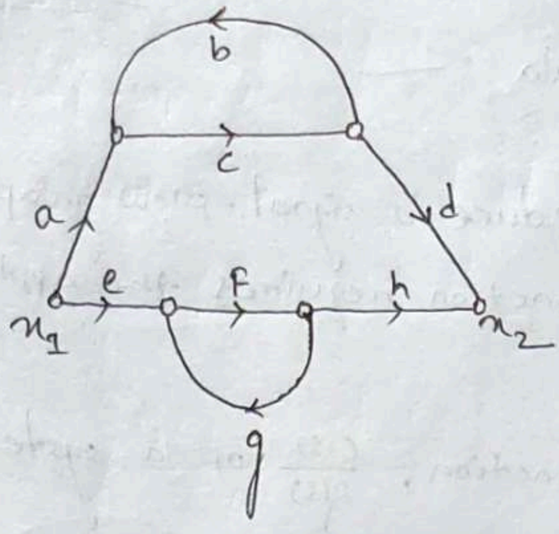
P_K = Path gain of k^{th} forward path

Δ_K = Determinant or path factor associated with k^{th} forward path.

Δ = Determintant

Δ = 1 − (sum of all possible loop gains)
+ (sum of gain product of all possible pair of non-touching loops)
− (sum of gain product all possible triple non-touching loops)
+ . . . .

Q-1



Two forward paths,

$P_1 = acd$ & $P_2 = efh$

$P_1 = acd =$ Path gain 1st path.

$P_2 = efh =$ Path gain 2nd path.

Two loops are, L_1 & L_2

$L_1 = bc$

& $L_2 = gf$

So, we use Mason's gain formula,

$$T = \sum_{K=1}^{K} \frac{P_K \Delta_K}{\Delta}$$

then, $\Delta = 1 - (L_1 + L_2) + L_1 L_2$

$= 1 - (bc + gf) + bcgf$

$\Delta_1 = 1 - L_2$

$= 1 - gf$

$\Delta_2 = 1 - L_1$

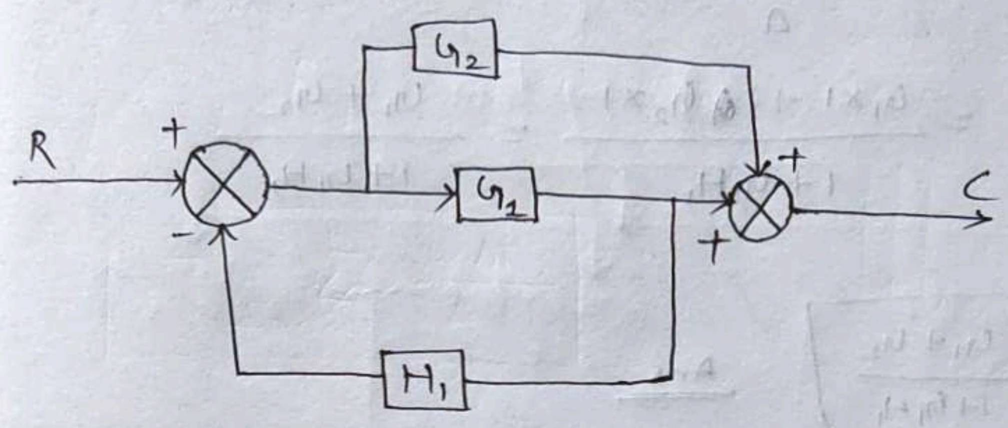
$= 1 - bc$

Now,

$$T = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{acd(1-gf) + efh(1-bc)}{1-(bc+gf)+bcgf}$$ Ans

(2) -



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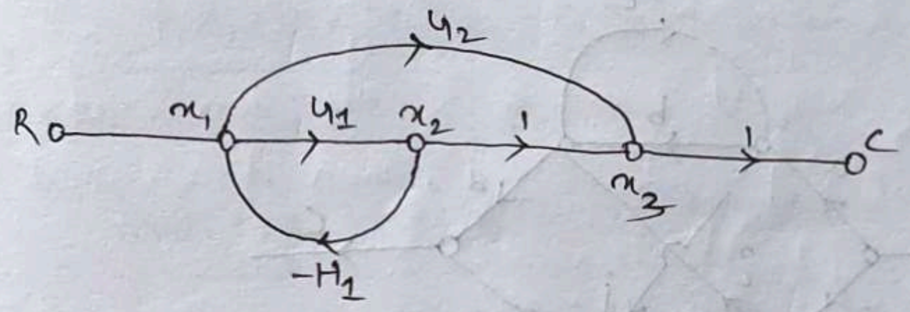

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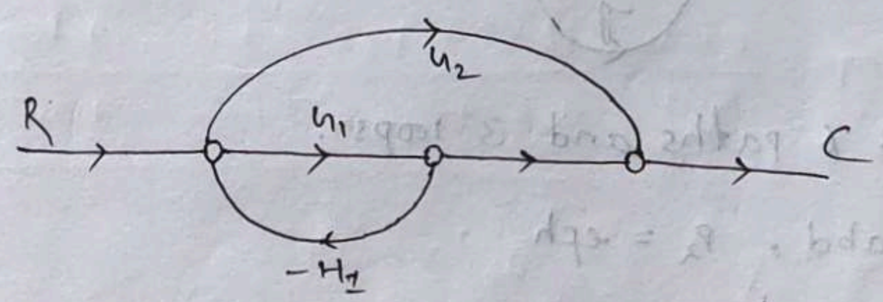


$P_1 = G_1 \times 1 \times 1 = G_1$

$P_2 = G_2$

$L = -G_1 H_1$

$$\Delta = 1 - L$$
$$= 1 - (-G_1 H_1)$$
$$= 1 + G_1 H_1$$

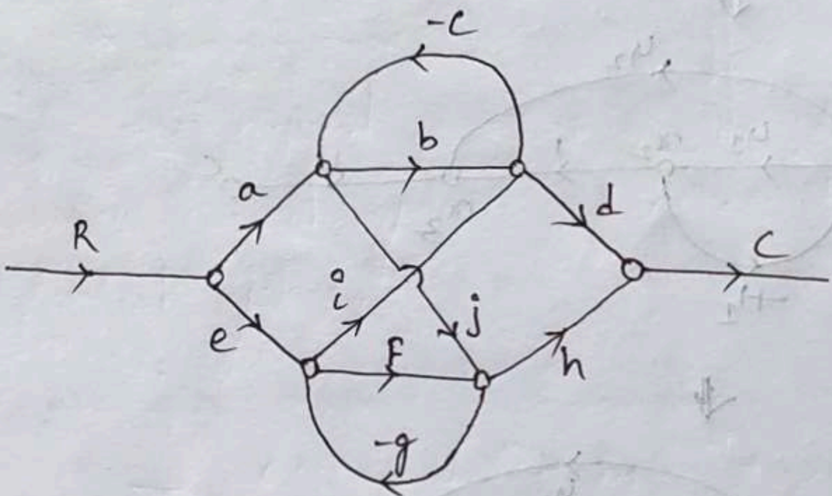
$\Delta_1 = 1$ & $\Delta_2 = 1$

$$T = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 \times 1 + G_2 \times 1}{1 + G_1 H_1} = \frac{G_1 + G_2}{1 + G_1 H_1}$$

$$\boxed{T = \frac{G_1 + G_2}{1 + G_1 H_1}}$$ Ans

(3))-



It has 6 paths and 3 loops.

$P_1 = abd$, $P_2 = efh$

$P_3 = ajh$, $P_4 = eid$

$P_5 = aj(-g)id$, $P_6 = ei(-c)jh$

$= -ajgid$ $\quad = -eicjh$

& Loops are,

$L_1 = -bc$, $L_2 = -gf$ & $L_3 = i(-c)j(-g)$

$= ijcg$

$$\Delta = 1 - [L_1 + L_2 + L_3] + [L_1 L_2]$$

$$= 1 - [(-bc) + (-gf) + (ijcg)] + [(-bc) \times (-gf)]$$

$$= 1 + bc + gf - ijcg + bcgf$$

$$\Delta_1 = 1 - L_2$$

$$= 1 - (-gf) = 1 + gf$$

$$\Delta_2 = 1 - L_1$$

$$= 1 - (-bc) = 1 + bc$$

$\Delta_3 = 1$, $\Delta_4 = 1$, $\Delta_5 = 1$ & $\Delta_6 = 1$

Now,

$$T = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6}{\Delta}$$

$$= \frac{abd(1+gf) + efh(1+bc) + ajh + eid - ajgid - eicjh}{1 + bc + gf - ijcg + bcgf}$$

Ans

Dt :- 20.04.22

* Time Response Analysis :— (TRA)

The behaviour of output of a transfer function w.r.t time is called time response analysis.

- We know, control system defined by T.F.

Control system

→ 1st order C.S

→ 2nd order C.S

→ n order C.S

* 1st order C.S :—

If the highest power of 'S' the denominator of T.F of a control system is 1, then it is called 1st order control system.

$$\boxed{T = \frac{C(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_m s^0}{a_0 s^n + a_1 s^{n-1} + \cdots + a_n s^0}}$$

When, s^n & $n = 1$, $a_0 s^1 + a_n s^0$

then it is 1st order

& $n = 2$, then, $a_0 s^2 + a_1 s^1 + a_n s^0$ it is called 2nd order.

T.R.A

Transient — steady state

steady state → Constant output stage.

- Time response analysis is two types i.e,

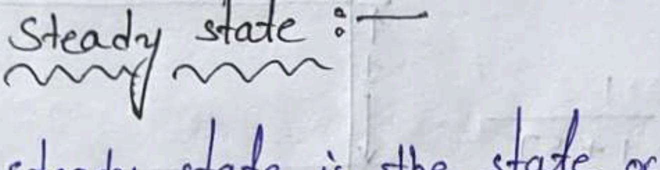
(i) - Transient

(ii) - steady state

* Steady state :—

steady state is the state of the output of a system after infinite interval of time after input signal intiated.

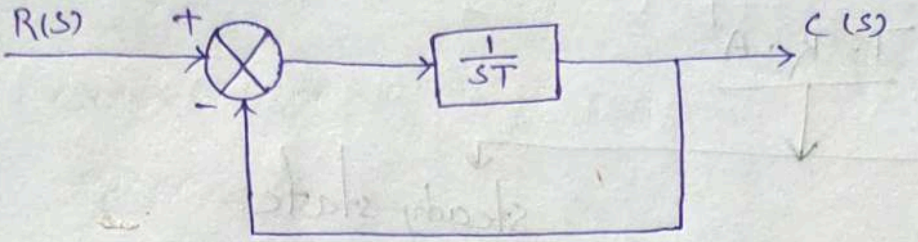
* 1st order Control System (Giving Unit step signal) :—



$H(s) = 1$

Assume,

$$G(S) = \frac{1}{ST}$$



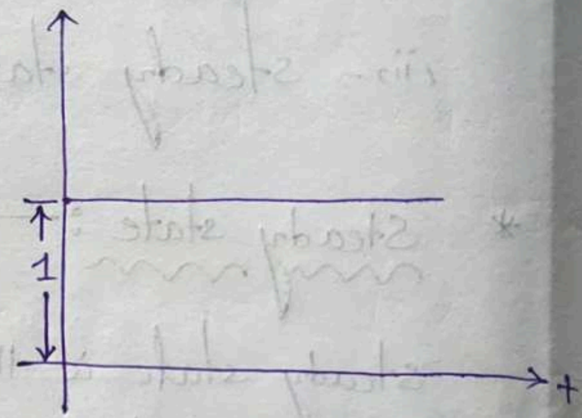
H(S) = 1

$$T = \frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S) \cdot H(S)}$$

$$= \frac{\frac{1}{ST}}{1 + \frac{1}{ST} \times 1} = \frac{1}{1 + ST}$$

$$\boxed{T = \frac{1}{1 + ST}}$$

$$T = \frac{C(S)}{R(S)} = \frac{1}{1 + ST}$$



Unit step Input

$$r(t) = u(t) = \begin{cases} A = 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$r(t) = 1$

$R(S) = L(1) = \frac{1}{S}$

$$T = \frac{C(S)}{R(S)} = \frac{1}{ST + 1}$$

$$\Rightarrow C(S) = R(S)\left(\frac{1}{ST + 1}\right)$$

$$\Rightarrow C(S) = \frac{1}{S(ST + 1)} \quad \left[\because R(S) = \frac{1}{S}\right]$$

$$\Rightarrow C(S) = \frac{A}{S} + \frac{B}{ST + 1}$$

$$\frac{1}{S(ST+1)} = \frac{A}{S} + \frac{B}{ST+1}$$

$$\Rightarrow \frac{1}{\cancel{S(ST+1)}} = \frac{A(ST+1)+BS}{\cancel{S(ST+1)}}$$

$$\Rightarrow 1 = AST + A + BS$$

$$\Rightarrow S(AT+B) + A = 0.S + 1$$

$$0 = AT + B \quad , \quad A = 1$$

$$\Rightarrow 0 = T + B$$

$$\Rightarrow B = -T$$

$$\Rightarrow C(S) = \frac{1}{S} - \frac{T}{ST+1}$$

Taking I.L.T on both side,

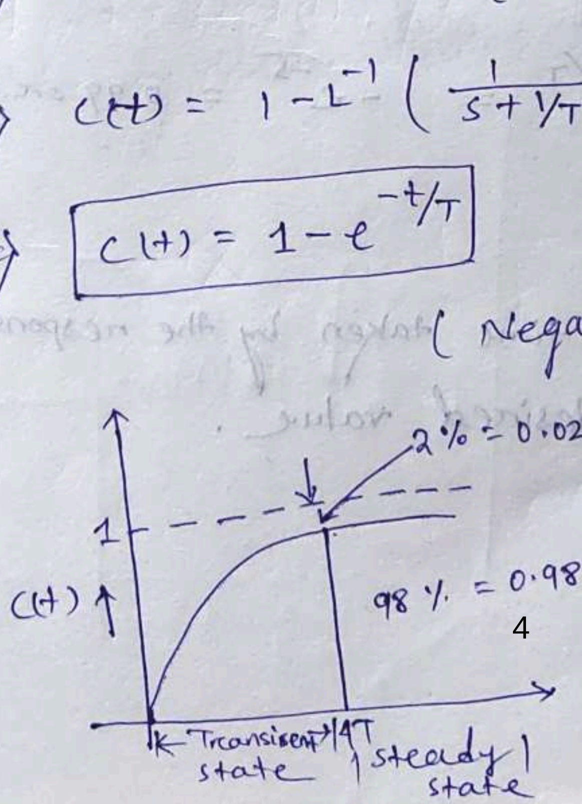
$$L^{-1}(C(S)) = L^{-1}\left(\frac{1}{S}\right) - L^{-1}\left(\frac{T}{ST+1}\right)$$

$$\Rightarrow C(t) = 1 - L^{-1}\frac{\cancel{T}}{\cancel{T}(S+\frac{1}{T})}$$

$$\Rightarrow C(t) = 1 - L^{-1}\left(\frac{1}{S+1/T}\right)$$

$$\Rightarrow \boxed{C(t) = 1 - e^{-t/T}}$$

(Negative Expontial)



$$c(t) = 1 - e^{-t/T}$$

$c(t)$ is the response in Time domain.

When,

$t = 0$

$$c(t) = 1 - e^{-0/T}$$

$$= 1 - e^{-0} = 1 - e^{\frac{1}{0}} = 0$$

When, $t = T$

$$c(t) = 1 - e^{-T/T}$$

$$= 1 - e^{-1} = 0.632 \text{ or } 63.2\%$$

When, $t = 2T$

$$c(t) = 1 - e^{-2T/T} = 1 - e^{-2} = 0.864 \text{ or } 86.4\%$$

When, $t = 3T$

$$c(t) = 1 - e^{-3T/T} = 1 - e^{-3} = 0.95 \text{ or } 95\%$$

When, $t = 4T$

$$c(t) = 1 - e^{-4T/T} = 1 - e^{-4}$$

$$= 0.98 \text{ or } 98\%$$

When, $t = 5T$

$$c(t) = 1 - e^{-5T/T} = 1 - e^{-5} = 0.99 \text{ or } 99\%$$

Time constant :—

Time constant is the time taken by the response to reach 63.2% of desired value.

I.M.P *

steady state Errror :—

$T = 3 m.s$

$t \to \infty , \ 4T = 12 m.s$

$$e(t) = r(t) - c(t)$$

$$= 1 - \left(1 - e^{-t/T}\right)$$

$$= e^{-t/T}$$

- steady state errror is symbolised by e_{ss} :

then

$$e_{ss} = \lim_{t \to \infty} e(t)$$

$$= \lim_{t \to \infty} e^{-t/T}$$

$$= \lim_{t \to \infty} \frac{1}{e^{t/T}} = \frac{1}{\infty} = 0$$

$$\Rightarrow \boxed{e_{ss} = 0}$$

steady state errror approaches to zero.

* Time Response Analysis :—

Time response of a control means how a system behaves an accordance with time when specified input test signal is applied.

* Transient state :—

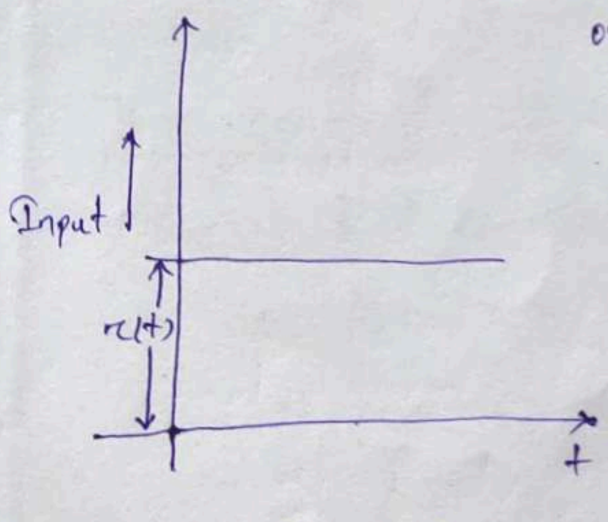
Initial part of time response of a control system transient appears.

- The transient part of time the response reveals the nature of the response (i.e, oscillating or over damped) and speed.

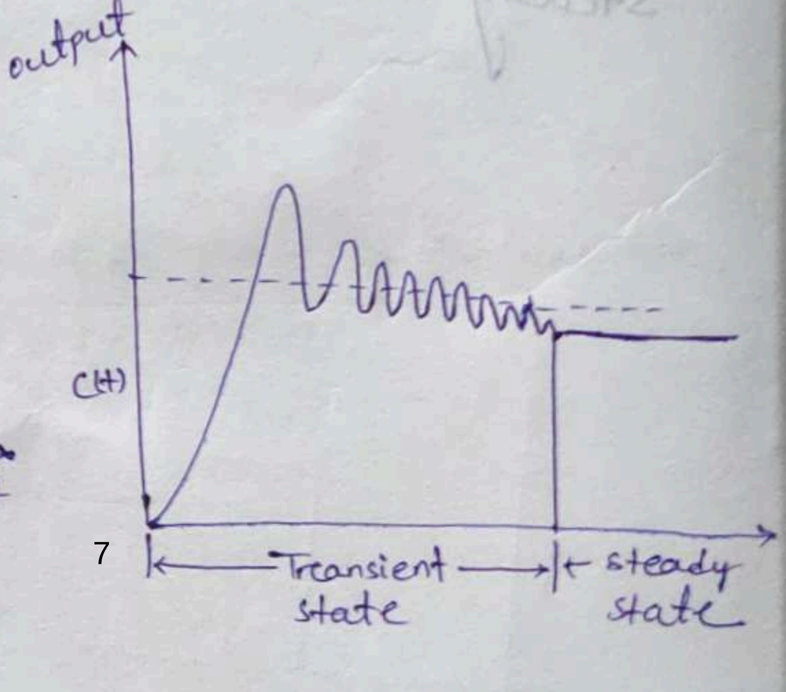
* steady state :—

After transient, steady state is achieved. steady state means state of the output of the control system as the time approaches infinity.

- It reveals (steady state) accuracy of a control system. steady state error is observed if the actual output doesn't match with the input.



Input Test Signal



(b) When Unit Impulse Input Is Given :-

We know the output expression,

$$C(s) = R(s)\,\frac{1}{1+sT}$$

As input to the system is a unit impulse

$$R(s) = 1$$

$$C(s) = R(s)\cdot\frac{1}{1+sT}$$

$$= 1\cdot\frac{1}{1+sT}$$

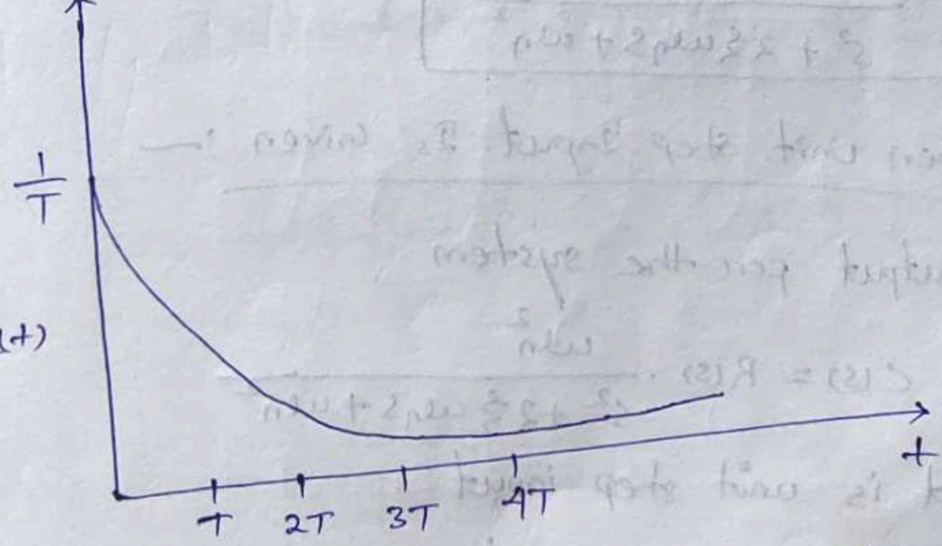
$$= \frac{1}{1+sT} \Rightarrow \frac{1}{T\left(s+\frac{1}{T}\right)}$$

$$C(s) = \frac{1}{T}\left(\frac{1}{s+\frac{1}{T}}\right)$$

Taking I.L.T

$$\mathcal{L}^{-1}C(s) = \mathcal{L}^{-1}\frac{1}{T}\left(\frac{1}{s+\frac{1}{T}}\right)$$

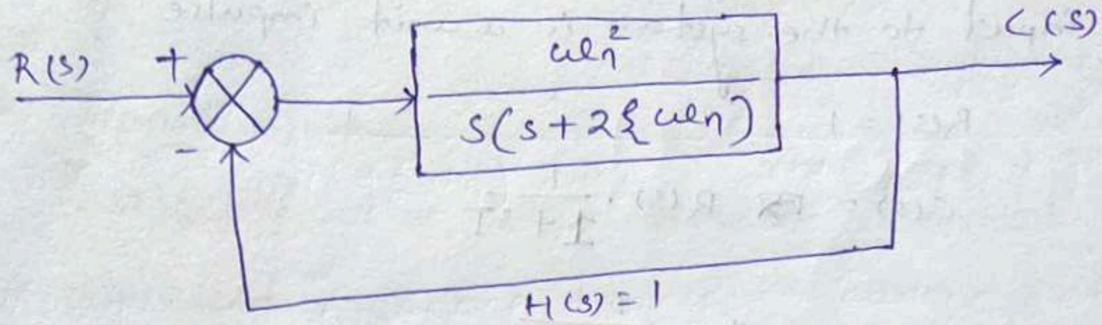
$$\Rightarrow \boxed{c(t) = \frac{1}{T}e^{-t/T}}$$



Time response of 1st order C.S for unit impulse input.

* Time Response Of A Second Order C.S :—

In second order control system highest power of s of characteristics eq^n is 2.



Here, $G(s) = \dfrac{\omega_n^2}{s(s+2\xi\omega_n)}$ & $H(s) = 1$

$$\frac{C(s)}{R(s)} = \frac{\dfrac{\omega_n^2}{s(s+2\xi\omega_n)}}{1+\dfrac{\omega_n^2}{s(s+2\xi\omega_n)} \times 1}$$

$$= \frac{\dfrac{\omega_n^2}{s(s+2\xi\omega_n)}}{\dfrac{s^2+2\xi\omega_n s+\omega_n^2 \cdot 1}{s(s+2\xi\omega_n)}}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}}$$

(a) – When Unit step Input Is Given :—

output for the system

$$C(s) = R(s) \cdot \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

As input is unit step input

$$r(t) = 1 \quad \& \quad R(s) = \frac{1}{s}$$

$$C(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$= \frac{A}{s} + \frac{Bs + C}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\frac{A}{s} + \frac{Bs + C}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

$$\Rightarrow A(s^2 + 2\xi\omega_n s + \omega_n^2) + Bs^2 + Cs = \omega_n^2$$

$$\Rightarrow (A+B)s^2 + (2\xi\omega_n A + C)s + A\omega_n^2 = \omega_n^2$$

Comparing co-efficient of s^2, s, & constant term on both side of the eq^n.

$$A + B = 0, \quad 2\xi\omega_n A + C = 0, \quad A = 1$$

$$\Rightarrow B = -1 \qquad C = -2\xi\omega_n$$

$$C(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Making perfect square of denominator of second part

$$C(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{s^2 + 2\cdot\xi\omega_n s + (\xi\omega_n)^2 - (\xi\omega_n)^2 + \omega_n^2}$$

$$\Rightarrow C(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{(s + \xi\omega_n)^2 + \omega_n^2(1 - \xi^2)}$$

Put $\omega_d = \omega_n\sqrt{1 - \xi^2}$

$$C(s) = \frac{1}{s} - \frac{s + 2\xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2}$$

$$\Rightarrow C(s) = \frac{1}{s} - \frac{s + \xi\omega_n + \xi\omega_n}{(s+\xi\omega_n)^2 + \omega_d^2}$$

$$= \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}$$

$$\Rightarrow C(s) = \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n}{\omega_d}\,\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}$$

Taking inverse laplace transform ~~eq~~ on both sides of the eqn,

$$L^{-1}C(s) = L^{-1}\left(\frac{1}{s}\right) - L^{-1}\frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n}{\omega_d}L^{-1}\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}$$

$$\Rightarrow c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin\omega_d t$$

$$= 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin\omega_d t$$

$$= 1 - e^{-\xi\omega_n t}\left(\cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right)$$

$$\Rightarrow c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left(\sqrt{1-\xi^2}\cdot\cos\omega_d t + \xi\sin\omega_d t\right)$$

Put $\xi = \cos\phi$, $\sqrt{1-\xi^2} = \sin\phi$

$$\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}(\sin\phi\cos\omega_d t + \cos\phi\sin\omega_d t)$$

$$\Rightarrow \boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \phi)}$$

- The error is given as,

$$e(t) = r(t) - c(t)$$

$$r(t) = 1,$$

$$e(t) = 1 - \left[1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)\right]$$

$$= \cancel{1} - \cancel{1} + \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin\left(\omega_n \sqrt{1-\xi^2}\, t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

$$\Rightarrow e(t) = \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin\left(\omega_n \sqrt{1-\xi^2}\, t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

steady state error, $e_{ss} = \lim_{t\to\infty} e(t)$

$$\boxed{e_{ss} = \lim_{t\to\infty} \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin\left(\omega_n \sqrt{1-\xi^2}\, t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)}$$

- The response or output depends upon the value of ξ. For $\xi < 1$, the response represents exponentially decaying oscillations having frequency,

$$\omega_n \sqrt{1-\xi^2} = \omega_d$$

Time Constant, $T = \frac{1}{\xi \omega_n}$

ω_n = Natural frequency of oscillations

$\omega_d = \omega_n \sqrt{1-\xi^2}$ = damped frequency of oscillations

ξ = Affect damping, called damping ratio.

$\xi \omega_n$ = Damping factor or Actual damping or Damping co-efficient

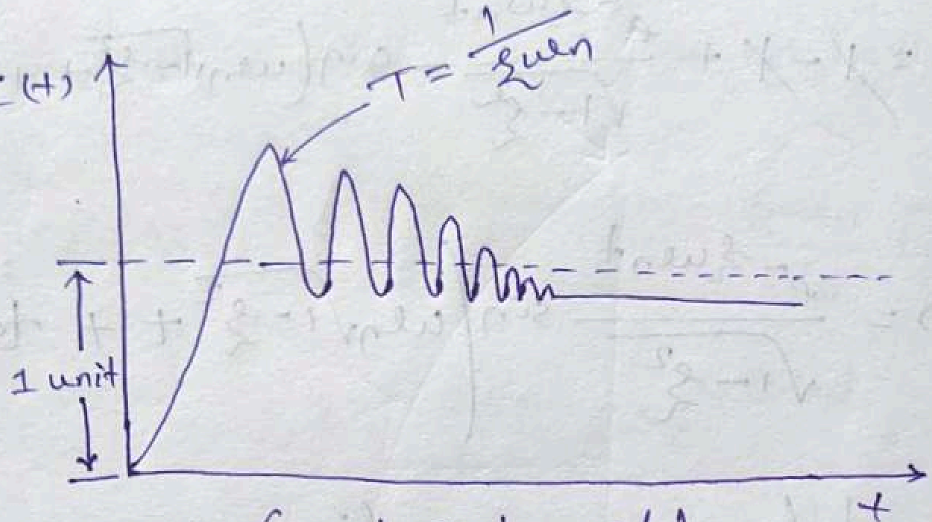
$\xi < 1$, $c(t)$ = Under damped response given damped oscillation.

$\xi = 0$, $c(t)$ = Undamped response.

$\xi = 1$, $c(t)$ = Critically damped.

$\xi > 1$, $c(t)$ = Over damped

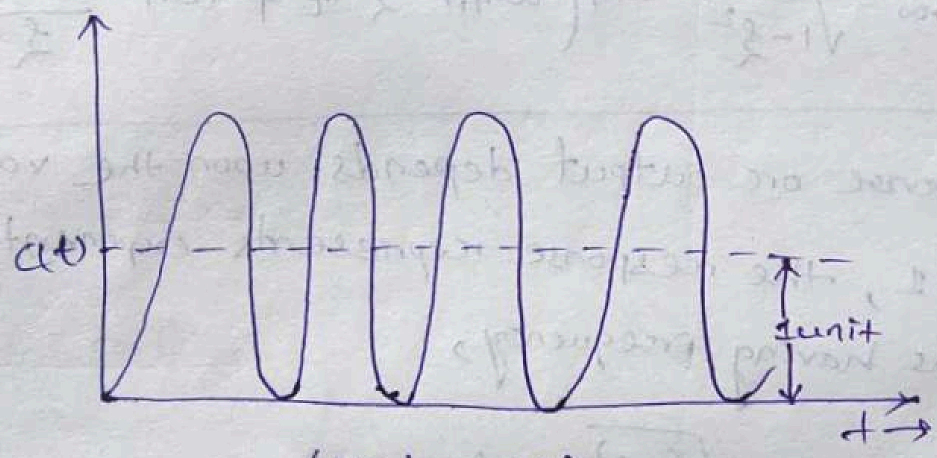
$\xi < 1$,



(under damped)

$\xi = 0$,

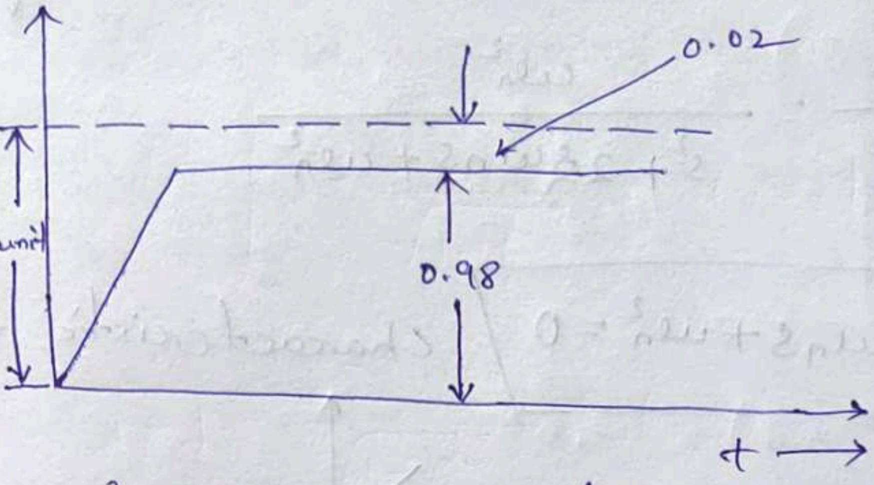
$c(t) = 1 - \cos\omega_n t$



(Undamped)

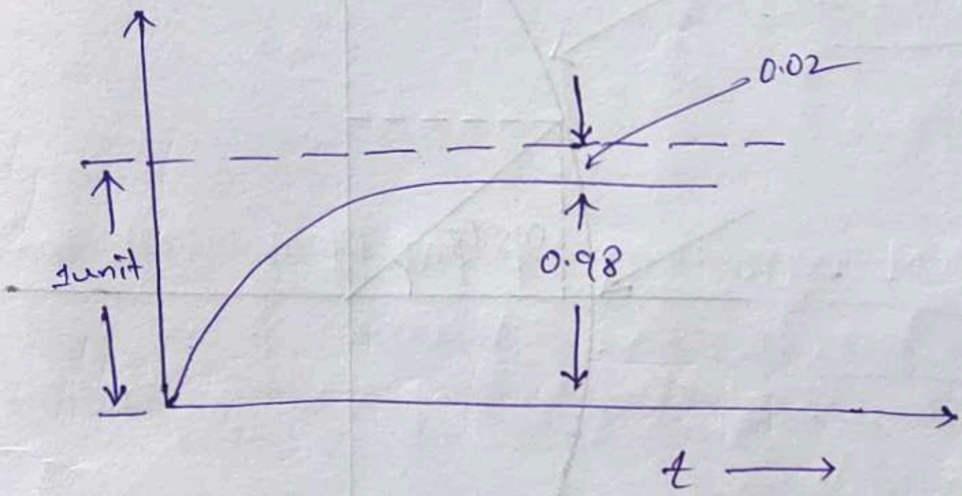
$\xi = 1$

$$c(t) = 1 - e^{-\xi \omega_n t}(1 + \omega_n t)$$



(critically damped)

$\xi > 1$,



(over damped)

$$c(t) = 1 - \frac{e^{-(\xi - \sqrt{\xi^2 - 1})\omega_n t}}{2\sqrt{\xi^2 - 1}\,(\xi - \sqrt{\xi^2 - 1})} + \frac{e^{-(\xi + \sqrt{\xi^2 - 1})\omega_n t}}{2\sqrt{\xi^2 - 1}\cdot(\xi + \sqrt{\xi^2 - 1})}$$

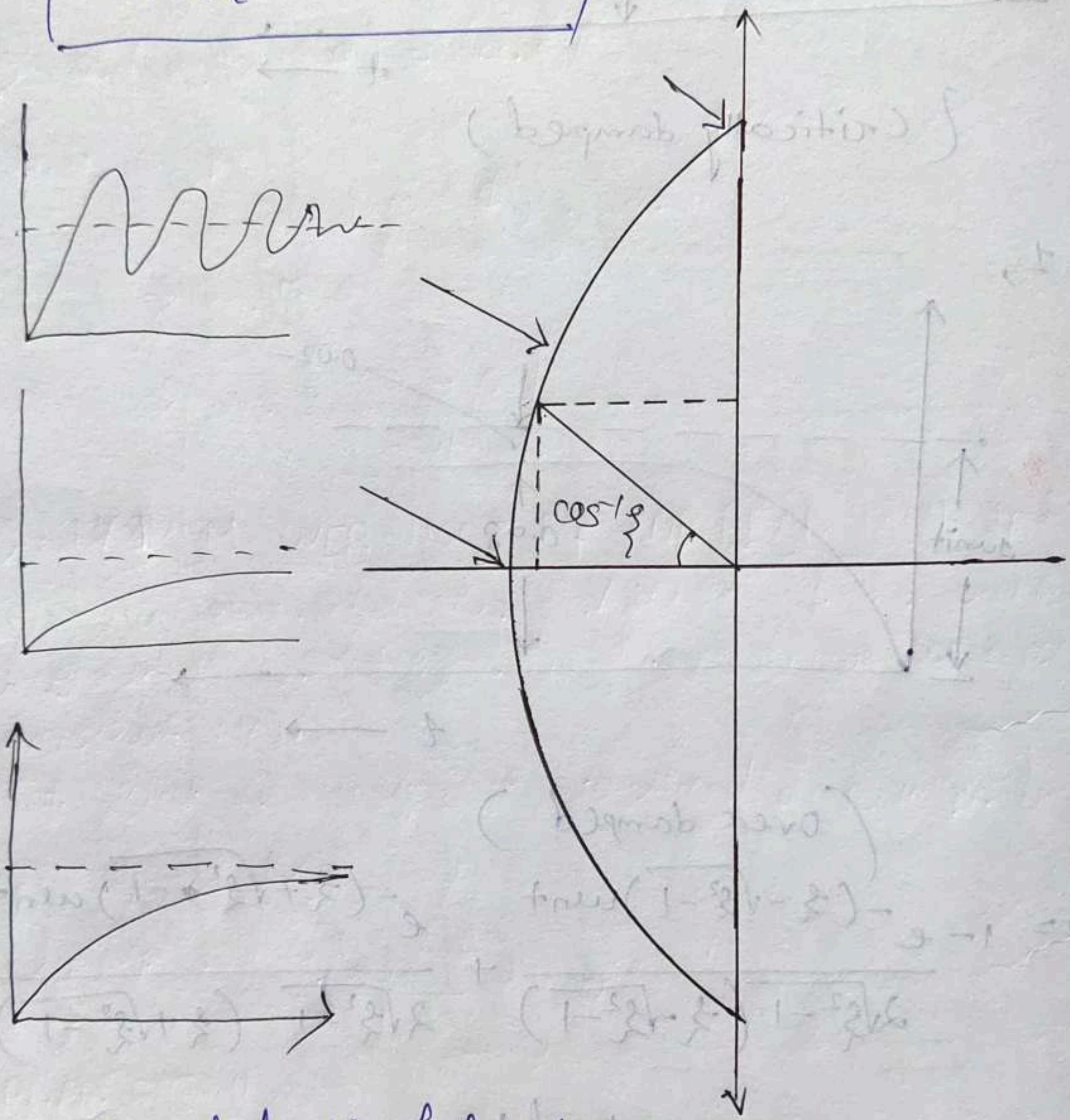
ξ = damping ratio = $\dfrac{\text{Actual damping}}{\text{critical damping}} = \dfrac{\xi \omega_n}{\omega_n}$

*CHARACTERISTICS EQUATION :—

Transfer function of second order control system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\boxed{s^2 + 2\zeta\omega_n s + \omega_n^2 = 0}$$ Characteristic eqn.

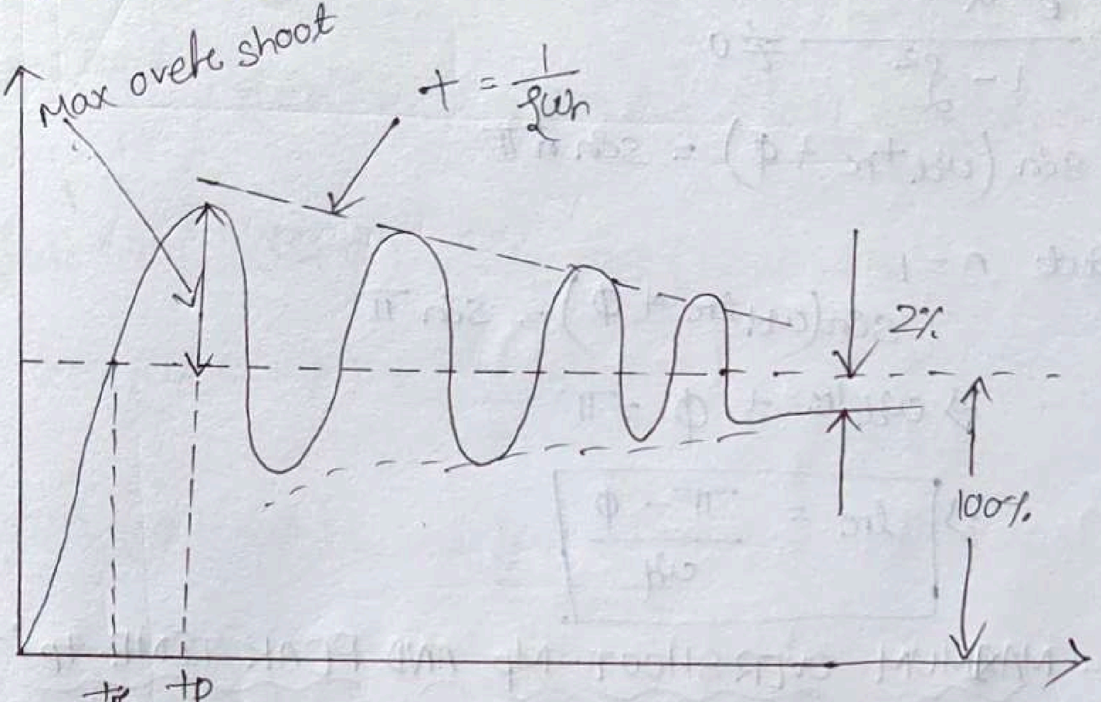


Location of roots of the characteristic equation and corresponding time responses.

* TRANSIENT RESPONSE SPECIFICATION OF SECOND ORDER CONTROL SYSTEM :-

→ The time response of an underdamped control system exhibits damped occillations prior to reaching the stady states.

→ The specifications pertaining to time respon during transient part.



* RISE TIME (t_r) :-

→ The ries time in the time taken by the response 0 to 100% or 10% to 90% of the desired value of the output at the vaty front instant.

→ 0% to 100% for under damped systems

90% to 10% for over damped systems

For under damped system :-

we know $c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$

At the first instant when $c(t)$ become 1

$t = t_r$.

$$1 = 1 - \frac{e^{-\xi \omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \phi)$$

$$\Rightarrow \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \phi) = 0$$

$$\Rightarrow \sin(\omega_d t_r + \phi) = \frac{0}{\frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}}} = 0$$

As, $\frac{e^{-\xi\omega_n t_r}}{1-\xi^2} \neq 0$

$$\Rightarrow \sin(\omega_d t_r + \phi) = \sin n\pi$$

Put $n=1$

$$\sin(\omega_d t_r + \phi) = \sin \pi$$

$$\Rightarrow \omega_d t_r + \phi = \pi$$

$$\Rightarrow \boxed{t_r = \frac{\pi - \phi}{\omega_d}}$$

* (2) - MAXIMUM OVERSHOOT M_p AND PEAK TIME t_p :-

→ The maximum positive deviation of the output with respect to its desired value is known as maximum overshoot (M_p).

→ If input is unit step. Desired output in unity.

$$M_p = c(t)_{max} - 1$$

$$\% M_p = \frac{c(t)_{max} - 1}{1} \times 100$$

* PEAK TIME :-

- The time needed to reach the maximum overshoot is called peak time and denoted by t_p.

For $c(t)$ becomes $c(t)_{max}$.

$$\frac{d\,c(t)}{dt} = 0$$

$$\Rightarrow \frac{d}{dt}\left(1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi)\right) = 0$$

$$\Rightarrow \frac{d(1)}{dt} - \frac{1}{\sqrt{1-\zeta^2}} \frac{d}{dt}\left(e^{-\zeta\omega_n t} \cdot \sin(\omega_d t + \phi)\right) = 0$$

$$\Rightarrow 0 - \left(\frac{1}{\sqrt{1-\zeta^2}} \left(-\zeta\omega_n e^{-\zeta\omega_n t} \sin(\omega_d t + \phi) + e^{-\zeta\omega_n t} \cos(\omega_d t + \phi)\,\omega_d\right) = 0\right.$$

$$\Rightarrow \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left(\zeta\omega_n \sin(\omega_d t + \phi) - \omega_d \cos(\omega_d t + \phi)\right) = 0$$

$$\Rightarrow \zeta\omega_n \sin(\omega_d t + \phi) = \omega_d \cos(\omega_d t + \phi)$$

$$\Rightarrow \tan(\omega_d t + \phi) = \frac{\omega_d}{\zeta\omega_n}$$

$$\Rightarrow \tan(\omega_d t + \phi) = \frac{\cancel{\omega_n}\sqrt{1-\zeta^2}}{\zeta\,\cancel{\omega_n}}$$

$$\Rightarrow \tan(\omega_d t + \phi) = \frac{\sqrt{1+\zeta^2}}{\zeta} = \tan\phi$$

$$\Rightarrow \frac{\tan\omega_d t + \tan\phi}{1 - \tan\omega_d t - \tan\phi} = \tan\phi$$

$$\Rightarrow \tan\omega_d t + \tan\phi = \tan\phi$$

$$\Rightarrow \tan\omega_d t_p = 0$$

$$\Rightarrow \tan\omega_d t_p = \tan\pi$$

But $n = 1$

$\omega_d t_p = \pi$

$$\Rightarrow \boxed{t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}}$$

$$c(t)_{max} = 1 - \frac{e^{-\zeta\omega_n t_p}}{\sqrt{1+\zeta^2}} \sin(\omega_d t_p + \phi)$$

$$= 1 - \frac{e^{-\zeta\omega_n \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}}}{\sqrt{1-\zeta^2}} \sin\left(\omega_d \times \frac{1}{\omega_d} + \phi\right)$$

$$= 1 - \frac{e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}}{\sqrt{1-\zeta^2}} \sin(\pi + \phi)$$

$$= 1 - \frac{e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}}{\sqrt{1-\zeta^2}} (-\sin\phi)$$

$$C(t)_{max} = 1 + \frac{e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}}{\sqrt{1-\zeta^2}} \sin\phi$$

$$c(t)_{max} = 1 + \frac{e^{\frac{-\zeta\pi}{\sqrt{1+\zeta^2}}}}{\cancel{\sqrt{1-\zeta^2}}} \cancel{\sqrt{1-\zeta^2}}$$

$$c(t)_{max} = 1 + e^{\frac{-\zeta\pi}{\sqrt{1+\zeta^2}}} \quad \left(\because \sin\phi = \sqrt{1-\zeta^2}\right)$$

$$M_p = c(t)_{max} - 1$$

$$M_p = 1 + e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} - 1$$

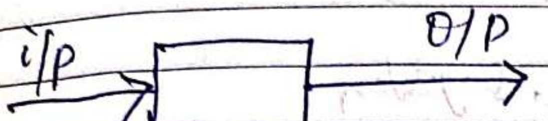
$$\boxed{M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}}$$

$$\% M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$$

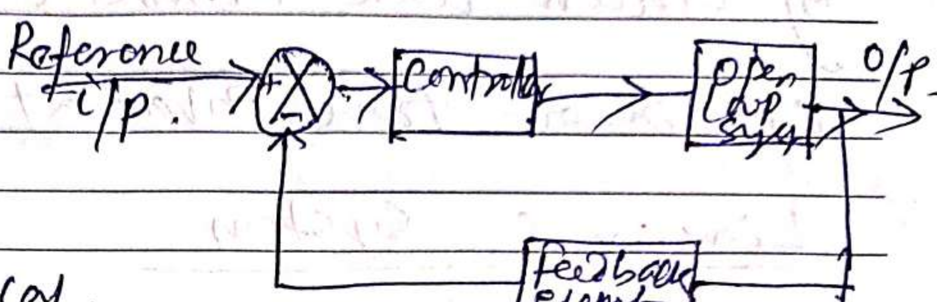
Control system

A system which consist of numbers of components connected together to perform a specific function, in which the output is controlled by input.

Open Loop C. system



Closed Loop. C. system



Advantage (Open Loop)

1. Simple & Economical.
2. Easier to construct.
3. Stable.

Disadvantage

1. Inaccurate and unreliable.
2. Change in output are not corrected automatically.

Advantage (Closed Loop)

1) ~~Complex & Costlier~~ Accurate & reliable.
2) Less ~~stable~~ effected by noise.

Disadvantage

1) Complex & Costlier
2) feedback reduces the overall gain of the system.
3) Stability is a major problem.

Types of control system.

1) Linear C. System 2) Non Linear C System.
3) Time variant C. system 4) Time invariant C. Sy.
5) Linear time variant C. System
6) Linear time invariant C. System

Linear C. System

Control system obey linearity & homogenity.

Non-Linear C. System

Control system doesn't obey linearity & homogenity

Time Variant C. System

If output is varying with time.

~~Linear~~ Time invariant Syst

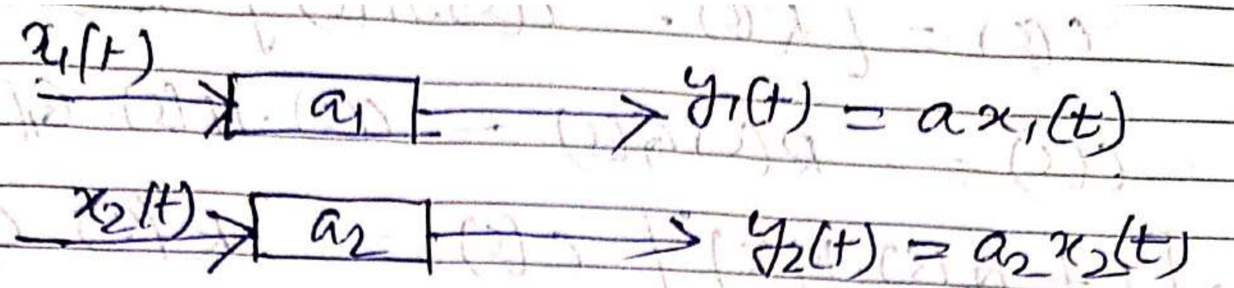
If output is not varying with time.

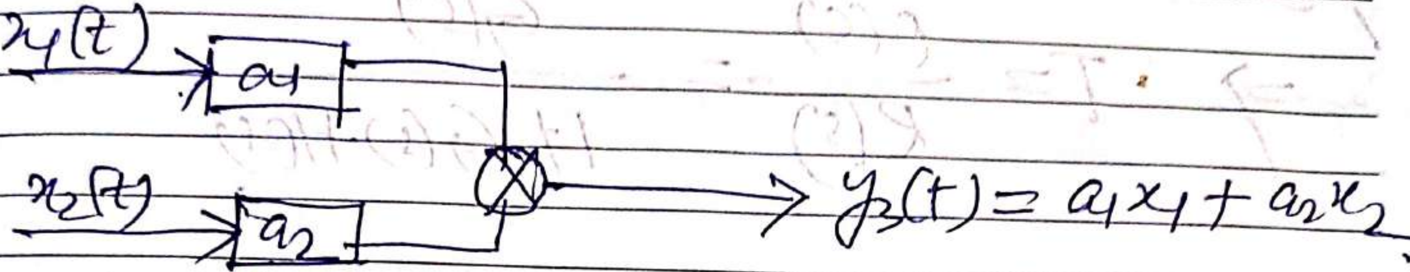
Linear time variant Control System

If the control system is both linear and time variant, then it is called LTV C.Sy

~~Non~~ Linear Time invariant Cont Syst

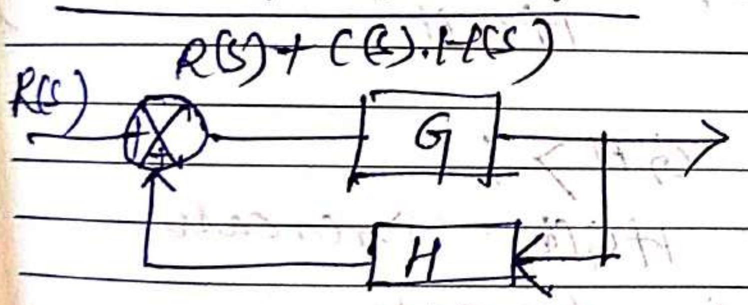
If the control system is both linear and time invariant, then it is called LTI. C-Sys





TYPES OF FEEDBACK.

Positi feed back.



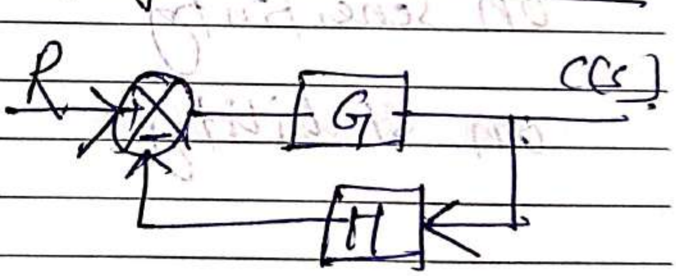
Adding feedback element to the reference input.

$$C(s) = [R(s) + C(s) H(s)] G(s)$$

$$[1 - G(s) \cdot H(s)] \, C(s) = R(s) \cdot G(s)$$

$$\Rightarrow T = \frac{C(s)}{R(s)} = \frac{G(s)}{1 - G(s) \cdot H(s)}$$

Negative feedback.



Subtracting feedback element from the reference input.

$C(s) = [R(s) - C(s).H(s)] G(s)$

$C(s) = R(s) G(s) - C(s).H(s).G(s)$

$\Rightarrow [1 + G(s).H(s)] C(s) = R(s).G(s)$

$\Rightarrow T = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s).H(s)}$

EFFECT OF FEEDBACK

on gain.

on Sensitivity

on stability.

Gain

$T = \frac{G(s)}{1+GH}$ | For -ve feedback

If $GH > 1$

$1+G(s)$ increase

Gain decreases.

$1+GH$ decrease

Gain increase.

sensitivity

Sensitivity $S = \frac{\text{\% change in } T}{\text{\% change in } G}$

$= \frac{\frac{\partial T}{T}}{\frac{\partial G}{G}} = \frac{\partial T}{\partial G} \frac{G}{T}$ — (1)

$$\frac{\partial T}{\partial G} = \frac{\partial}{\partial G}\left[\frac{G}{1+GH}\right] = \frac{(1+GH) - GH}{(1+GH)^2}$$

$$= \frac{1+GH-GH}{(1+GH)^2} = \frac{1}{(1+GH)^2}$$

$$S = \frac{1}{(1+GH)^2}\frac{G}{T} \quad \left(\text{putting value of } \frac{\partial T}{\partial G} \text{ in eqn } 1\right)$$

$$= \frac{1}{(1+GH)^2} \times \frac{G}{\frac{G}{1+GH}} = \frac{1}{1+GH}$$

$$S^T_G = \frac{1}{1+GH}.$$

Stability

O/P is more controllable the system is stable.

SERVOMECHANISM.

Automatic control of any physical quantity (position, velocity, displacement) is called Servomechanism.

The word Servo means controlling Mechanical position or derivatives of position like velocity and acceleration

It is an automatic device that uses the error sensing negative feedback to correction of performance of mechanism.

A Servo drive is a special electronic amplifier used to power electric Servomechanisms

Servo mechanism uses negative feedback to control mechanical position

position control Servomechanism used in hydraulic and pneumatic machines to control the position.

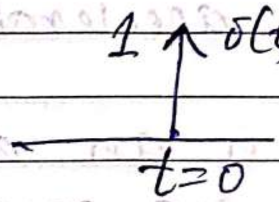
It is used in automatic machine tools, Statellite tracking antenna, Air craft system and Navigation system.

A Servomechanism primarily consist of 3-basic components.

1) feedback system.
2) Error detector.
3) Electric motor.

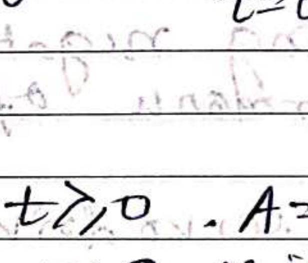
Test Signals

a) Impulse: $\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$



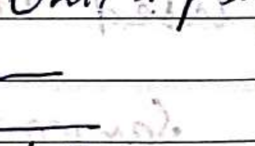
b) Step Signal: $u(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$. A=1 Unit step Sig.

A step is a signal whose value changes from one level to another level A in zero time.



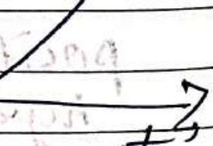
c) Ramp Signal. $r(t) = \begin{cases} At & t \geq 0 \\ 0 & t < 0 \end{cases}$

A=1, Unit ramp.



d) Parabolic Signal.

$$x(t) = \begin{cases} \frac{At^2}{2} & t \geq 0 \\ 0 & t < 0. \end{cases}$$



when A=1, Unit parabolic Signal.

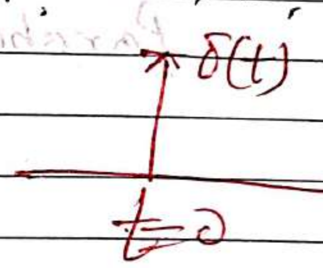
Impulse Signal

An unit impulse is defined as a signal which has zero value everywhere except at $t=0$ where its magnitude is infinite.
It is generally called δ-function.

$$\delta(t)=0,\ t\neq 0,\quad \int_{-\epsilon}^{+\epsilon}\delta(t)\,dt=1 \quad \epsilon\to 0,$$

$$\delta(t)=\dot{u}(t)=\frac{d}{dt}u(t)$$

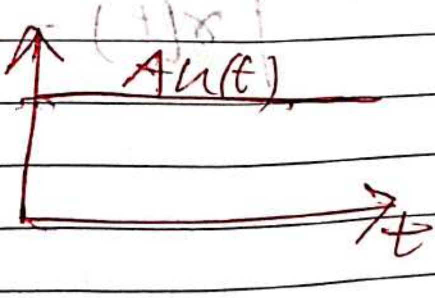
$$\mathcal{L}\,\delta(t)=(1)= R(s)$$



Step signals:

The step is a signal whose value changes from one level (usually zero) to another level A in zero time.

$$r(t)=A\,u(t)$$
$$u(t)=1,\quad t\geq 0$$
$$=0\quad t<0$$



$$L\,r(t)=A/s=R(s)$$

Ramp Signal

The ramp is a signal which starts at a value of zero and increases linearly with time.

$$r(t)=At;\ t\geq 0$$
$$=0\quad t<0$$

$$L\,r(t)=A/s^2=R(s)$$

31 |29 |30 |31 |

Parabolic Signal

athematically, $x(t) = At^2/2 \quad t > 0$

$= 0 \quad t < 0$

$R(s) = A/s^3$.

Parabolic signal is the integral of ramp signal

Relation

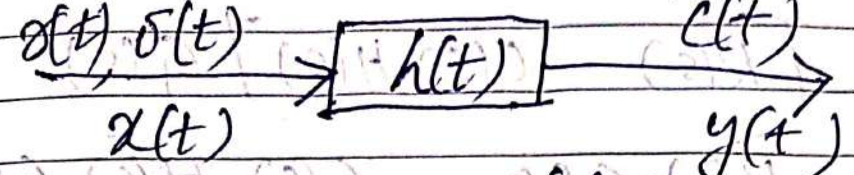
$\int \delta(t) = u(t) \Rightarrow \delta(t) = \frac{d}{dt} u(t)$

$\int u(t) = r(t) \Rightarrow u(t) = \frac{d}{dt} r(t)$

$\int r(t) = x(t) \Rightarrow r(t) = \frac{d}{dt} x(t)$

Impulse Response of a System

The response of the system for an impulse is called the impulse response of the system.



$$y(t) = x(t) * h(t)$$

$$Y(w) = X(w) \cdot H(w)$$

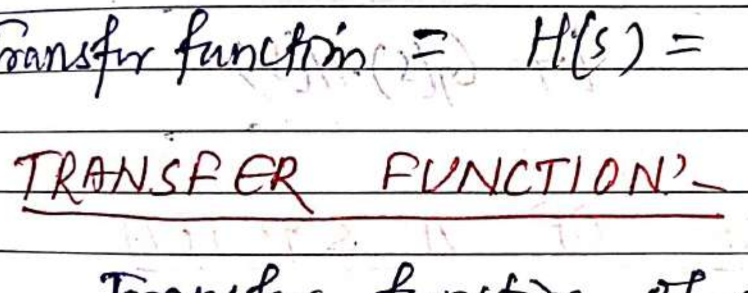
$$\Rightarrow H(w) = \frac{Y(w)}{X(w)} \quad \text{(Fourier series)}$$

$$\text{Transfer function} = H(s) = \frac{Y(s)}{X(s)} \quad \text{Laplace Transform)}$$

TRANSFER FUNCTION:-

Transfer function of a control system is the ratio of Laplace Transform of output to Laplace Transform of input.

$$\text{i.e. Transfer function} = \frac{\text{L.T. of O/P}}{\text{L.T. of I/P}}$$ with zero initial condition.



$$C(s) = [R(s) - C(s)H(s)]\,G(s)$$

$$T \cdot F = \frac{C(s)}{R(s)}$$

$$C(s) = \left[R(s) - C(s) \cdot H(s)\right] G(s)$$

$$C(s) = R(s) \cdot G(s) - C(s) \cdot H(s) \cdot G(s)$$

$$\Rightarrow C(s) + C(s) \cdot H(s) \cdot G(s) = R(s) \cdot G(s)$$

$$\Rightarrow C(s)\left[1 + H(s) \cdot G(s)\right] = R(s) \cdot G(s)$$

$$\Rightarrow T \cdot F = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s) \cdot H(s)}$$

IMPULSE RESPONSE OF A SYSTEM

The response of the system for an impulse is called as impulse response of the system.

Generally this can be represented with $h(t)$ or $h(n)$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$y(t) = x(t) * h(t)$$

$$Y(\omega) = X(\omega) \cdot H(\omega)$$

$$H(w) = \frac{Y(w)}{X(w)}$$

$$F^{-1}(H(w)) = h(t)$$

$$R(s) \longrightarrow \boxed{H(s)} \longrightarrow C(s)$$

$$C(s) = R(s) \cdot H(s)$$

$$H(s) = \frac{C(s)}{R(s)}$$

$$L^{-1} H(s) = h(t)$$

<u>problem</u>

$$H(s) = \frac{s^2+2s-4}{s^2-4} = 1 + \frac{2s}{s^2-4}$$

$$s^2-4 \,)\, s^2+2s-4 \,(\, 1$$
$$s^2-4$$
$$2s$$

$$= 1 + \frac{2s}{s^2-4}$$

$$\delta(t) \xrightarrow{L.T} 1$$

$$L^{-1}(1) = \delta(t)$$

$$H(s) = 1 + \frac{2s}{(s+2)(s-2)}$$

$$= 1 + \frac{s+s+2-2}{(s+2)(s-2)}$$

$$= 1 + \frac{(s+2)+(s-2)}{(s+2)(s-2)}$$

$$H(s) = 1 + \frac{1}{s+2} + \frac{1}{s-2}$$

$$\mathcal{L}^{-1}H(s) = \mathcal{L}^{-1}(1) + \mathcal{L}^{-1}\left(\frac{1}{s+2}\right) + \mathcal{L}^{-1}\left(\frac{1}{s-2}\right)$$

$$h(t) = \delta(t) + \left[e^{-2t} + e^{2t}\right]u(t)$$

USES OF TRANSFER FUNCTION

Transfer function are used

a) ~~SISO~~ Analysis of SISO filters in the field of signal processing.

b) Communication Theory.

c) Control Theory.

d) Used exclusively LTI System.

ADVANTAGE & DISADVANTAGE OF T.F

Advantage

a) If T.F. of a system is known, the response of the system to any input can be determined easily.

b) A. T.F is the mathematical model and give gain of the system.

c) Since it involves L.T. (Laplace Transform) the terms are simple algebraic expression and no differential terms are present.

d) poles and zeros of the system can be determined from the knowledge of T.F.

Disadvantage

a) T.F doesn't take onto account ~~into~~ the initial condition of the system.

b) T.F can be defined only for linear system.

c) No inference can be drawn about the physical structure of the system.

d) It is applicable for SISO system.

e) To find frequency response, we need to shift the system into Fourier Domain

PROPERTIES OF TRANSFER FUNCTION:—

1) Mathematical model expressing the differential equation that relates the output and the input of the system.

2) Independent of the magnitude and nature of input.

3) Doesn't provide any information about the physical structure of the system.

4) Transfer function of physically different systems can be identical.

5) If the transfer function is known, the output response can be studied for various input to understand the nature of the system.

POLES AND ZEROS OF A TRANSFER FUNCTION

$$G(s) = \frac{Y(s)}{X(s)} = \frac{C(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_n}$$

Transfer function of a control system can be written in above form.

$$G(s) = \frac{(s-z_0)(s-z_1)\cdots(s-z_m)}{(s-P_0)(s-P_1)\cdots(s-P_n)}$$

Roots of numerator polynomial → Zeros.

$z_0, z_1, z_2, \ldots z_m$ $\quad z_i, i = 0,1,2, \ldots m$.

Roots of denominator polynomial → Poles

$P_0, P_1, P_2, \ldots P_n$ $\quad P_j, j = 0,1,2 \ldots n$

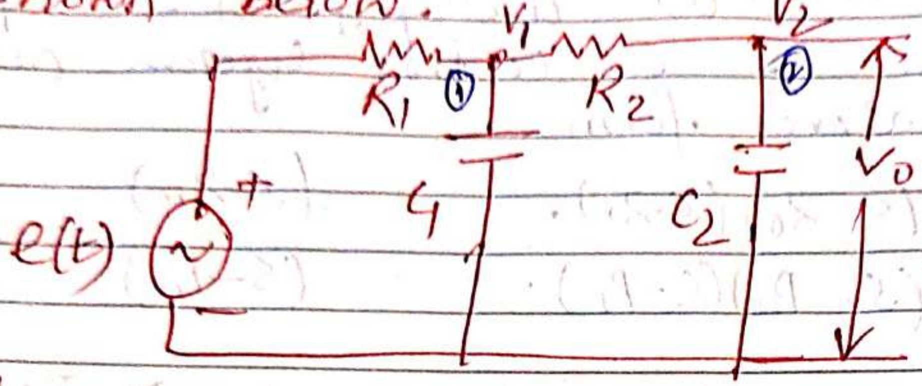
CHARACTERSTICK EQUATION OF TRANSFER FUNCTION.

$$G(s) = \frac{C(s)}{R(s)} = \frac{b_0 s^m + b_1 s^{m-1} \cdots + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_n}$$

If equate denominator polynomial to zero of T.F, it is called the characteristic equation of the Transfer function.

Problem

1) Obtain the transfer function of Electric network shown below.



ns

Applying nodal analysis for node. 1

$$\frac{V_1(t) - e(t)}{R_1} + C_1\frac{dV_1(t)}{dt} + \frac{V_1(t) - V_2(t)}{R_2} = 0.$$

Taking Laplace Transform on both sides

$$\frac{V_1(s) - E(s)}{R_1} + C_1 s V_1(s) + \frac{V_1(s) - V_2(s)}{R_2} = 0.$$

$$\Rightarrow V_1(s)\left(\frac{1}{R_1} + C_1 s + \frac{1}{R_2}\right) = \frac{E(s)}{R_1} + \frac{V_2(s)}{R_2} \quad \rightarrow (1)$$

Applying nodal analysis for node 2.

$$\frac{V_2(t) - V_1(t)}{R_2} + C_2\frac{dV_2(t)}{dt} = 0.$$

Taking Laplace Transform on both sides

$$\frac{V_2(s) - V_1(s)}{R_2} + sC_2 V_2(s) = 0.$$

$$\Rightarrow \frac{V_1(s)}{R_2} = \frac{V_2(s)}{R_2} + sC_2 V_2(s)$$

$$\Rightarrow \frac{V_1(s)}{R_2} = \left(\frac{1}{R_2} + sC_2\right) V_2(s)$$

$$\Rightarrow V_1(s) = \left(\frac{R_2}{R_2} + sR_2C_2\right) V_2(s)$$

$$V_1(s) = (1 + sC_2R_2) V_2(s) \quad \text{— (2)}$$

Putting value of $V_1(s)$ from equ^n (2) to equ^n(1)

$$(1 + R_2C_2s) V_2(s) \left(\frac{1}{R_1} + C_1s + \frac{1}{R_2}\right) = \frac{E(s)}{R_1} + \frac{V_2(s)}{R_2}$$

$$\Rightarrow V_2(s)\left[(1 + C_2R_2s)\left(\frac{1}{R_1} + C_1s + \frac{1}{R_2}\right) - \frac{1}{R_2}\right]$$

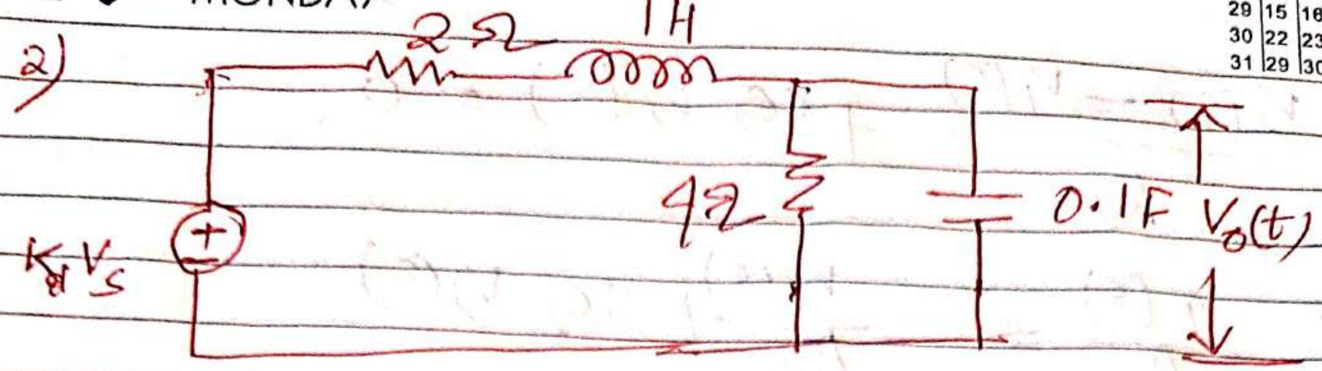
$$= \frac{E(s)}{R_1}$$

$$\Rightarrow$$

$$\frac{V_2(s)}{E(s)} = \frac{1}{R_1\left[(1 + C_2R_2s)\left(\frac{1}{R_1} + C_1s + \frac{1}{R_2}\right) - \frac{1}{R_2}\right]}$$

$$= \frac{R_2}{\left[(1 + C_2R_2s)(R_2 + C_1sR_1R_2 + R_1) - R_1\right]}$$

2)



$L \to SL = S$

$C \to \frac{1}{SC} = \frac{1}{\frac{S}{10}} = \frac{10}{S}$

Applying nodal analysis to node 1

$$\frac{KV_s - V_o}{2+S} = \frac{V_o}{4} + \frac{V_o}{\frac{10}{S}}$$

$$\Rightarrow \frac{K}{2+S} V_s = V_o\left(\frac{1}{4} + \frac{S}{10} + \frac{1}{2+S}\right)$$

$$\Rightarrow \frac{K}{2+S} V_s = V_o \cdot \left[\frac{\cancel{50+20} \; 5(S+2) + 2S(S+2) + 20}{20(2+S)}\right]$$

$$\Rightarrow \frac{K}{(2+S)} V_s = V_o\left(\frac{5S+10+2S^2+4S+20}{\cancel{20+20S} \; 20(2+S)}\right)$$

$$\Rightarrow \frac{V_o}{V_s} = \frac{2S^2+9S+30}{20K}$$

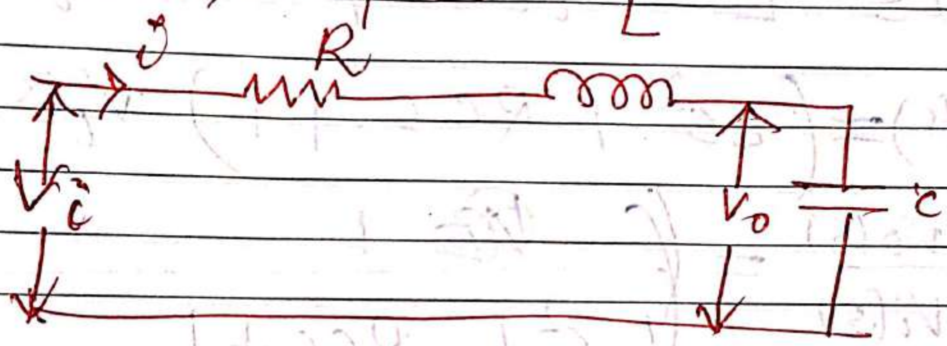
3) Mathematical Model of control System

a) Differential equation model.

b) Transfer function model.

c) State space model.

Taking an example.



$$V_i = Ri + L\frac{di}{dt} + V_0.$$

$\Rightarrow$ But $i = C\frac{dV_0}{dt}$

$$V_i = RC\frac{dV_0}{dt} + L\frac{d^2V_0}{dt^2} + V_0$$

$$V_i = L\frac{d^2V_0}{dt^2} + RC\frac{dV_0}{dt} + V_0. \quad (1)$$

Differential equation model.

27	1	2	3	4	5	6	7
28	8	9	10	11	12	13	14
29	15	16	17	18	19	20	21
30	22	23	24	25	26	27	28
31	29	30	31				

<u>Transfer function model</u>

$$\cancel{V_i(t) = Ri + L\frac{di}{dt} + V_0(t)}$$

$$V_i = L\frac{d^2 v_0(t)}{dt^2} + RC\frac{dv_0}{dt} + V_0$$

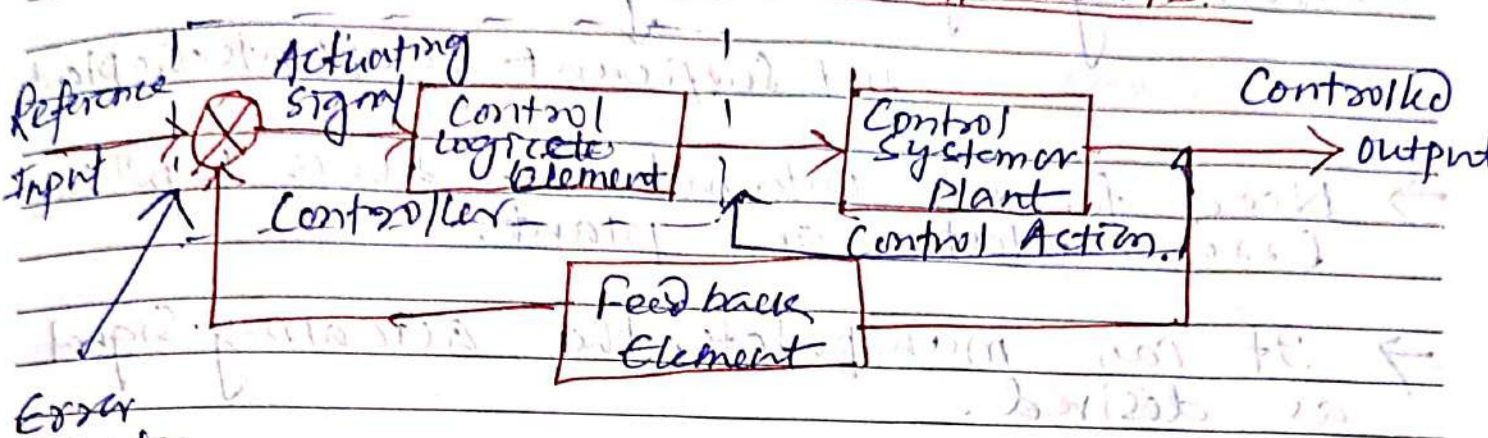
Taking Laplace Transform on both sides

$$V_i(s) = L\,s^2 V_0(s) + RCs\,V_0(s) + V_0(s)$$

$$V_i(s) = \left(Ls^2 + RCs + 1\right) V_0(s)$$

$$\Rightarrow \frac{V_0(s)}{V_i(s)} = \left(\frac{1}{Ls^2 + RCs + 1}\right)$$

CONTROL SYSTEM COMPONENTS.



Block diagram representation of a closed loop control system

Basic Elements
a) Feedback element
b) Controller
c) Control ~~Action~~ System.

Feed back Elements
The feedback element is used to feed back the output signal to the error detector for comparison with the input.

Controller:- It consists of the error detector and the control logic elements.

Error detector:- Receives the measured signal (feedback/output) and compare it with reference/input and determines the error signal also known as Actuating signal.

→ Actuating Signal → Low power level
↓
not Sufficient to operate the plant

→ Need for an intermediate device betn the error detector and plant.

→ It can manipulate the actuating signal as desired.

→ Manipulation in the form of amplification or generation of desired function.

→ Control system components → manipulation is done by them (components).

CONTROL SYSTEM COMPONENTS

→ Employed or introduced in a system to perform a specific function/purpose in the system.

→ Components can be mechanical, electrical, hydraulic, Pneumatic, thermal or any other type.

→ Modern control systems uses sensors and encoders as control system components.

Following devices - sensors. :
(1) Potentiometers.
(2) AC servomotors.
(3) D.C. Servomotors.
(4) Stepper motors.
(5) Tacho Generators.

POTENTIOMETERS.

A potentiometer is an electromechanical transducer which converts the mechanical energy (displacement) (either linear or rotational) into electrical energy. (voltage)

It is also called as Error detecting device because, It is used as an error detector in control system.

Sunday 04

Error - Find the difference betwn O/P signal and i/p signal

→ Inexpensive and easy to apply and use.

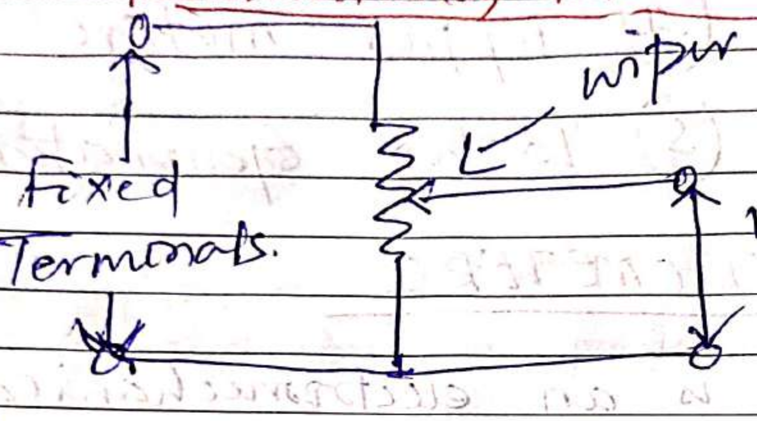
Types of

32	5	6	7	1 8	2 9
33	12	13	14	15	16
34	19	20	21	22	23
35	26	27	28	29	30

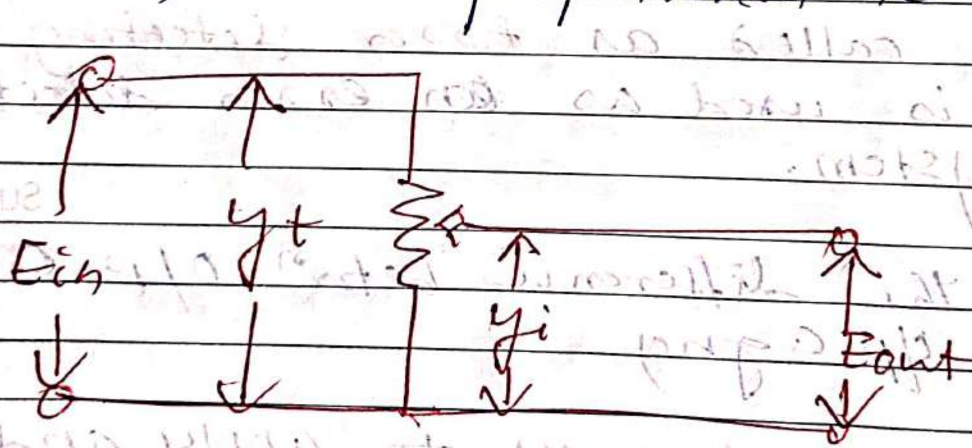
Types of potentiometers:

1) Translational (Linear) potentiometers
2) Rotational potentiometers

Translational potentiometer



When voltages is applied across the fixe terminals of the potentiometer, the output Voltage which is measured across the vari terminal is proportional to input displac



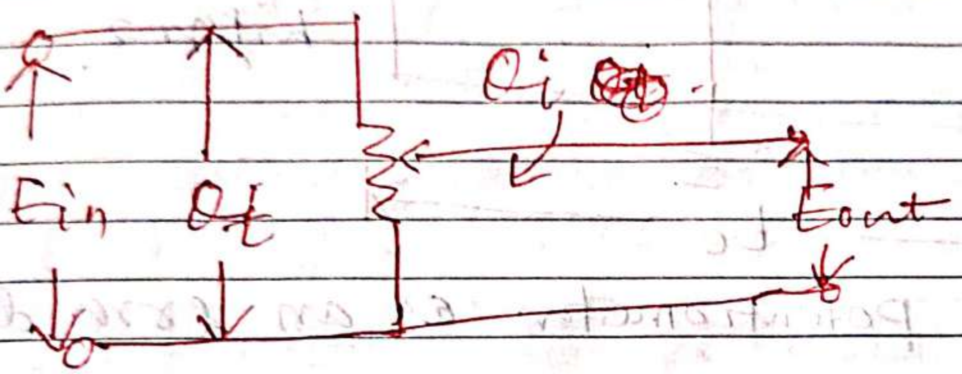
Under ideal conditions the ratio betw output voltage and input voltage is giv

$$\boxed{\frac{E_{out}}{E_{in}} = \frac{y_i}{y_t}}$$

y_i = displacement from Zero position.

y_t = Total length of the translational potentiometer.

Rotational potentiometer

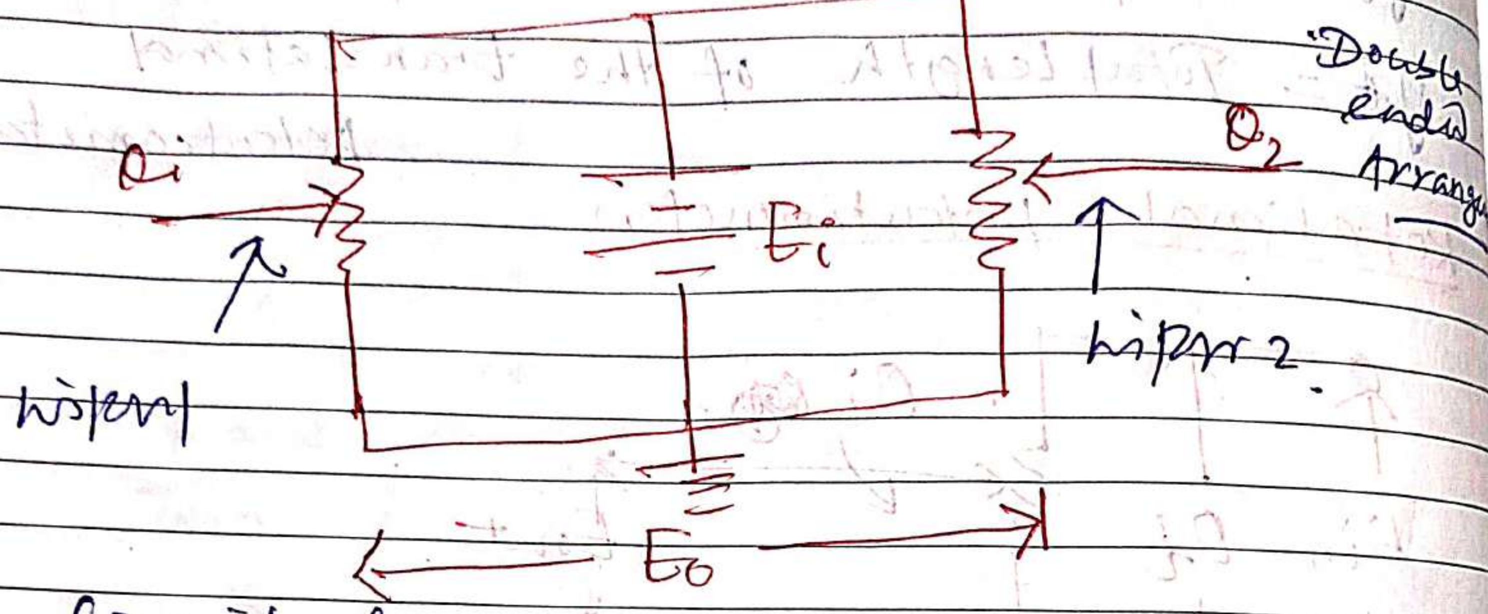


$$\boxed{\frac{E_{out}}{E_{in}} = \frac{\theta_i}{\theta_t}}$$

θ_i = Input angular displacement.

θ_t = Total length of the wiper.

→ Potentiometer can be used as a error detector to compare the positions of two remotely located shafts.



Circuit for potentiometer as an error detector

→ The output voltage E_0 is given by.

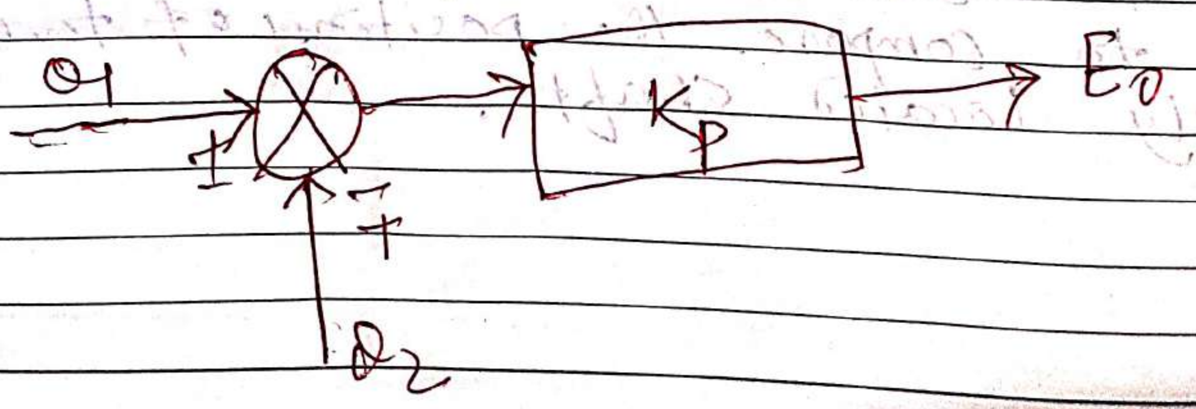
$$E_0 = K_p(\theta_1 - \theta_2)$$

normally D.C volt

K_p = $\frac{\text{The ratio of the input of excitation volt}}{\text{Total angle of rotation}}$

$= \frac{E_i}{\theta_T}$

$$E_0 = \frac{E_i}{\theta_T}(\theta_1 - \theta_2)$$



polarity of output voltage describe the relative position of the shaft.

In case of a.c, the phase difference will find the relative positions of the shafts.

Resolution of potentiometers:-

It is defined as the ratio of change in the output voltage in one step to the supply voltage.

$$\% \text{ Resolution} = \frac{\Delta E}{E_s} \times 100$$

$$\% \text{ Resolution} = \frac{100}{\text{No. of turns}}$$

TACHOGENERATORS

→ It is an electromechanical device which produces an output voltage that is proportional to its shaft speed → Electrical signal.
Mechanical signal

→ It works on the principle of induction motor.

→ Two types of Tachogenerators →

a) A.C. Tachogenerator.

b) D.C. Tachogenerator.

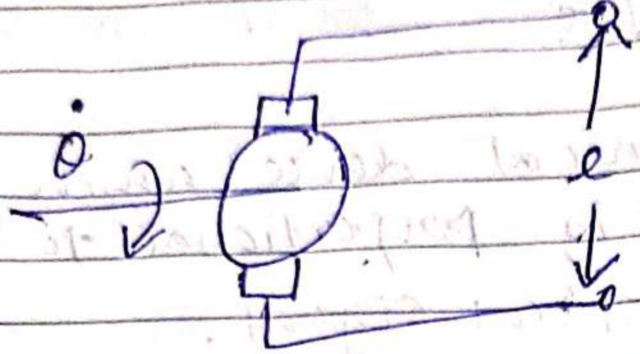
D.C. Tachogenerator

The D.C. Tachogenerator resembles a small motor in that it comprises of a stator with a permanent magnetic field, a rotating armature circuit and a commutator and brush assembly.

→ The rotor is connected to the shaft.

→ The output voltage is proportional to the angular velocity of the shaft.

→ polarity of the output voltage depends on the direction of the rotation of the shaft.



Dynamics of D.C. Tachogenerator can be represented by the equation.

$$e(t) = K_t \frac{d\theta(t)}{dt} = K_t \dot{\theta}$$

e = output voltage (volts)
θ = rotor displacement (radians)
K_t = sensitivity of the Tachogenerator. (volts per rad/sec)

Problem:-
a) High-frequency ripple generated by the commutator-brush assembly
b) Maintenance is difficult.

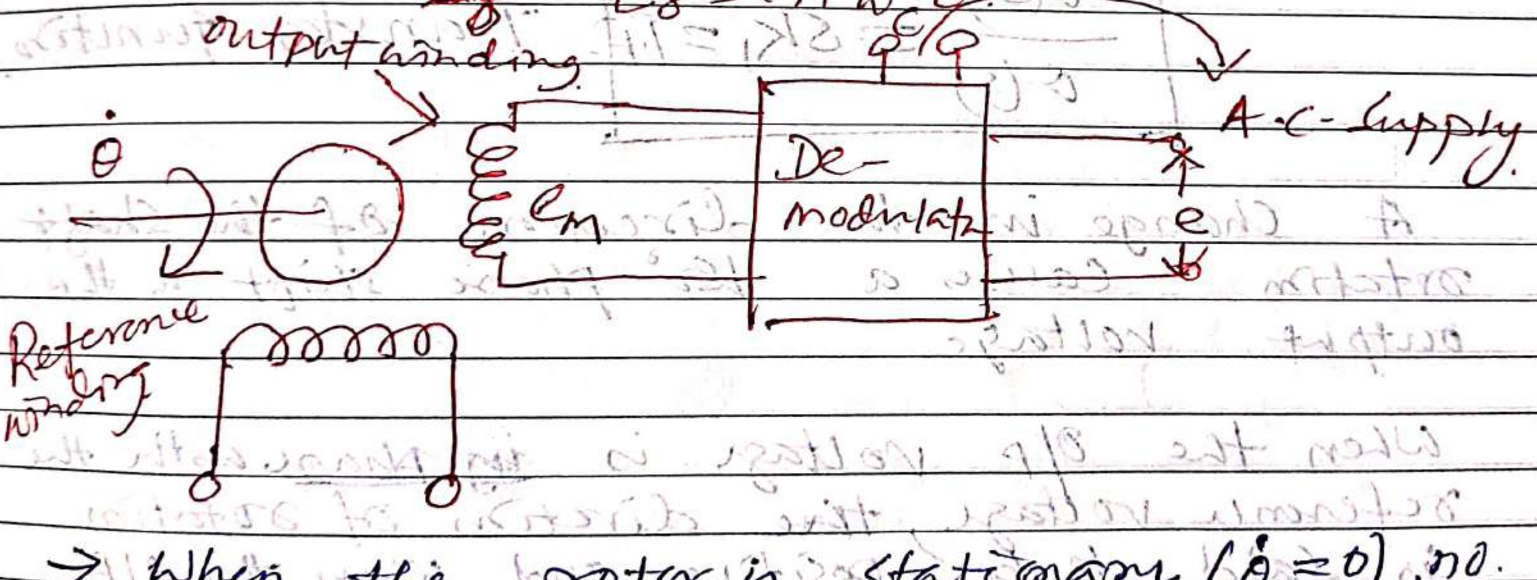
A.C. Tachogenerator:

→ Resembles two phase induction motor.
→ Comprises of → a) two stator windings
(Arranged in space quadrature)

b) Rotor is not conductively connected to an external ckt.

→ A sinusoidal voltage is applied to the excitation winding (reference)

$$e_r = E_r \sin \omega_c t$$



→ When the rotor is stationary $(\dot{\theta} = 0)$, no emf is induced in the output winding and therefore the output voltage is zero.

→ When the motor rotates, a voltage at the reference a voltage at the reference frequency ω_c is induced.

→ The magnitude of the output voltage is proportional to the rotational speed.

$$e \propto \omega$$
$$e = K_1 \omega$$
$$e = K_1 \frac{d\theta}{dt}$$

Taking Laplace Transform we get

output $E(s) = K_1 s \theta(s)$ input.

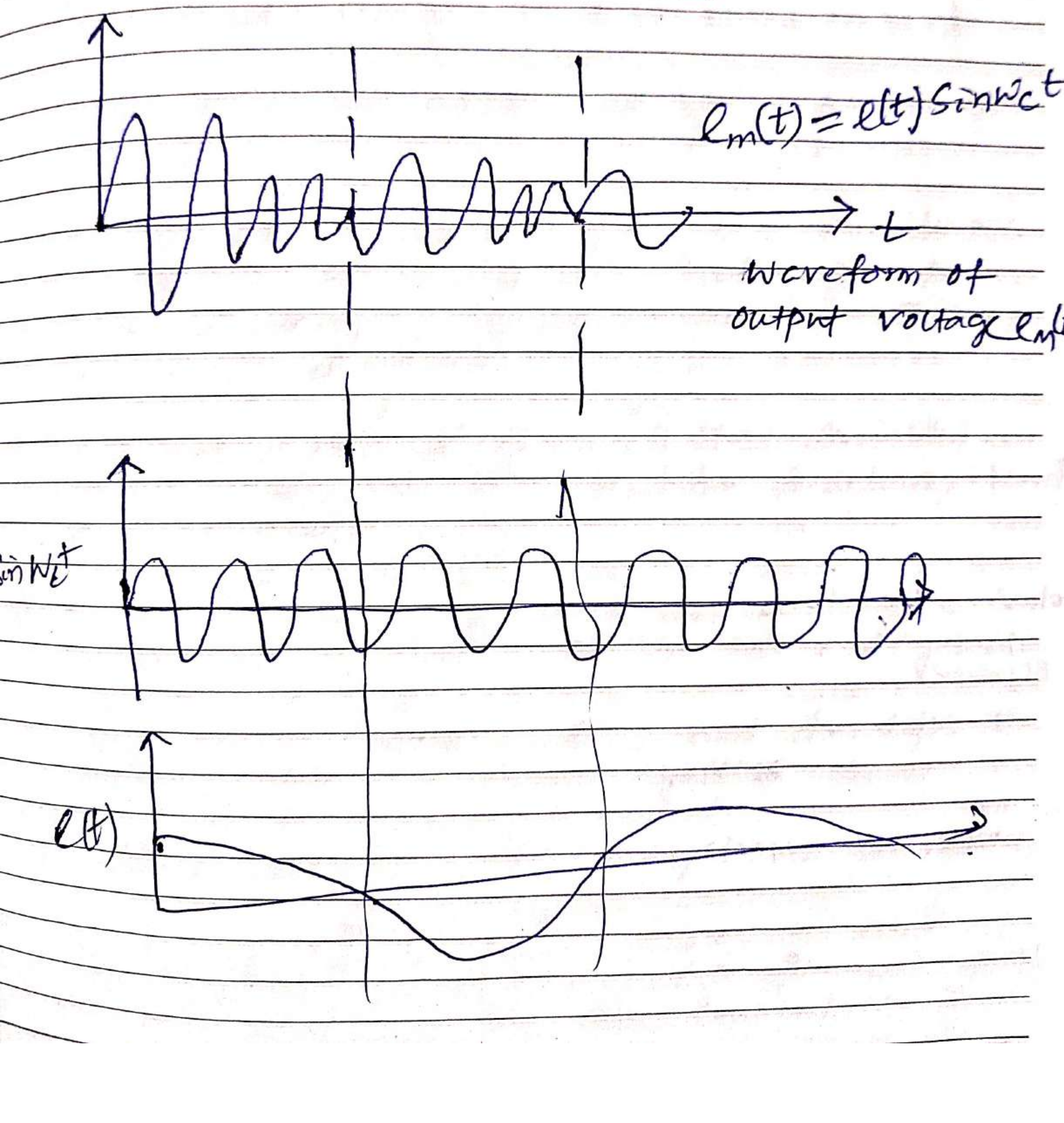
$$\boxed{\frac{E(s)}{\theta(s)} = sK_1 = T.F}$$ Transfer function.

A change in the direction of the shaft rotation causes a 180° phase shift in the output voltage.

When the o/p voltage is in phase with the reference voltage, the direction of rotation is said to be positive and when the o/p voltage is 180° out of phase with the reference voltage the dirn of rotation is said to be negative.

The output of an A.C. Tachogenerator is thus in modulated form:

$$\boxed{e_m(t) = e(t)\sin w_c t}$$



SERVOMOTORS

Servomechanism.
↓
Servo + mechanism.
↓
Servant
(slave)

⇒ Servo mechanism is defined as a closed loop control system in which a small input power controls a larger output power in a strictly proportionate manner.

⇒ The controlled variable (output variable) is some mechanical variable like position, velocity or acceleration.

⇒ Servo systems are used in automatic control system which works on the error signals.

⇒ The error signals are used to drive the motor used in servo systems.

⇒ Motors used in servo systems are called SERVO MOTORS.

Servomotors usually drive a final control element. These motors are coupled to the output shaft i.e load through gear train for power matching.

These motor are used to convert electrical signal applied into the angular velocity or movement of shaft.

Requirement of a Good Servomotor.

→ Inertia of the rotor should be as low as possible.

→ Its response of the servomotor should be as fast as possible. for quickly changing error signal it must react with good response.

This is achieved by keeping the torque weight high.

→ It should have linear torque-speed characteristics

→ Linear relationship between electrical control signal and rotor speed over a wide range.

→ It should be easily reversible.

→ Its operation should be stable without any oscillation or overshoot.

→ The motor should withstand frequent starting operation

TYPES OF SERVOMOTORS.

Classified depending upon the nature of the electric supply to be used for its operation.

Servomotors.
- A.C. Servomotors.
- D.C. Servomotors.
  - Armature Controlled D.C. Servomotors.
  - Field controlled D.C. Servomotors.

D.C. Servomotor

⇒ More or less same as normal d.c motor.

⇒ D.C. Servomotor behaves like a mechanical transducer which convert d.c. voltage into mechanical signal i.e angular displacement

⇒ All d.c. Servomotors are essentially separately excited type. This ensure torque–speed characteristics.

⇒ The control of d.c. Servomotor can be from field side and armature side.

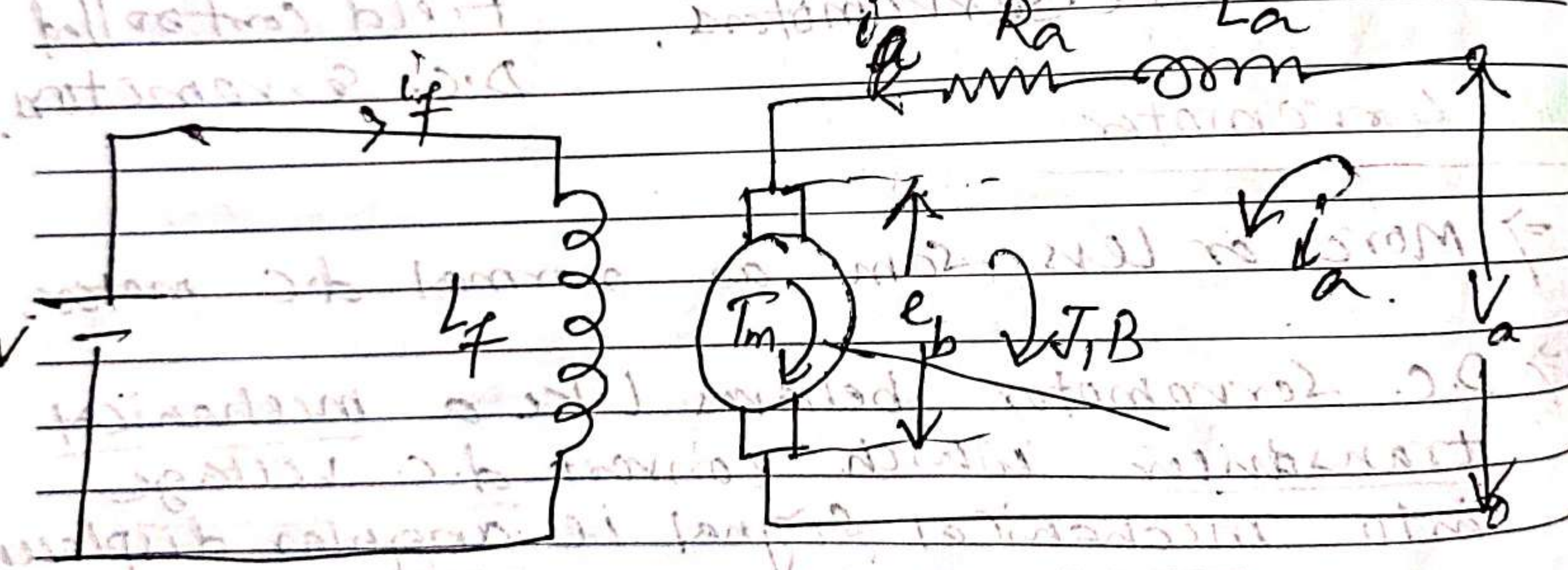
⇒ Depending upon this d.c. servomotor ~~can be~~ from field side are classified

a) field controlled D.C. servomotor.

b) Armature controlled D.C. servomotor.

Armature controlled D.C. Servomotor:

⇒ In this motor, the field current is held constant and armature current is varied to control the torque.



Circuit diagram for Armature controlled D.C. servomotor.

Let

R_a = Armature Resistance.

L_a = Armature Inductance.

i_a = Armature Current.

V_a = Armature voltage.

W_m = Angular velocity.
e_b = back emf.
J = moment of Inertia
i_f = field current.
L_f = field inductance.

Now, air flux ϕ is proportional to field current

$$\phi \propto i_f$$

$$\phi = K_f i_f \quad \text{--- (8)}$$

i_f = const, armature current i_a produces the Torque T_m (due to application of V_a) which in turn produces angular shaft in the motor shaft.

produced torque T_m is proportional to flux ϕ and and armature current i_a.

$$T_m \propto \phi i_a$$

$$T_m \propto K_f i_f i_a \quad (\because \phi = K_f i_f)$$

$$T_m = \underbrace{K_t K_f i_f}\, i_a.$$

$$\boxed{T_m = K_1 i_a} \qquad K_1 = K_t K_f i_f$$

=> As the speed of the motor shaft increases a back emf (e_b) is induced in the armature circuit.

=> This back emf (e_b) is proportional to the speed of the motor shaft and direction of the back emf is opposite to the armature i/p voltage. V_a.

$$e_b \propto \omega$$

$$e_b = K_b \frac{d\theta}{dt} \quad \text{--- (1)}$$

Applying KVL in the armature ckt, we get

$$V_a = i_a R_a + L_a \frac{di_a}{dt} + e_b \quad \text{--- (2)}$$

The load-torque equation is given by.

$$J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} = T_m = K_1 i_a \quad \text{--- (3)}$$

Taking Laplace Transform of equ^n (1), (2) & (3).

$$E_b(s) = K_b \, s \, \theta(s) \quad \text{--- (4)}$$

$$V_a(s) = I_a(s) R_a + s L_a I_a(s) + E_b(s)$$

$$\Rightarrow V_a(s) - E_b(s) = (R_a + sL_a) I_a(s) \text{ --- (5)}$$

$$s^2 J \theta(s) + sB\theta(s) = T_m(s) = K_1 I_a(s)$$

$$[Js^2 + Bs]\,\theta(s) = T_m(s) = K_1 I_a(s) \quad —(6)$$

from equ^n (5)

$$I_a(s) = \frac{V_a(s) - E_b(s)}{R_a + sL_a}$$

$$I_a(s) = \frac{V_a(s) - K_b s\theta(s)}{R_a + sL_a}$$

Substitute $I_a(s)$ in equ^n (6) we get

$$[Js^2 + Bs]\,\theta(s) = K_1 \frac{[V_a(s) - K_b s\theta(s)]}{sL_a + R_a}$$

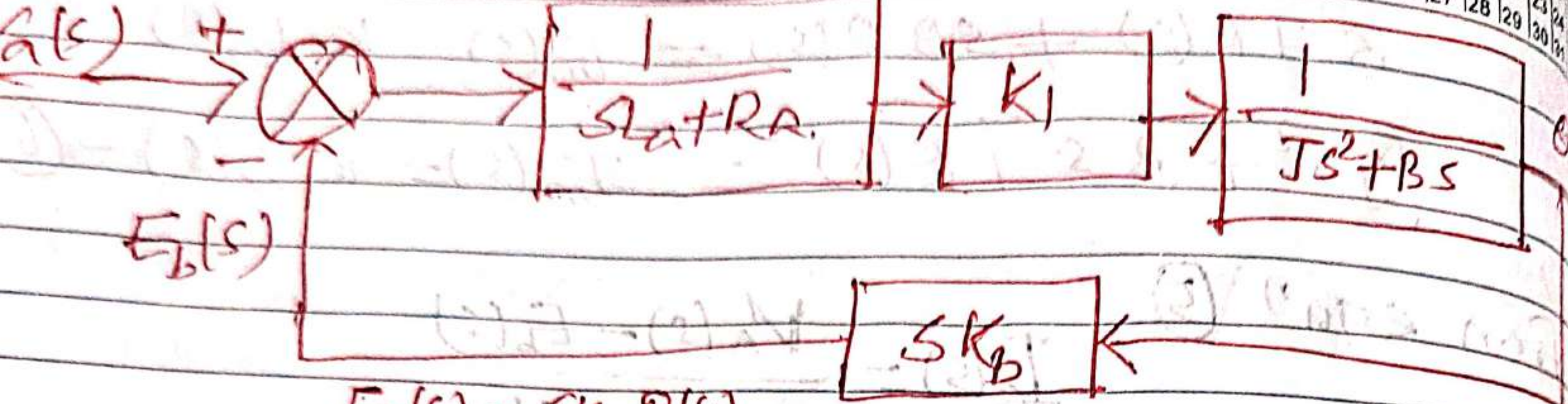
$$\Rightarrow \theta(s)\left[\frac{(Js^2 + Bs)(sL_a + R_a)}{K_1} + K_b s\right] = V_a(s)$$

$$\frac{\theta(s)}{V_a(s)} = \frac{K_1}{(Js^2 + Bs)(sL_a + R_a) + K_1 K_b s}$$

$$\boxed{\frac{\theta(s)}{V_a(s)} = \frac{\dfrac{K_1}{(Js^2 + Bs)(sL_a + R_a)}}{1 + \dfrac{K_1 K_b s}{(Js^2 + Bs)(sL_a + R_a)}}}$$

Tra[nsfer]
fun[ction]

Sunday



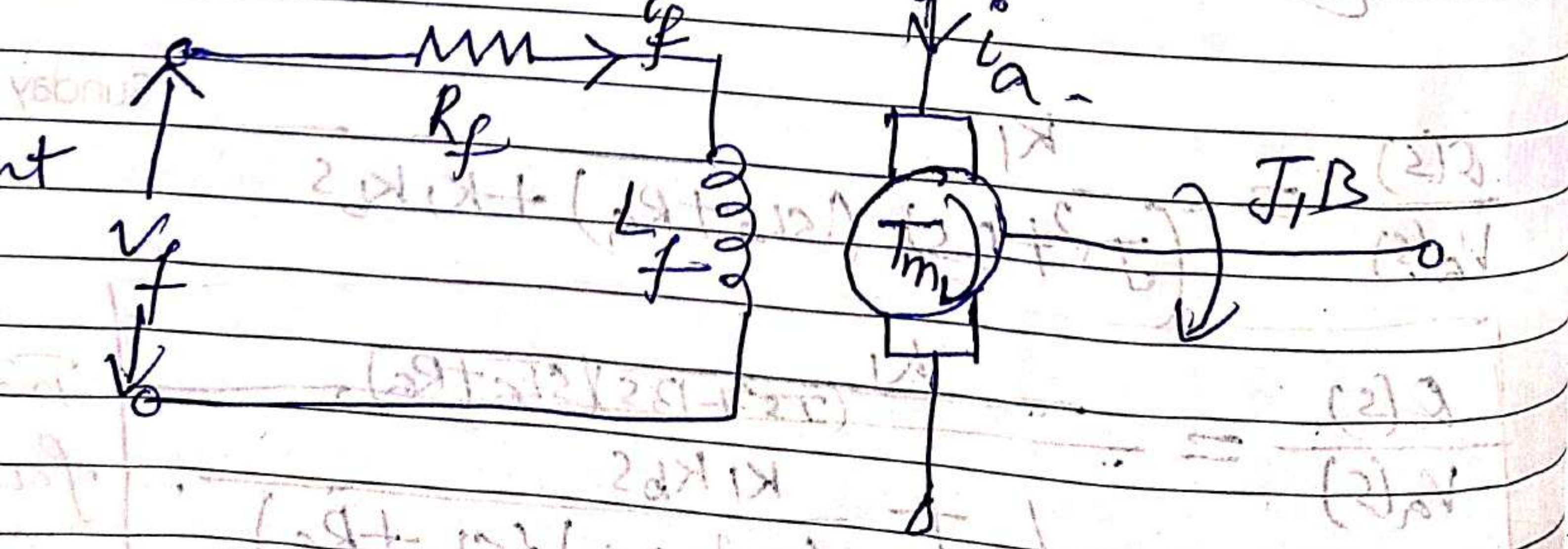
$E_b(s) = sK_b\,\theta(s)$

Block diagram Armature Controll D.C. Servo

Field controlled D.C. Servomoter.

⇒ In the field controlled D.C. Servomotor variable input voltage (field voltage V_f) is applied to field winding and armature current (i_a) is kept cons[t]

⇒ The output is the angular shift in the motor shaft.



Let R_f = field resistance,
L_f = field inductance.
i_f = field current.
V_f = Variable field voltage.
θ = Angular displacement of motor shaft.
T_m = Torque developed by the motor.
B = Co-efficient of viscous friction.
J = Moment of inertia.
i_a = armature current is kept const and the motor shaft is controlled by the input voltage V_f.

As the i/p voltage is applied a current i_f flows which produces flux in the machine
↓
torque at the motor shaft.
↓
Angular shift in the motor shaft

$$T_m \propto i_f$$

$$T_m = K_f i_f \quad \text{--- (1)} \quad K_f = \text{motor torque const.}$$

field equation

$$V_f = i_f R_f + L_f \frac{di_f}{dt} \quad \text{--- (2)}$$

Torque equation:

$$J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = T_m \quad \text{— (3)}$$

Taking Laplace Transform of eqn, we get

$$T_m(S) = K_f I_f(S)$$

$$(SL_f + R_f)\, I_f(S) = V_f(S) \quad \text{— (4)}$$

$$(S^2J + BS)\,\theta(S) = T_m(S) = K_f I_f(S) \quad \text{— (5)}$$

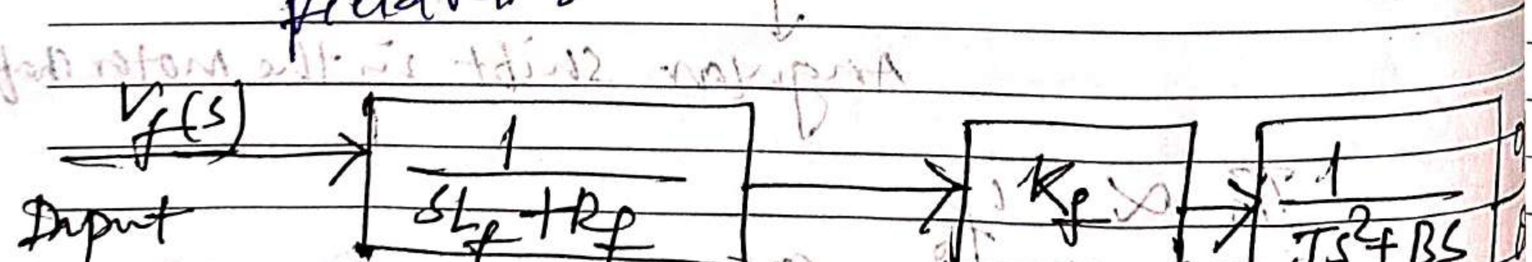
$$(S^2J + BS)\,\theta(S) = K_f \frac{V_f(S)}{(SL_f + R_f)}$$

Angular shift

$$\Rightarrow \boxed{\frac{\theta(S)}{V_f(S)} = \frac{K_f}{(SL_f + R_f)(S^2J + BS)}}$$

Field voltage

Transfer function



Block diagram representation of field controlled D.C. servomotor.

Armature controlled D.C. Servomotor	Field Controlled D.C. Servomotor
1) Better performance is expected due to closed loop.	1) Poor performance due to open loop structure.
2) The inductance of the armature ckt is small and hence T_a is negligible. This reduces the order of the system equn also.	2) The inductance of the field ckt is not negligible. It offers significant T_f
3) Speed of response of the motor to changing current is fast.	3) Speed of response of the motor to changing current is slow
4) The damping due to the armature resistance and the motor friction an extra damping is produced. Increased damping improves the transient response of the system.	4) Improve damping is not possible.

A.C. SERVOMOTOR

A.C servomotor
- 2-φ A.C servomotor.
- 3-φ A.C servomotor.

⇒ These motor has two parts.
(i) Stator. (ii) Rotor.

⇒ These are two phase induction motor, Stator has two distributed windings.

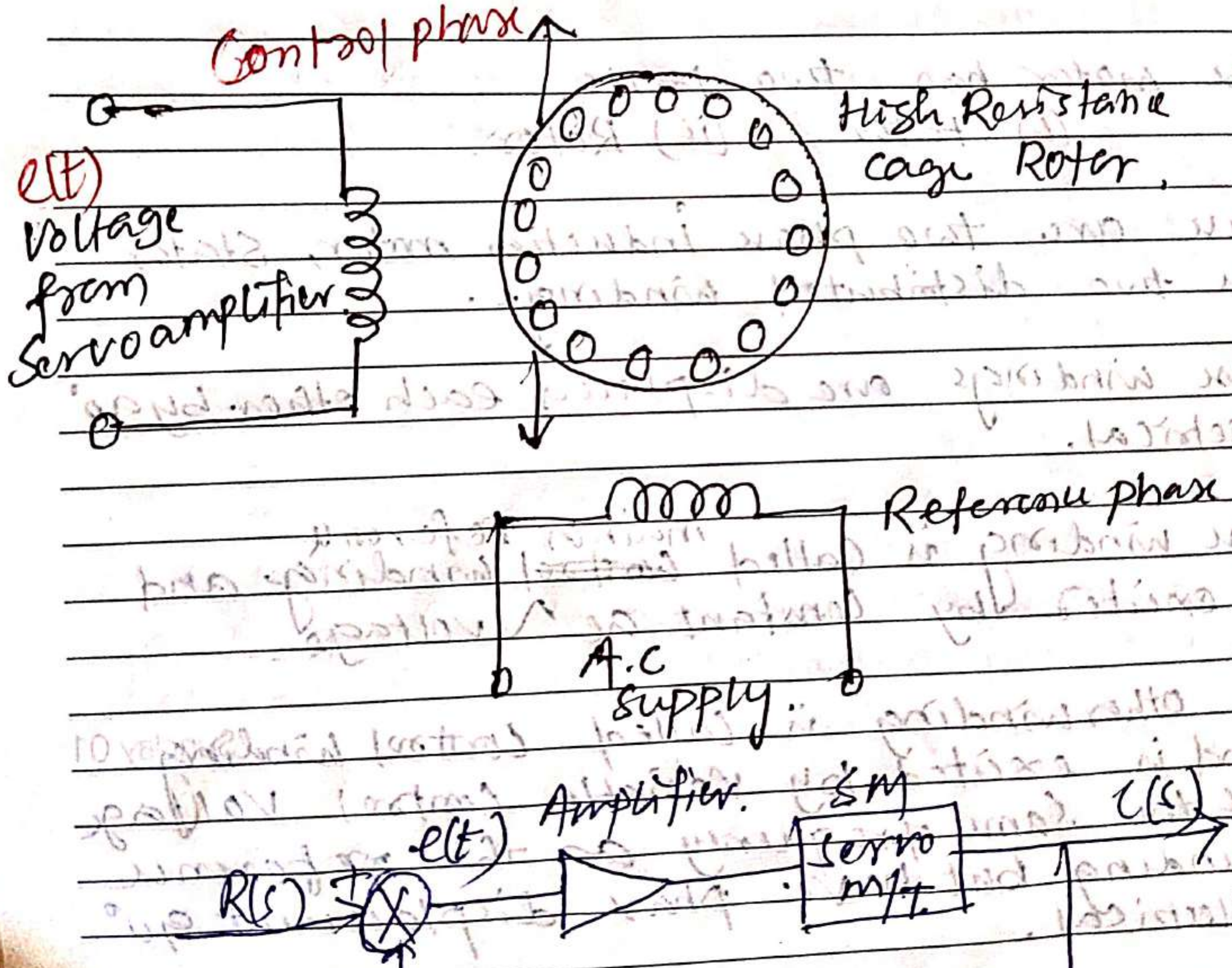
⇒ These windings are displaced each other by 90° electrical.

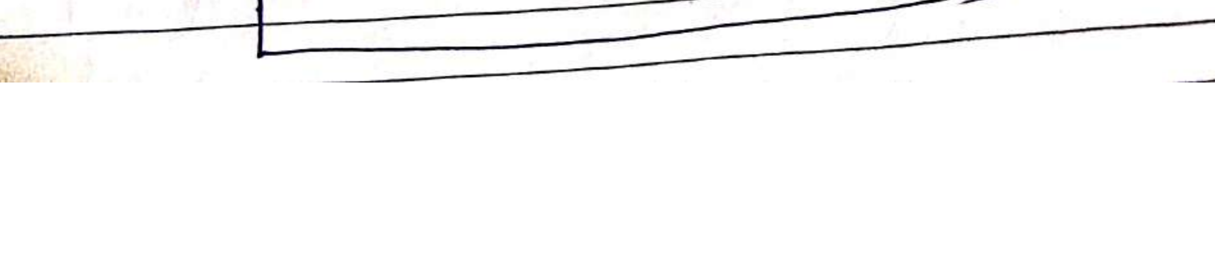
⇒ One winding is called ~~Control~~ main or reference winding and is excited by constant ac voltage

⇒ The other winding is called control winding and is excited by variable control voltage of the same frequency as the reference winding but have phase displacement 90° electrical.

=> The variable control voltage for control winding is obtained from servo amplifier.

=> The direction of the rotation of the rotor depends upon phase relationship of

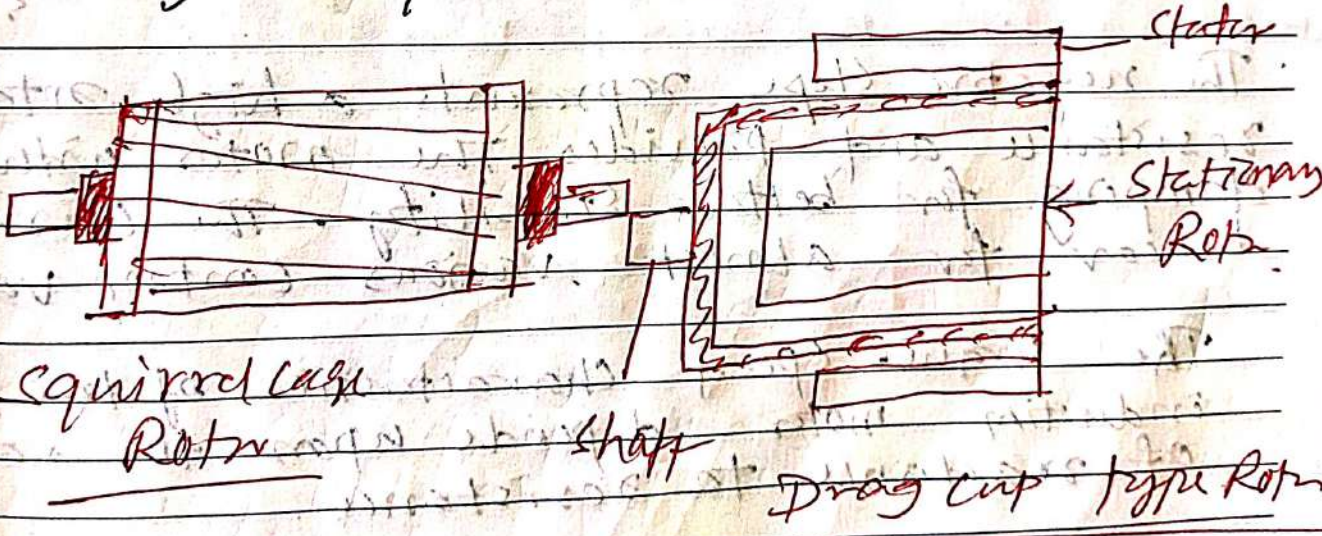




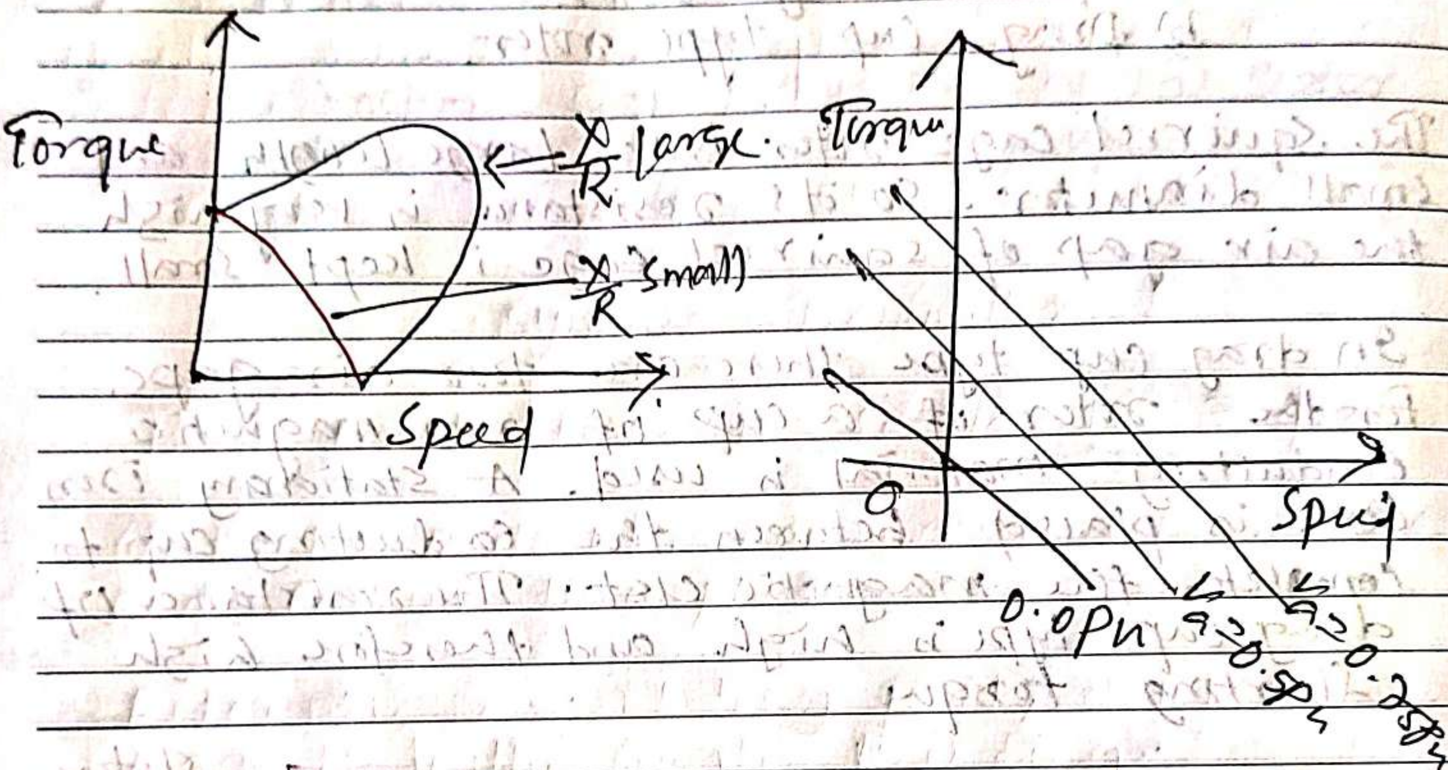
Two types of rotor.
a) Squirrel cage rotor.
b) Drag cup type rotor.

The squirrel cage rotor have large length and small diameter. So its resistance is very high. The air gap of squirrel cage is kept small.

In drag cup type there are two air gaps. For the rotor of a cup of non-magnetic conducting material is used. A stationary iron core is placed between the conducting cup to complete the magnetic ckt. The resistance of drag cup type is high and therefore high starting torque



Torque-Speed characteristics



The negative slope represents a high rotor resistance and provides the motor with positive damping for better stability. The curve is linear for almost various control voltages.

The torque-speed characteristics of two phase induction motor depends upon the ratio of reactance to resistance.

For high resistance and low reactance, the characteristic is linear for large ratio X to R it becomes non-linear as shown.

Three phase AC Servomotors.

⇒ Three phase induction motors with the voltage control are used as a servo motor for the applications in the high power servo systems.

⇒ A three phase squirrel cage Induction motor is a highly nonlinear coupled ckt device. It is used as a linear decoupled machine by using a control method known as a vector control or Field Oriented control.

⇒ The current in this type of machine is controlled in such a way that the torque and flux are decoupled. The decoupling result in high speed and high torque respon

SYNCHROS

A synchros is an electromagnetic transducer which converts the angular position of a shaft into electrical signal.

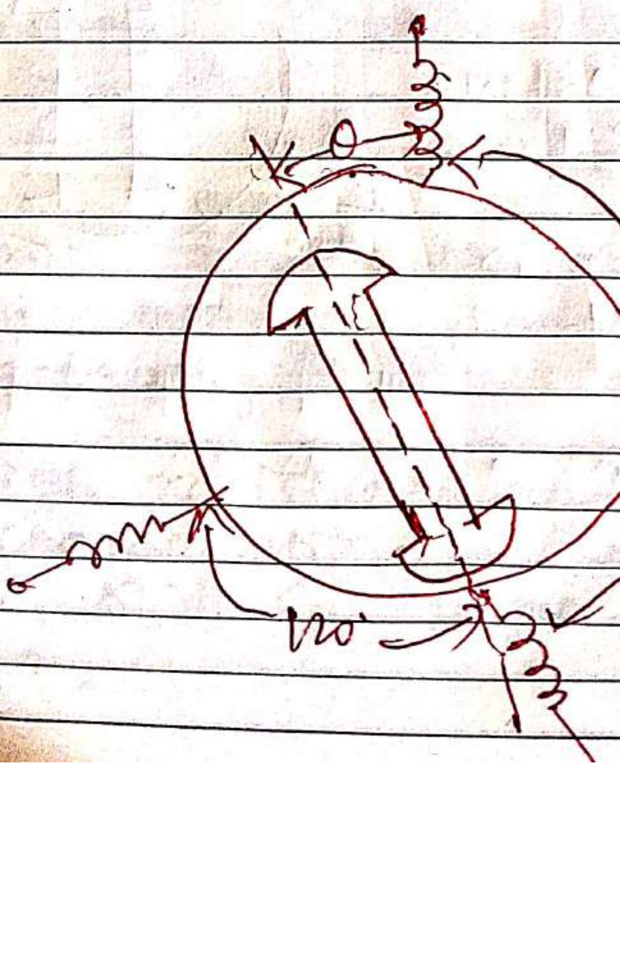
Synchros are used as detectors and encoders

Synchro Transmitter

The construction is similar to 3-ϕ alternator

Stator is made of laminated silicon steel and carries three phase star connected winding

Rotor is a rotating part, dump-bell shaped magnet with single winding.



A single phase AC voltage is applied to rotor through slip ring.

Let the voltage applied be

$$e_r = E_r \sin w_0 t$$

Magnetizing current will flow in the rotor coil. It produces sinusoidal varying flux and distributed in air gap, because of transformer action voltage get induced in all stator coil which is proportional to cosine of angle between stator and rotor coil axis.

Now consider rotor of synchro transmitter is at an angle θ, the voltages in each stator coil with respect to neutral are

$$E_{an} = KE_r \sin w_0 t \cos\theta$$

$$E_{bn} = KE_r \sin w_0 t \cos(\theta + 120^\circ)$$

Sunday 08

$$E_{cn} = KE_r \sin w_0 t \cos(\theta + 240^\circ)$$

Magnitude of stator terminal voltages are

$$E_{cb} = E_{cn} - E_{bn}$$

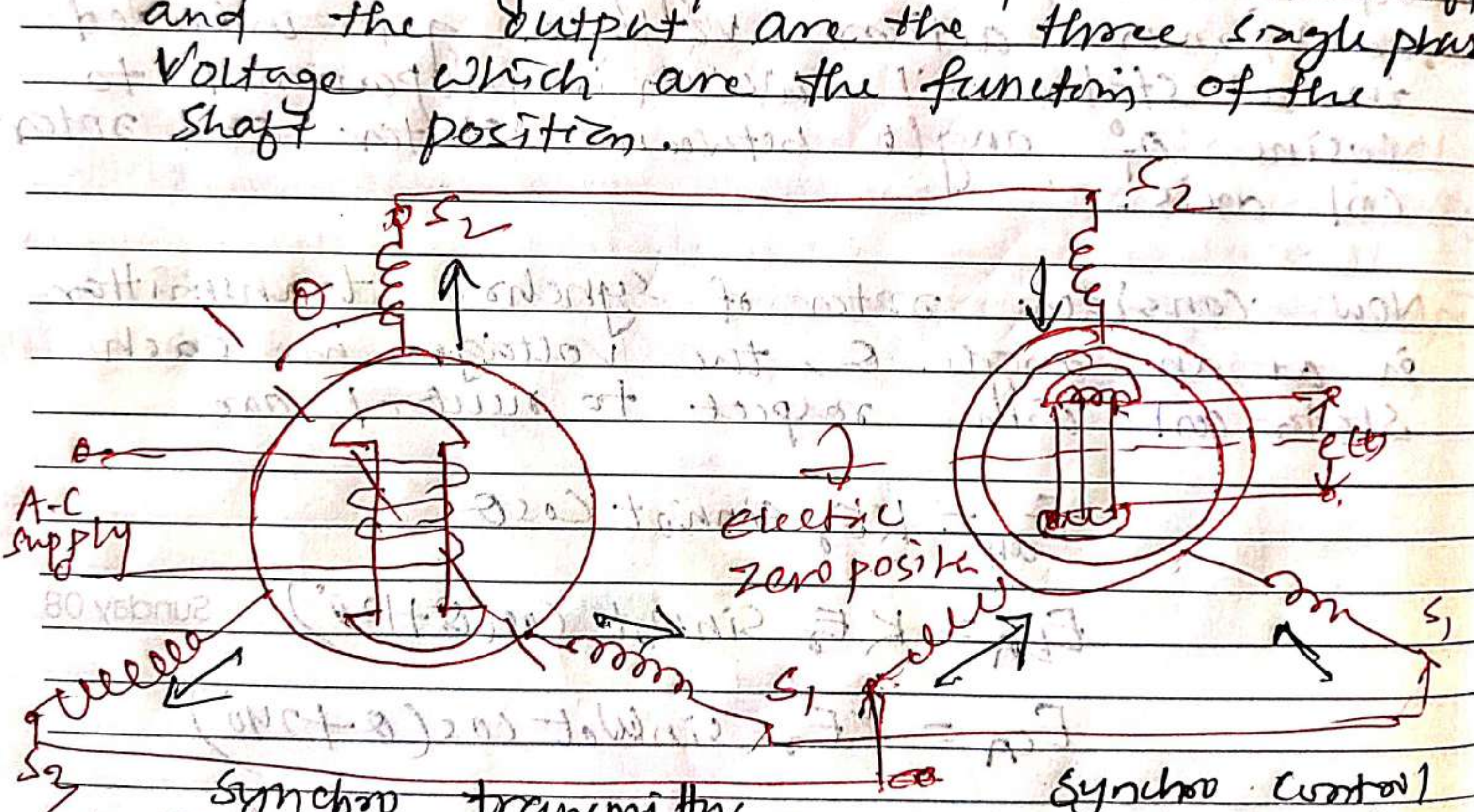
$$E_{cb} = \sqrt{3} KE_r \sin w_0 t \sin(\theta)$$

$$E_{ac} = \sqrt{3} KE_r \sin w_0 t \sin(\theta + 120^\circ)$$

$$E_{ba} = \sqrt{3} KE_r \sin w_0 t \sin(\theta + 240^\circ)$$

When $\theta = 0$, the maximum induced voltage will be. E_{an} and E_{cb} will be zero. This position of the rotor is defined as electrical zero of the transmitter and is used as the reference for indicating angular position of the rotor.

Thus the input to the synchro transmitter is the angular position of the rotor shaft and the output are the three single phase voltage which are the functions of the shaft position.



Synchro error detector.

principle of operation of synchro control transformer is same as that of synchro transmitter.

Rotor of synchro control transformer is cylindrical type.

The combination of synchro transmitter and synchro control transformer is used as error detector.

The function of error detector is to convert the difference of two shaft position into electrical signal.

The o/p of synchro transmitter is i/p to synchro control transformer.

Same current will flow in the stator winding of synchro control transformer but in opposite dirn.

The voltage across the rotor terminals of control transformer is

$$e(t) = K_1 V_r \cos\phi \sin \omega_0 t$$

ϕ = angular displacement betwn two rotor.

when $\phi = 90°$ $e(t) = 0$.

This position is called electrical zero position.

Let the transmitter rotate through an angle θ as dirn indicated and let control transformer rotate in the same dirn through an angle α.

Then $\phi = (90 - \theta + \alpha)$

putting ϕ

$$e(t) = K_1 V_r \sin(\theta - \alpha) \sin \omega_0 t.$$

We see that when the two rotor shaft are not in alignment the rotor voltage of control transformer is approximately a sine function of the difference between the two shaft angle

RULES FOR BLOCK DIAGRAM REDUCTION

Block diagram is the pictorial representation of components of control Engineering or control system.

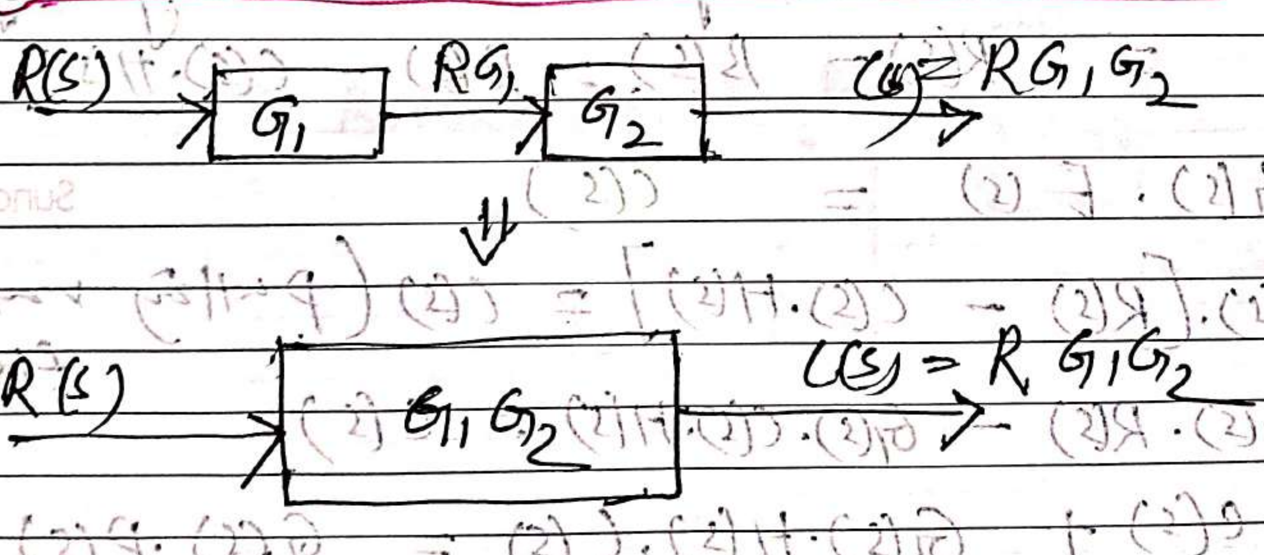
Complex system having more number of block diagram in complex form.

To get the Transfer function we need to simplify the block diagram of the C.S.

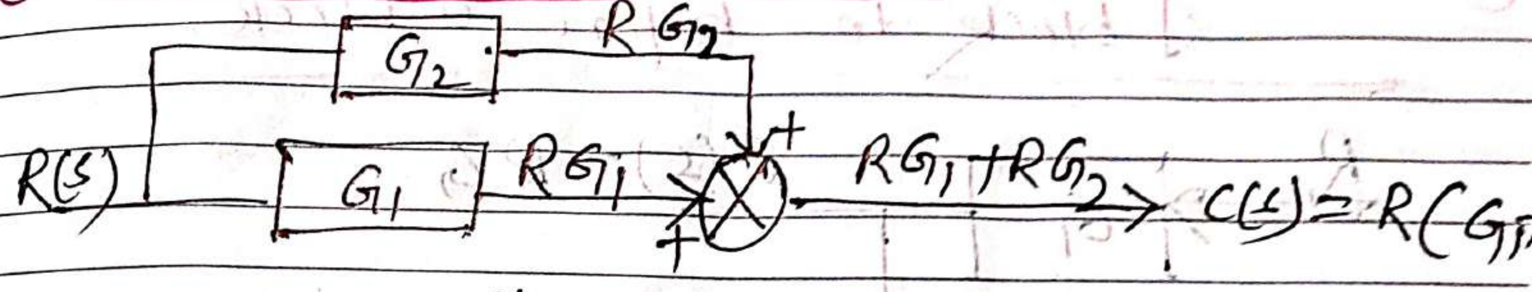
To reduce the block diagram, we should follow some rules mentioned below.

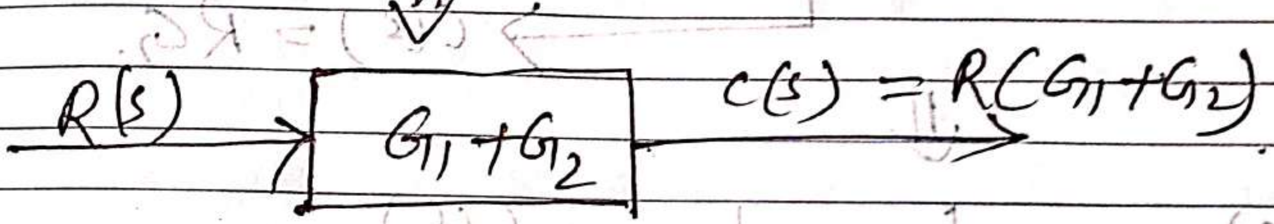
Rules :-

(1) Blocks are in Series or Cascade

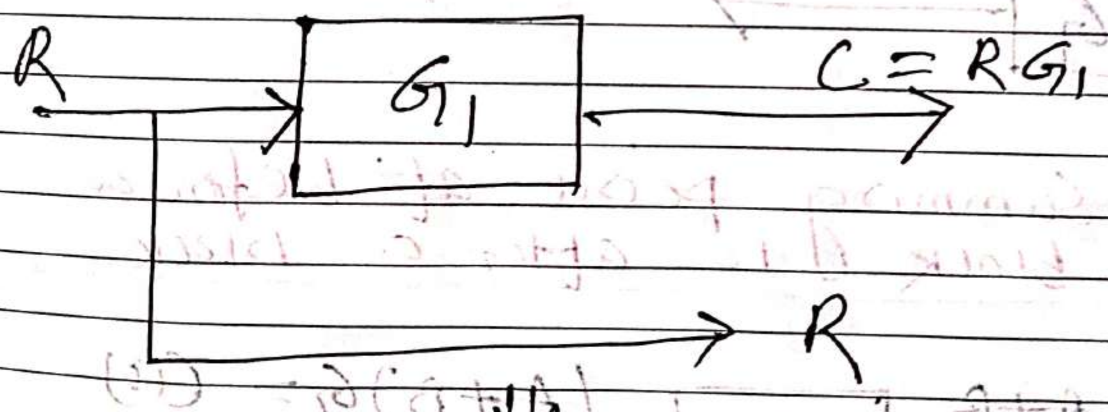


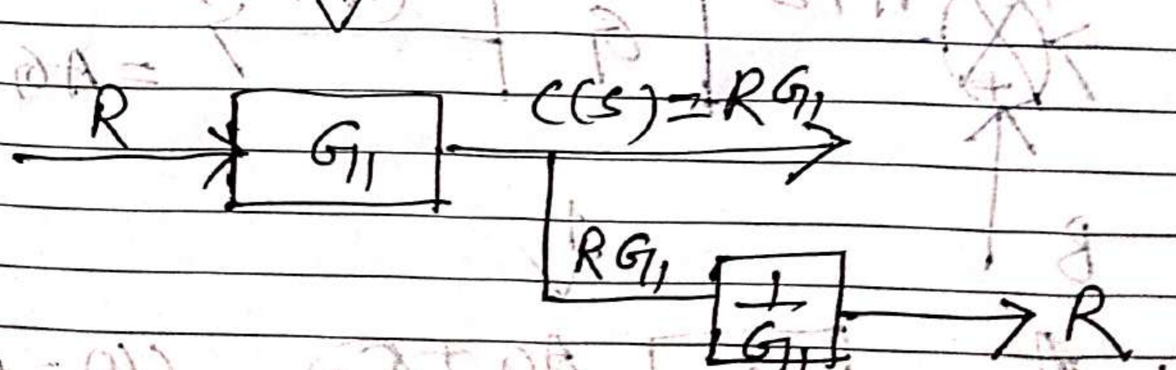
② Blocks are in parallel.



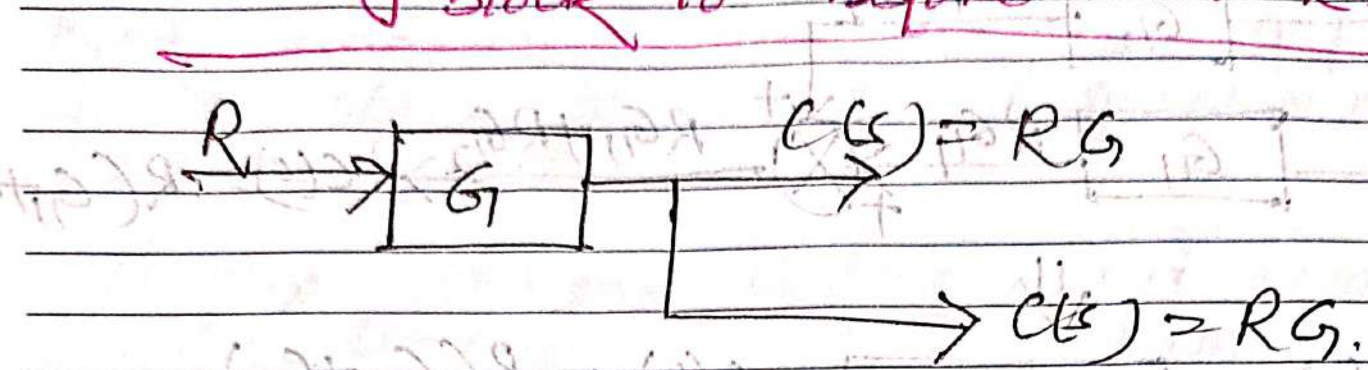


③ Moving Take off point before a block to after a block.

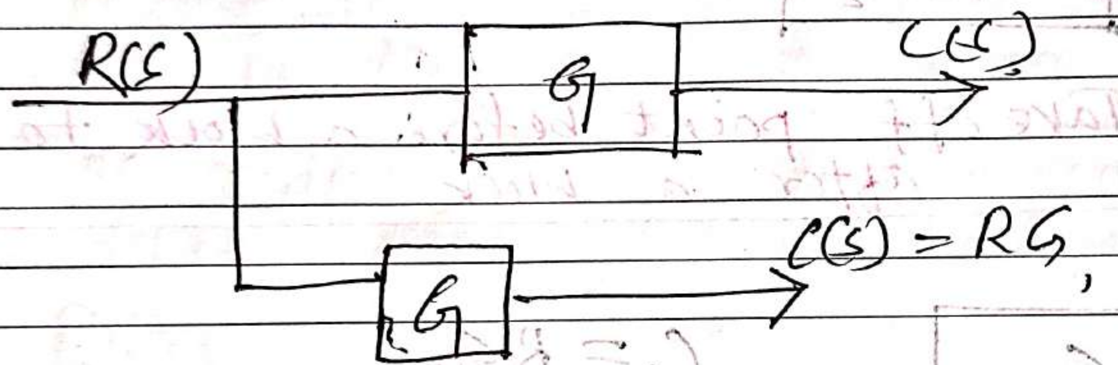




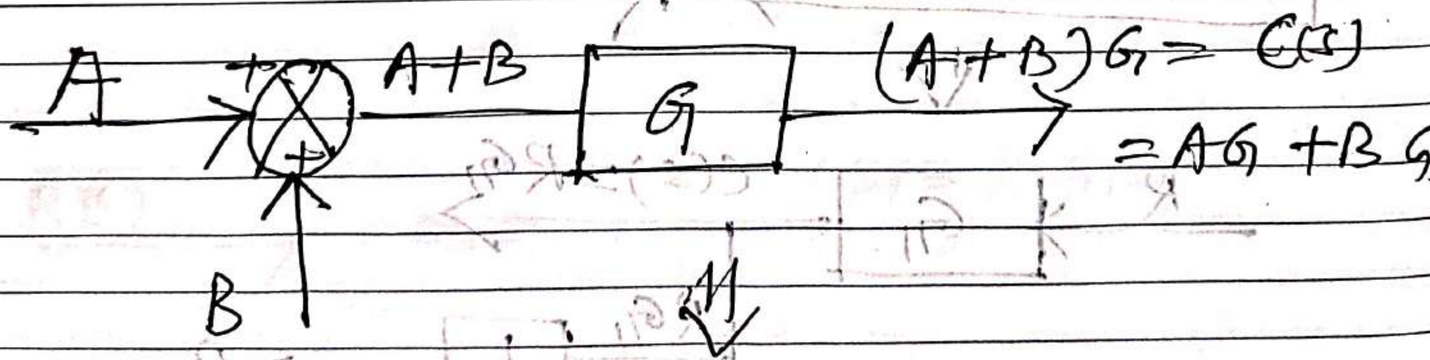
(4) Moving take off point ~~be~~ after a block to before a block.



⇓



(5) Moving summing point ~~at~~ before a block to after a block



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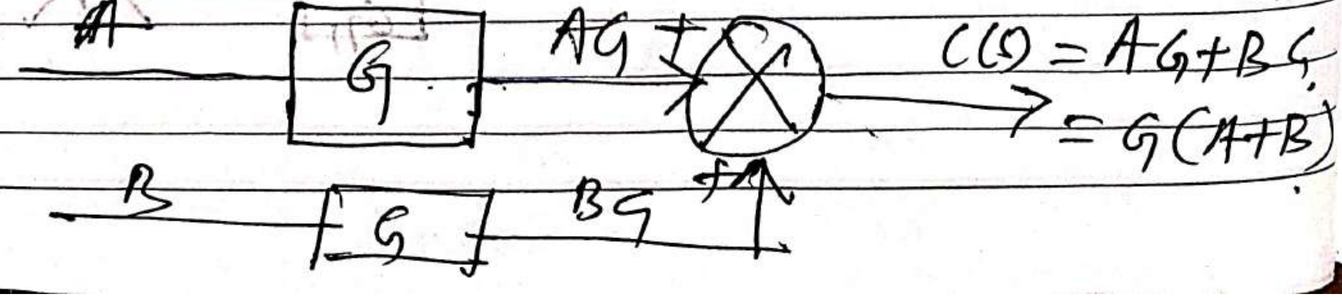
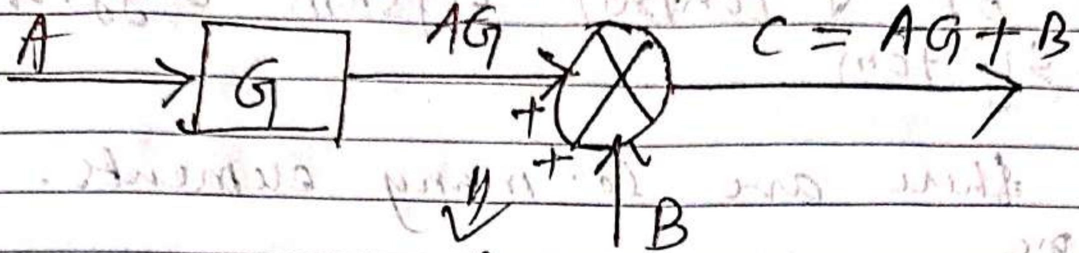


(6) Moving summing point after a block to before a block



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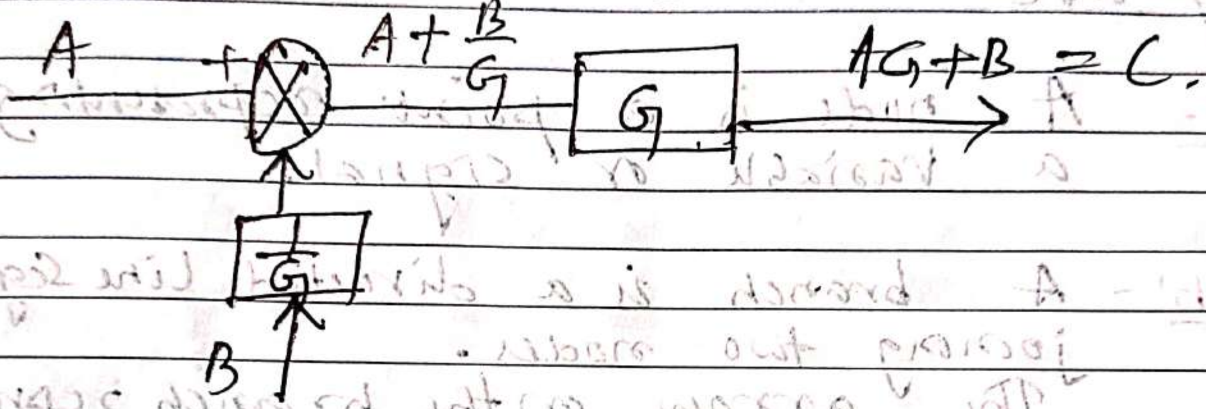
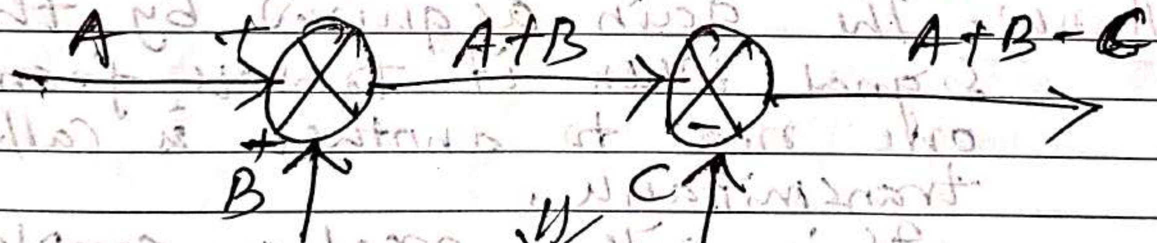


(7) Inter change of summing point



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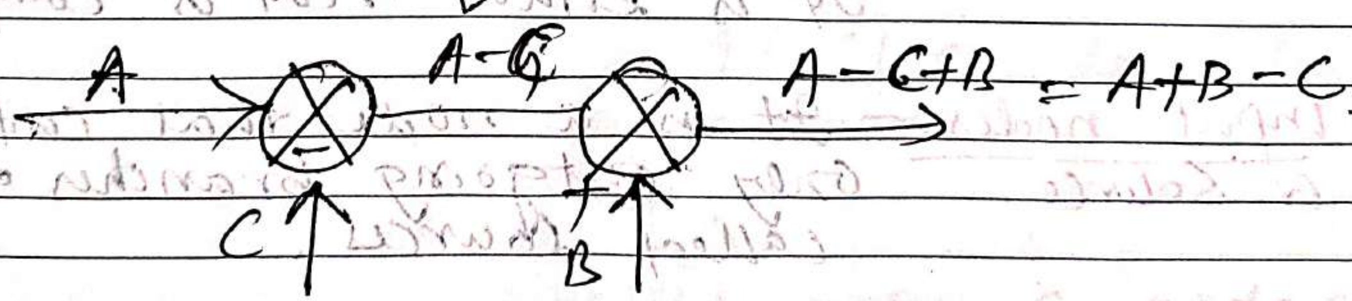


SIGNAL FLOW GRAPH.

SFG is a graphical representation of components of control system engg. i.e. control system.

In SFG there are so many elements. they are

a) **Node:-** A node is a point representing a variable or signal.

b) **Branch:-** A branch is a directed line segment joining two nodes.
The arrow on the branch represent the signal flow.

c) **Transmittance:-** The gain acquired by the signal when it travels from one node to another is called transmittance.
It is either real or complex.

d) **Input nodes:- or Source.** It is a node that contains only outgoing branches or called sources.

e) **Output node: or Sink.** It is a node that has only incoming branches.

f) Mixed node :- It is a node that has both incoming and outgoing branches

g) Path : A path is a traversal of connected branches in the direction of arrows. The path shouldn't cross a node more than one.

path is two types
(i) open path (ii) Closed path.

h) Forward Path or forward Path gain:-

Path from input to output is called forward Path.
product of all branch node is called forward Path gain.

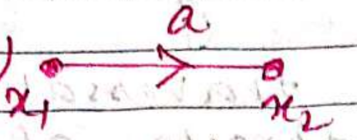
i) Loop Gain :- Product of all gains of Loop is called Loop gain.

(j) Non-touching Loop :- If Loop don't have common node.

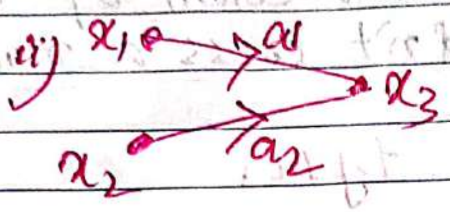
k) Individual Loop :- starting from a node and after moving certain distance on the graph and come to the same node and not touching node more than one

SIGNAL FLOW GRAPH ALGEBRA

Rule: 1

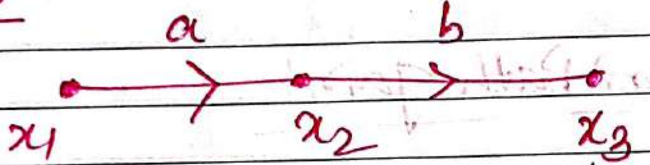
(i) 

$x_2 = ax_1$

(ii) 

$x_3 = a_1x_1 + a_2x_2$

Rule 2

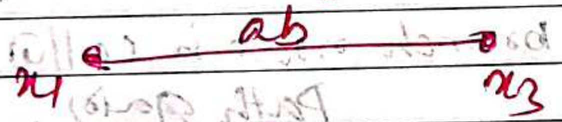


$x_2 = ax_1$

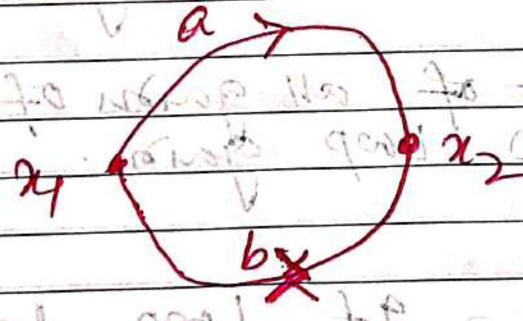
$x_3 = bx_2$

$= b(ax_1)$

$= abx_1$

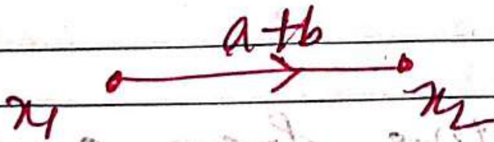


Rule 3

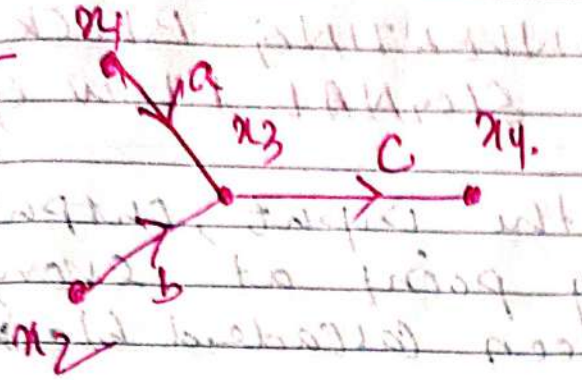


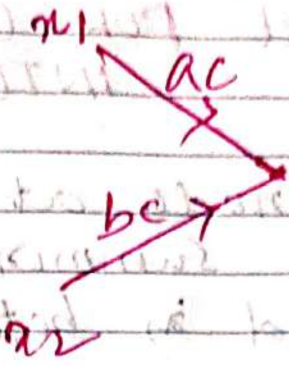
$x_2 = ax_1 + bx_1$

$x_2 = (a+b)x_1$



Rule: 4





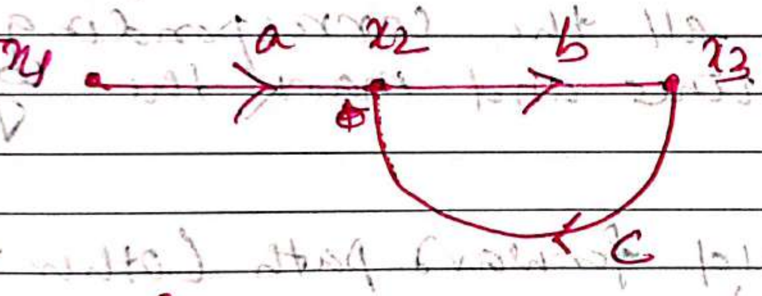
$x_4 = acx_1 + bcx_2$

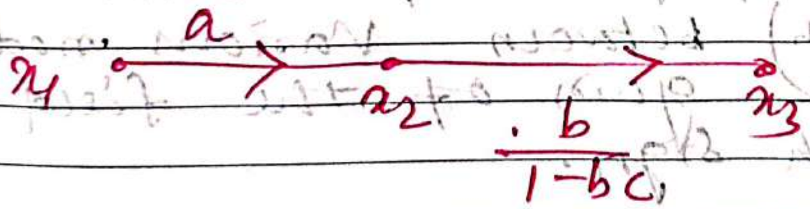
$$x_3 = ax_1 + bx_2$$

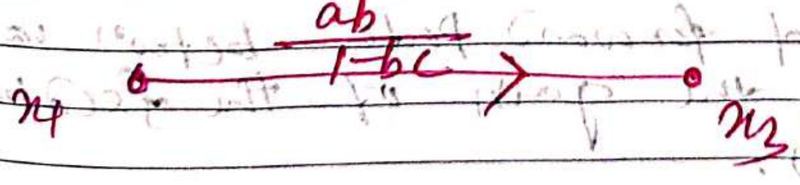
$$x_4 = cx_3 = c\,(ax_1 + bx_2)$$

$$x_4 = acx_1 + bcx_2$$

Rule: 5







PROCEDURE FOR CONVERTING BLOCK DIAGRAM INTO SIGNAL FLOW GRAPH.

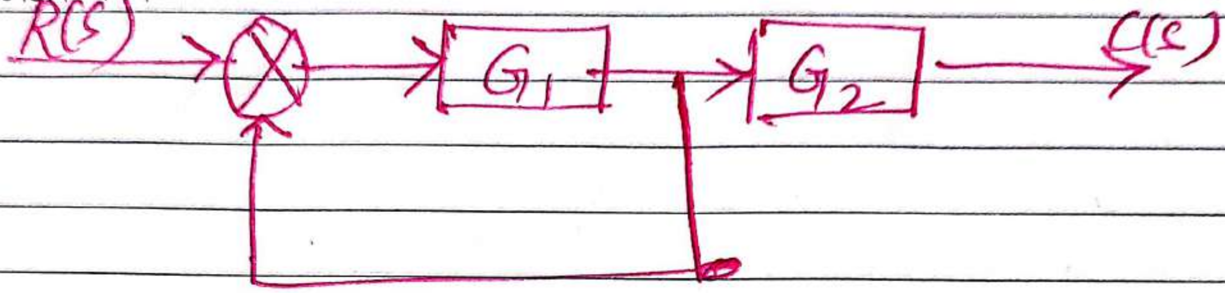
1) Assume node at the input, output, at every summing point at every branch point and in between cascaded blocks, takeoff.

2) Draw the nodes separately as small circle and number the circle in the order 1,2,3

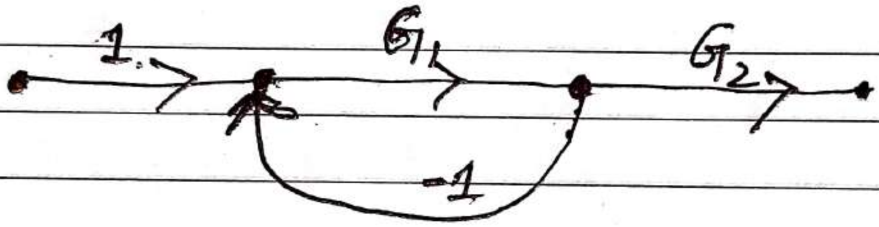
3) From the block diagram find the gain between each node in main forward path and connect all the corresponding circle
4) by straight lines and mark the gain on the node.

4) Draw the field forward path (other than main forward path) between various nodes and mark the gain of the field forward path along with sign.

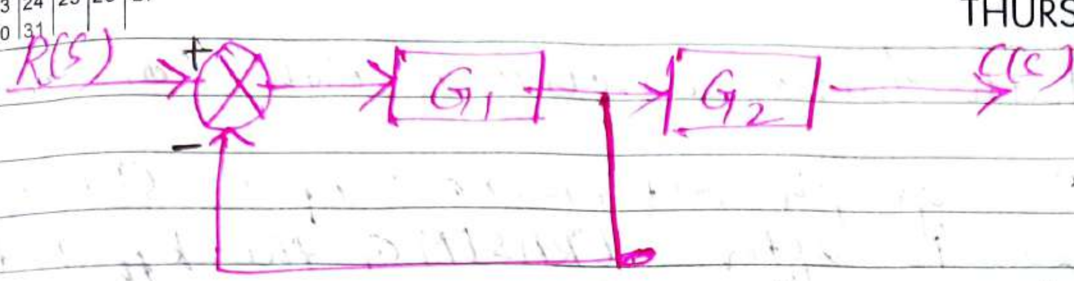
5) Draw the field forward path between various nodes and mark the gain of the feedback paths and sign.

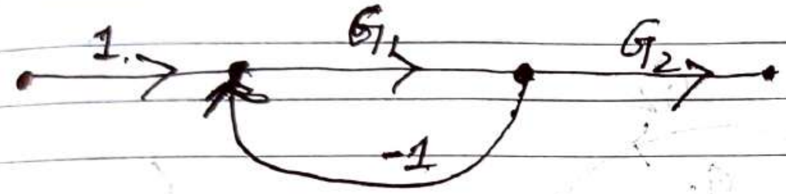


R(S)
G1
G2
C(S)



1
G1
G2
-1





MASON'S GAIN FORMULA

Let $R(s)$ = Input Signal.
$C(s)$ = Output Signal.

The overall transmittance (gain) can be determined by Mason's gain formula

$$T = \sum_{K=1}^{K} \frac{P_K \Delta_K}{\Delta}$$

P_K = Forward Path transmittance of K_{th} forward Path.

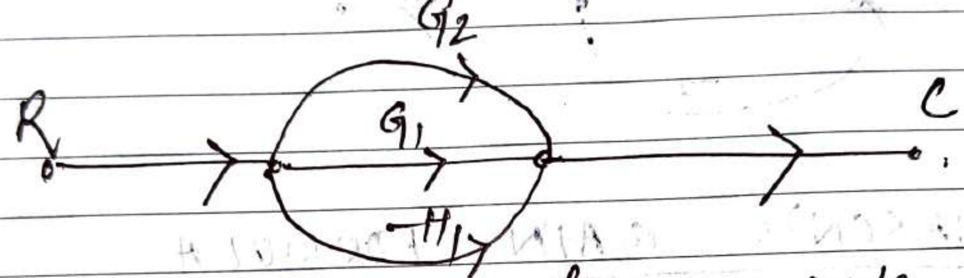
Δ = Graph determinant.

= 1 − (Sum of all individual Loop transmittance)
+ (Sum of loop transmittance products of all possible pairs of Non Touching Loops)
− (Sum of loop transmittance product of all possible triplets of Non-Touching Loops)
+ (- - -) − (- - -)

Wk	M	T	W	T	F	S	S
35	30						1
36	2	3	4	5	6	7	8
37	9	10	11	12	13	14	15
38	16	17	18	19	20	21	22
39	23	24	25	26	27	28	29

Δ_K = Path factor associated with concerned path
= The graph determinant of a SFG which exists after ERASING the K_{th} path from the graph.

Problem.1



There are two forward path.

$$P_1 = G_1 \quad \text{and} \quad P_2 = G_2.$$

Two closed loops are

$$L_1 = -G_1H_1 \quad \text{and} \quad L_2 = -G_2H_1$$

As both the loop are touching the both the forward paths.

$$\Delta_1 = 1, \qquad \Delta_2 = 1.$$

The graph determinant

$$\Delta = 1 - (L_1 + L_2)$$

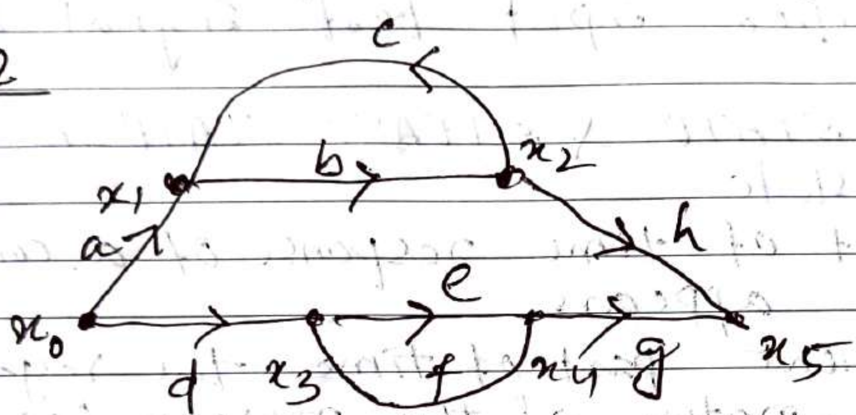
$$\Delta = 1 - (-G_1H_1 - G_2H_1)$$

$$\Delta = 1 + G_1H_1 + G_2H_1$$

$$\frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_1 \cdot 1 + G_2 \cdot 1}{1 + G_1H_1 + G_2H_1} = \frac{G_1 + G_2}{1 + G_1H_1 + G_2H_1}$$

Problem. 2



findout the overall transmittance using Mason's gain formula.

Soln In this SFG, there are two Path and two Loops.

Path $P_1 = abh$ = Path gain of Path P_1.

$P_2 = deg$ = Path gain of Path P_2.

$L_1 = bc$ = Loop gain of Loop 1.

$L_2 = ef$ = Loop gain of loop 2.

$$\Delta = 1 - (L_1 + L_2) + L_1L_2.$$

$$\Delta_1 = 1 - L_2, \quad \Delta_2 = 1 - L_1$$

$$T = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \frac{abh(1-ef) + deg(1-bc)}{1-(bc+ef)+bcef}.$$

—: TIME RESPONSE ANALYSIS :—

Time response of a control system means, how a system behaves in accordance with time when a specified input test signal is applied.

TRANSIENT STATE & STEADY STATE RESPONSE.

Transient State

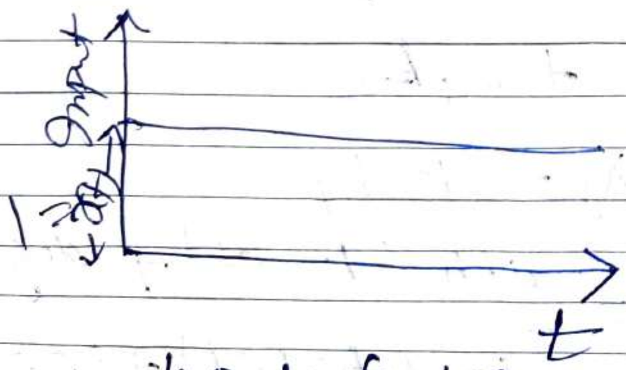
Initial part of time response of a control system transient appears.

The transient part of time the response reveals the nature of the response (i.e. oscillatory or overdamped) and speed.

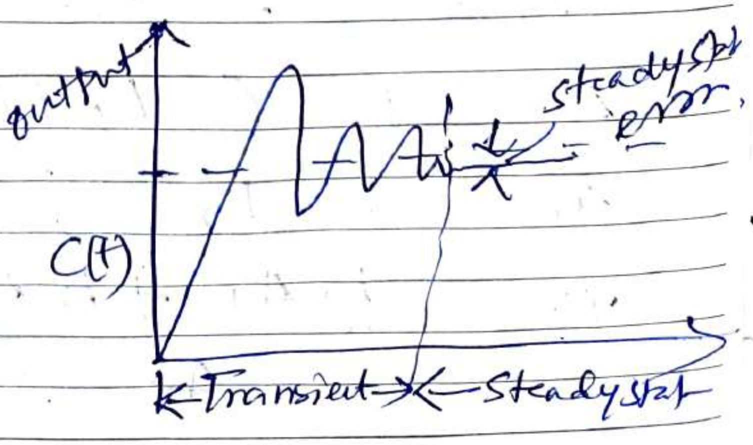
Steady State

After transient, steady state is achieved. Steady state means state of the output of the control system as the time approaches infinity.

It reveals (steady state) accuracy of a control system. Steady state error is observed if the actual output doesn't match with the input.

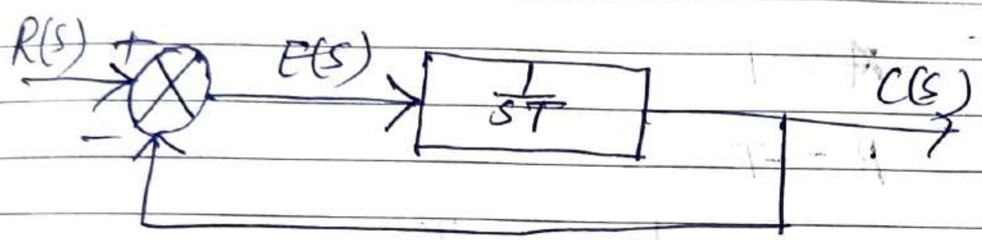


Input test signal



TIME RESPONSE OF A FIRST ORDER CONTROL SYSTEM

A control system is said to be first order if highest power of s of the characteristic eqn is one.



Here $G(s) = \frac{1}{ST}$ $\quad H(s) = 1$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)\cdot H(s)} = \frac{\frac{1}{ST}}{1+\frac{1}{ST}\cdot 1}$$

$$= \frac{\frac{1}{ST}}{\frac{ST+1}{ST}} = \frac{1}{1+ST}.$$

$$\frac{C(s)}{R(s)} = \frac{1}{1+ST}.$$

$C(s) = R(s)\frac{1}{1+ST}.$ = output of a 1st order control system.

a) WHEN UNIT STEP INPUT IS GIVEN:-

For unit step input $R(s) = \frac{1}{s} \leftarrow r(t) = 1$.

$$C(s) = R(s)\frac{1}{1+ST.}$$

$$= \frac{1}{s}\frac{1}{1+ST}.$$

$$C(s) = \frac{A}{s} + \frac{B}{1+sT} = \frac{1}{s(1+sT)}$$

$$= \frac{A(1+sT) + Bs}{s(1+sT)} = \frac{1}{s(1+sT)}$$

Put $s = 0$

$$A = 1.$$

Put $s = -\frac{1}{T}$ $\quad B = -T.$

$$C(s) = \frac{1}{s} - \frac{T}{1+sT}.$$

$$C(s) = \frac{1}{s} - \frac{1}{s+\frac{1}{T}}.$$

Taking inverse Laplace Transform on both sides.

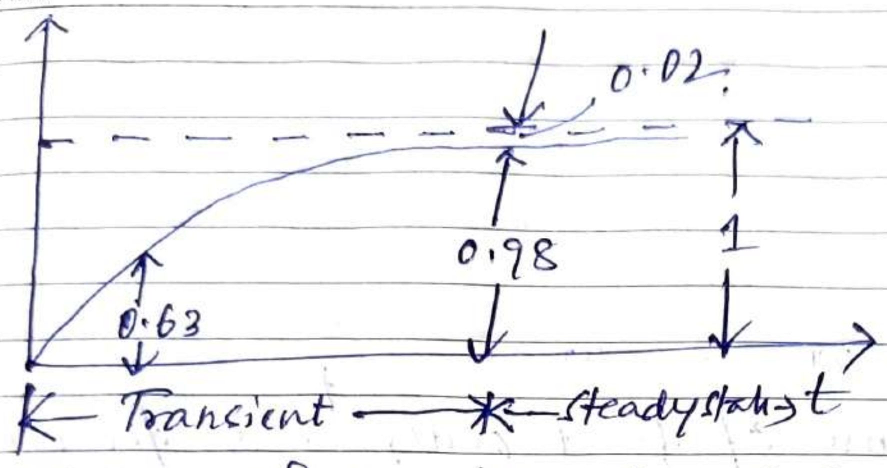
$$L^{-1} C(s) = L^{-1}\left(\frac{1}{s}\right) - L^{-1}\left(\frac{1}{s+\frac{1}{T}}\right)$$

$$\boxed{c(t) = 1 - e^{-t/T}}$$

$$\text{Error} = e(t) = r(t) - c(t) = 1 - \left(1 - e^{-t/T}\right).$$

$$e(t) = e^{-t/T}.$$

Steady state error $= e_{ss} = \lim_{t\to\infty} e^{-t/T} = 0.$



Time response of a 1st order C.S. for unit step input.

b) WHEN UNIT IMPULSE INPUT IS GIVEN.

We know the output expression

$$C(s) = R(s) \frac{1}{1+sT}.$$

As input to the system is a unit impulse
$R(s) = 1$.

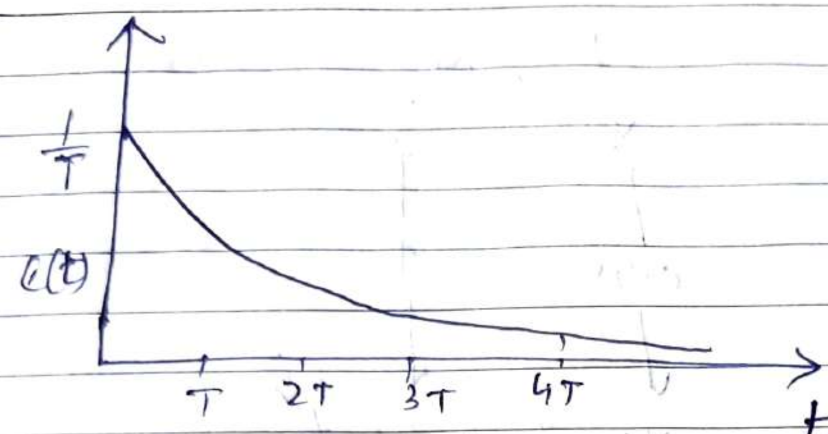
$$C(s) = 1 \cdot \frac{1}{1+sT}.$$

Taking inverse laplace Transform.

$$L^{-1}(C(s)) = L^{-1}\left(\frac{1}{1+sT}\right)$$

$$c(t) = L^{-1} \frac{1}{T}\left(\frac{1}{s+\frac{1}{T}}\right)$$

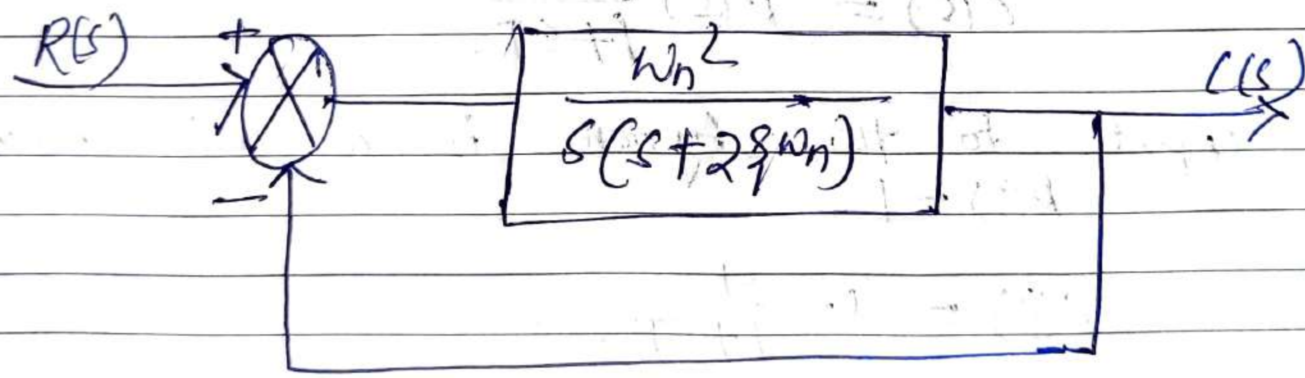
$$c(t) = \frac{1}{T} e^{-t/T}.$$



Time response of first order C.S. for unit impulse input.

TIME RESPONSE OF A SECOND ORDER C.S.

In second order control system highest power of s of characterstic equ^n is 2.



Here, $G(s) = \dfrac{\omega_n^2}{s(s+2\xi\omega_n)}$ $\quad H(s) = 1$.

$$\frac{C(s)}{R(s)} = \frac{\dfrac{\omega_n^2}{s(s+2\xi\omega_n)}}{1 + \dfrac{\omega_n^2}{s(s+2\xi\omega_n)}\cdot 1}$$

$$\frac{C(s)}{R(s)} = \frac{\dfrac{\omega_n^2}{s(s+2\xi\omega_n)}}{\dfrac{s^2+2\xi\omega_n s+\omega_n^2 \cdot 1}{s(s+2\xi\omega_n)}}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}}$$

a) WHEN UNIT STEP INPUT IS GIVEN.

output for the system.

$$C(s) = R(s) \cdot \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

As input is unit step input

$$r(t) = 1, \text{ and } R(s) = \frac{1}{s}.$$

$$C(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$= \frac{A}{s} + \frac{Bs+C}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\frac{A}{s} + \frac{Bs+C}{s^2+2\xi\omega_n s+\omega_n^2} = \frac{\omega_n^2}{s(s^2+2\xi\omega_n s+\omega_n^2)}.$$

$$A(s^2+2\xi\omega_n s+\omega_n^2) + Bs^2 + Cs = \omega_n^2.$$

~~Put s=0~~, $A = \cancel{\omega_n}\ 1$

Wk	M	T	W	T	F	S	S
40		1	2	3	4	5	6
41	7	8	9	10	11	12	13
42	14	15	16	17	18	19	20
43	21	22	23	24	25	26	27
44	28	29	30	31			

$$(A+B)s^2 + (2\xi\omega_n A + C)s + A\omega_n^2 = \omega_n^2.$$

~~A+B~~ = Comparing co-effient of s^2, s, and const. term on both side of the eqn.

$$A+B=0, \quad 2\xi\omega_n A + C = 0, \quad A=1.$$

$$B=-1 \qquad C = -2\xi\omega_n.$$

$$C(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{s^2+2\xi\omega_n s+\omega_n^2}.$$

making perfect square of denominator of second term

$$C(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{s^2+2\cdot\xi\omega_n s+(\xi\omega_n)^2-(\xi\omega_n)^2+\omega_n^2}$$

$$C(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{(s+\xi\omega_n)^2+\omega_n^2(1-\xi^2)}.$$

Put $\omega_d = \omega_n\sqrt{1-\xi^2}$

$$C(s) = \frac{1}{s} - \frac{s+2\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}.$$

$$C(s) = \frac{1}{s} - \frac{s+\xi\omega_n+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}$$

$$= \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}$$

$$C(s) = \frac{1}{s} - \frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2} - \frac{\xi\omega_n}{\omega_d}\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}$$

Taking inverse laplace Transform on both sides of the eqn.

$$L^{-1}C(s) = L^{-1}\left(\frac{1}{s}\right) - L^{-1}\left[\frac{s+\xi\omega_n}{(s+\xi\omega_n)^2+\omega_d^2}\right] - \frac{\xi\omega_n}{\omega_d}L^{-1}\left[\frac{\omega_d}{(s+\xi\omega_n)^2+\omega_d^2}\right]$$

$$c(t) = 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin\omega_d t$$

$$= 1 - e^{-\xi\omega_n t}\cos\omega_d t - \frac{\xi}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin\omega_d t$$

$$= 1 - e^{-\xi\omega_n t}\left(\cos\omega_d t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_d t\right)$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left(\sqrt{1-\xi^2}\cos\omega_d t + \xi\sin\omega_d t\right)$$

put $\xi = \cos\phi$, $\sqrt{1-\xi^2} = \sin\phi$.

$$\phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left(\sin\phi \cos\omega_d t + \cos\phi \sin\omega_d t\right)$$

$$\boxed{c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)}$$

The error is given as; $e(t) = r(t) - c(t)$

$r(t) = 1$, $e(t) = 1 - \left[1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)\right]$

$$= \cancel{1} - \cancel{1} + \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left(\omega_n\sqrt{1-\xi^2}\, t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

$$e(t) = \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left(\omega_n\sqrt{1-\xi^2}\, t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

Steady state error, $e_{ss} = \lim_{t\to\infty} e(t)$

$$\boxed{e_{ss} = \lim_{t\to\infty} \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin\left(\omega_n\sqrt{1-\xi^2}\, t + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)}$$

The response or output depends upon the value of ξ.

For $\xi < 1$, the response represents exponentially decaying oscillations having frequency $\omega_n\sqrt{1-\xi^2} = \omega_d$,

Time const $T = \frac{1}{\xi\omega_n}$

ω_n = Natural frequency of oscillations.

$\omega_d = \omega_n\sqrt{1-\xi^2}$ = damped frequency of oscillations.

ξ = Affect damping, called damping ratio.

$\xi\omega_n$ = damping factor.

Actual damping
Damping Co-efficient.

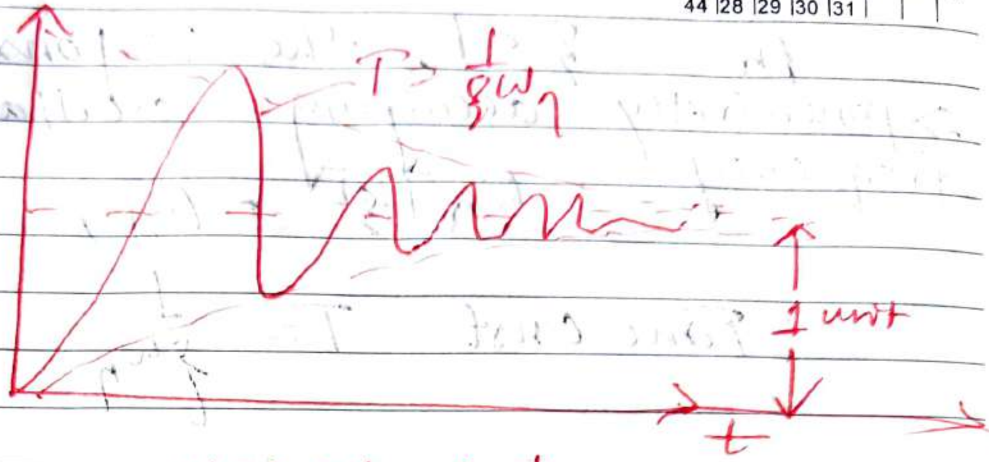
$\xi < 1$, $c(t)$ = Under damped response.
gives damped Oscillation.

$\xi = 0$, $c(t)$ = Undamped response.
Sustained Oscillation.

$\xi = 1$, $c(t)$ = Critically damped.

$\xi > 1$ $c(t)$ = Over damped.

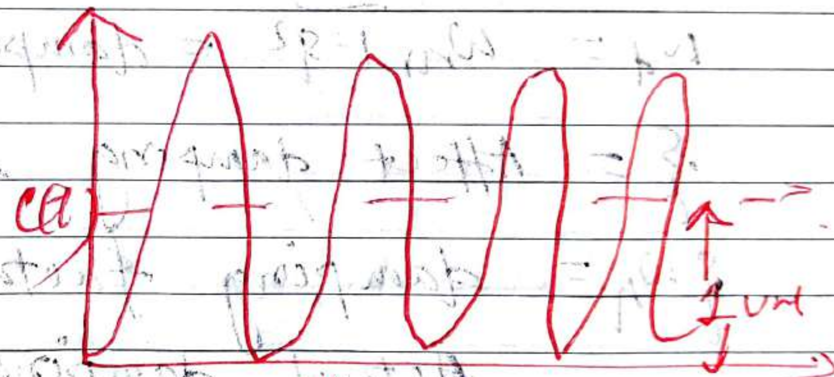
$\xi \leq 1$,



Underdamped

$\xi = 0$;

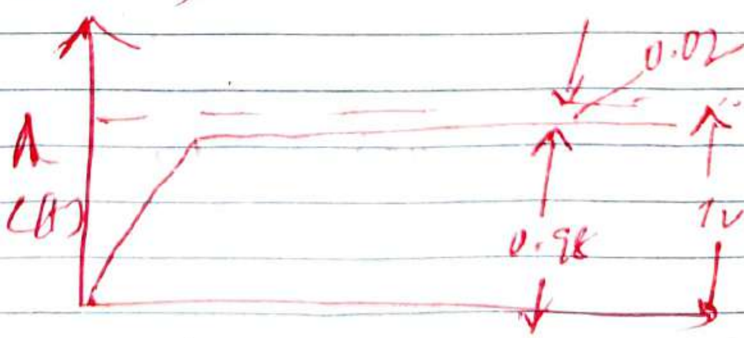
$c(t) = 1 - \cos\omega_n t$



Undamped

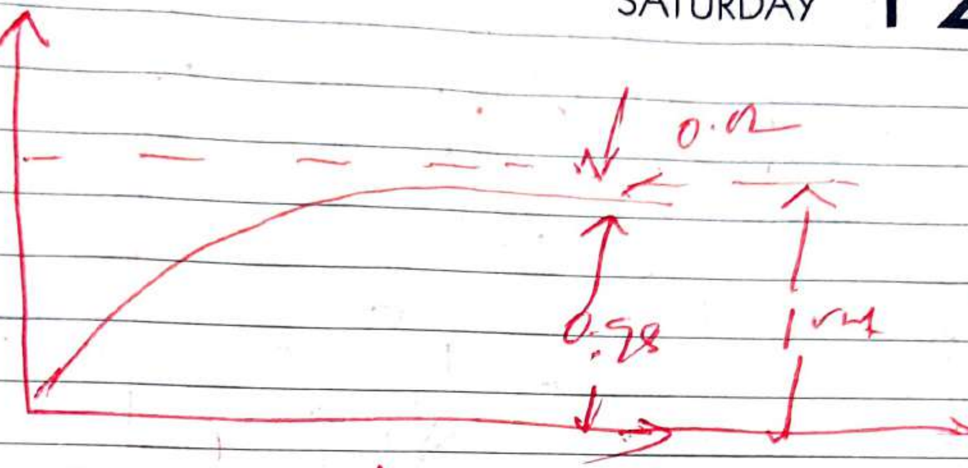
$\xi = 1$

$c(t) = 1 - e^{-\xi\omega_n t}(1 + \omega_n t)$



Critically damped

$\xi > 1$



Overdamped

$$c(t) = 1 - \frac{e^{-(\xi - \sqrt{\xi^2-1})\omega_n t}}{2\sqrt{\xi^2-1}\,(\xi - \sqrt{\xi^2-1})} + \frac{e^{-(\xi + \sqrt{\xi^2-1})\omega_n t}}{2\sqrt{\xi^2-1}\cdot(\xi + \sqrt{\xi^2-1})}$$

$$\xi = \text{damping ratio} = \frac{\text{Actual damping}}{\text{critical damping}} = \frac{\xi\omega_n}{\omega_n},$$

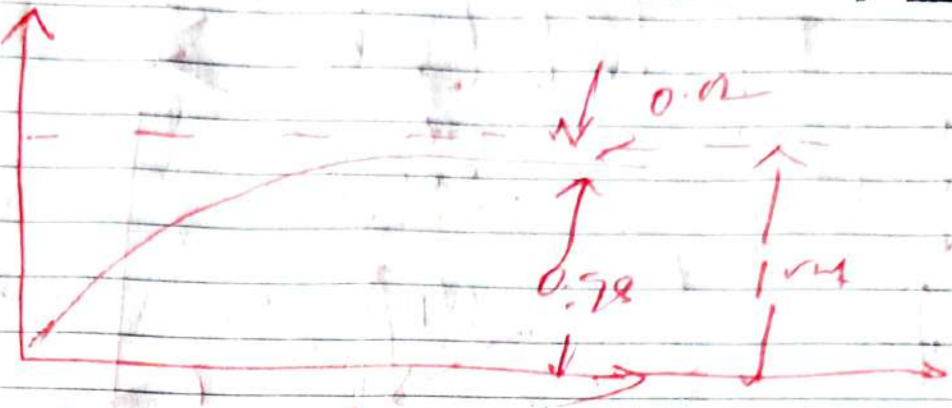
CHARACTERISTIC EQUATION:-

Transfer function of a second order control system

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad \text{characteristic equation.}$$

$\xi > 1$



Overdamped

$$c(t) = 1 - \frac{e^{-(\xi - \sqrt{\xi^2-1})\omega_n t}}{2\sqrt{\xi^2-1}\,(\xi - \sqrt{\xi^2-1})} + \frac{e^{-(\xi + \sqrt{\xi^2-1})\omega_n t}}{2\sqrt{\xi^2-1}\cdot(\xi + \sqrt{\xi^2-1})}$$

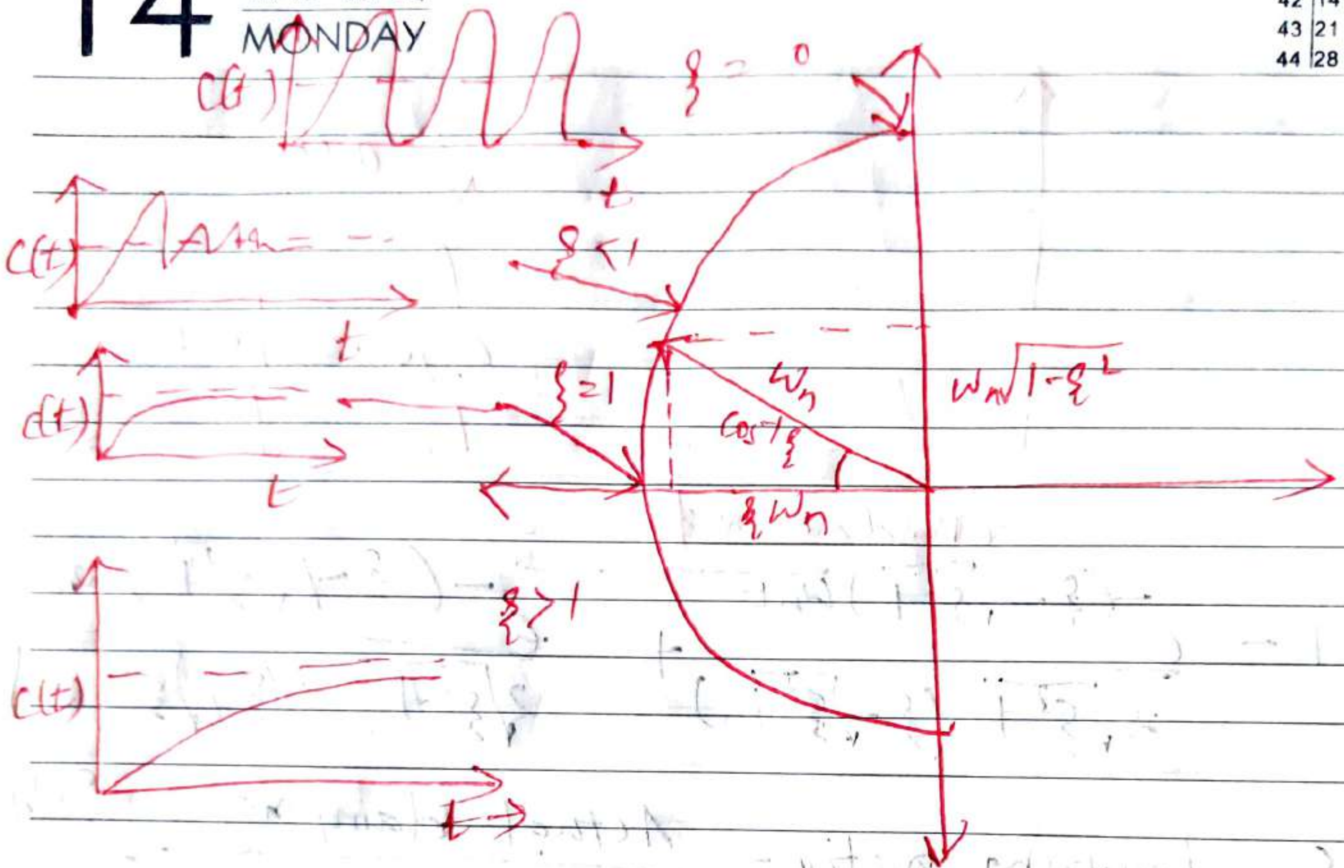
$$\xi = \text{damping ratio} = \frac{\text{Actual damping}}{\text{critical damping}} = \frac{\xi\omega_n}{\omega_n}.$$

CHARACTERISTIC EQUATION:-

Transfer function of a Second order Control System

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad \text{Characteristic equation.}$$

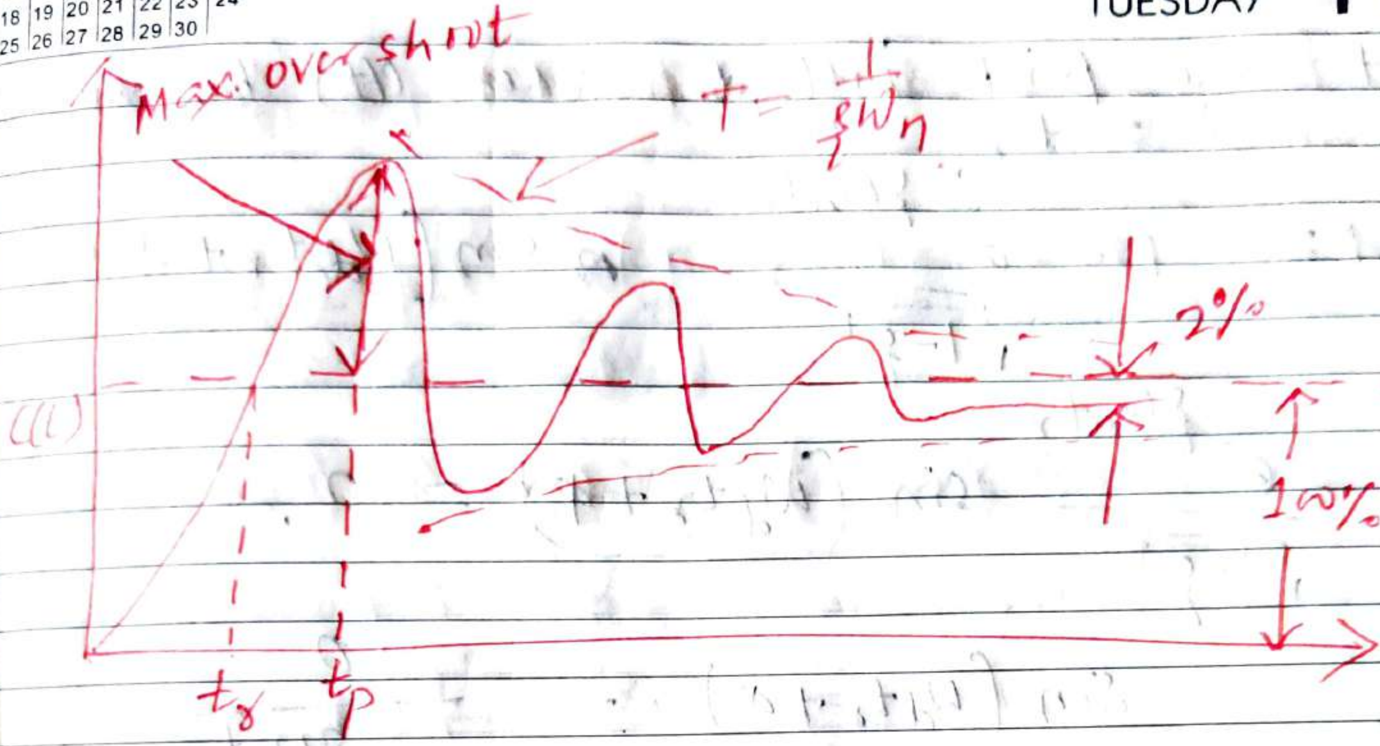


Location of roots of the characteristic equation and corresponding time response.

TRANSIENT RESPONSE SPECIFICATIONS OF SECOND ORDER CONTROL SYSTEM

The time response of an underdamped control system exhibits damped oscillations prior to reaching the steady state.

The specifications pertaining to time response during transient part.



(i) RISE TIME (t_r)

The rise time is the time taken by the response 0 to 100% or 10% to 90% ~~of the~~ desired value of the output at the very first instant.

0% to 100% for underdamped systems

10% to 90% for overdamped system.

For under damped system:-

We know $c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)$

At the first instant when $c(t)$ become 1

$$t = t_r$$

$$1 = 1 - \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \phi)$$

$$\Rightarrow \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \phi) = 0.$$

$$\Rightarrow \sin(\omega_d t_r + \phi) = \frac{0}{\frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}}} = 0.$$

As $\frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \neq 0.$

$$\Rightarrow \sin(\omega_d t_r + \phi) = \sin n\pi$$

But $n=1$ $\quad \sin(\omega_d t_r + \phi) = \sin \pi$

$$\Rightarrow \omega_d t_r + \phi = \pi$$

$$\Rightarrow \boxed{t_r = \frac{\pi - \phi}{\omega_d}}$$

2) ~~Maximu~~ MAXIMUM OVERSHOOT : M_p & PEAK TIME : t_p.

The maximum positive deviation of the output with respect to its desired value is known as maximum overshoot (M_p).

If input is unit step. Desired output is unity

$$M_p = c(t)_{max} - 1$$

$$\% M_p = \frac{c(t)_{max} - 1}{1} \times 100$$

PEAK TIME :- (t_p)

The time needed to reach the maximum overshoot is called Peak time and denoted by t_p.

For $c(t)$ becoms $c(t)_{max}$

$$\frac{d\, c(t)}{dt} = 0$$

$$\Rightarrow \frac{d}{dt}\left(1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \phi)\right) = 0$$

$$\Rightarrow \frac{d}{dt}(1) - \frac{1}{\sqrt{1-\xi^2}} \frac{d}{dt}\left(e^{-\xi\omega_n t} \cdot \sin(\omega_d t + \phi)\right) = 0$$

$$\Rightarrow 0 = \frac{1}{\sqrt{1-\xi^2}}\left(-\xi\omega_n e^{-\xi\omega_n t} \sin(\omega_d t + \phi) + e^{-\xi\omega_n t} \sin\omega_d t\phi\right)$$

$$\Rightarrow 0 - \left(\frac{1}{\sqrt{1-\xi^2}}\left(-\xi\omega_n e^{-\xi\omega_n t}\sin(\omega_d t+\phi) + e^{-\xi\omega_n t}\cos(\omega_d t+\phi)\,\omega_d\right)\right) = 0$$

$$\Rightarrow \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\left(\xi\omega_n \sin(\omega_d t+\phi) - \omega_d\cos(\omega_d t+\phi)\right) = 0$$

$$\Rightarrow \xi\omega_n \sin(\omega_d t+\phi) = \omega_d\cos(\omega_d t+\phi)$$

$$\Rightarrow \tan(\omega_d t+\phi) = \frac{\omega_d}{\xi\omega_n}$$

$$\Rightarrow \tan(\omega_d t+\phi) = \frac{\omega_n\sqrt{1-\xi^2}}{\xi\omega_n}$$

$$\Rightarrow \tan(\omega_d t+\phi) = \frac{\sqrt{1-\xi^2}}{\xi} = \tan\phi$$

$$\Rightarrow \frac{\tan\omega_d t + \tan\phi}{1-\tan\omega_d t\,\tan\phi} = \tan\phi$$

$$\Rightarrow \tan\omega_d t + \tan\phi = \tan\phi$$

$$\Rightarrow \tan\omega_d t_p = 0$$

$$\Rightarrow \tan\omega_d t_p = \tan n\pi$$

But $n=1$

$$\omega_d t_p = \pi$$

$$\Rightarrow \boxed{t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}$$

$$c(t)_{max} = 1 - \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \phi)$$

$$= 1 - \frac{e^{-\xi\omega_n \frac{\pi}{\omega_n\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin\left(\omega_d \times \frac{\pi}{\cancel{\omega_n\sqrt{1-\xi^2}}\ \omega_d} + \phi\right)$$

$$= 1 - \frac{e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin(\pi + \phi)$$

$$= 1 \cancel{\phi} - \frac{e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} (-\sin\phi)$$

$$c(t)_{max} = 1 + \frac{e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}{\sqrt{1-\xi^2}} \sin\phi.$$

$$c(t)_{max} = 1 + \frac{e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}\sqrt{1-\xi^2}}{\sqrt{1-\xi^2}}$$

$$c(t)_{max} = 1 + e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \qquad (\because \sin\phi = \sqrt{1-\xi^2})$$

$$M_p = c(t)_{max} - 1$$

$$M_p = 1 + e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} - 1$$

$$\boxed{M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}}}$$

$$\% M_p = e^{-\frac{\xi\pi}{\sqrt{1-\xi^2}}} \times 100$$

STEADY STATE ERROR

Steady state error is defined as the difference betwn the reference input (i.e desired output) and actual output at steady state.

As the steady state error is the index of accuracy of a control system. So steady state error should be minimum as far as possible.

The magnitude of the steady state error depends upon the types of input and open loop transfer function $G(s).H(s)$ of a closed loop control system.

TYPES OF THE SYSTEM:-

The product of the forward path transfer function and feedback path transfer function of a control system is known as open loop transfer function.

In General

$$G(s).H(s) = \frac{K(1+sT_a)(1+sT_b)\cdots}{s^N(1+sT_1)(1+sT_2)\cdots}$$

K = forward path gain.

$-\frac{1}{T_a}, -\frac{1}{T_b} \cdots$ are the zeros

$-\frac{1}{T_1}, -\frac{1}{T_2} \cdots$ are the poles.

N is the number of poles at origin.

Wk	M	T	W	T	F	S	S
40		1	2	3	4	5	6
41	7	8	9	10	11	12	13
42	14	15	16	17	18	19	20
43	21	22	23	24	25	26	27
44	28	29	30	31			

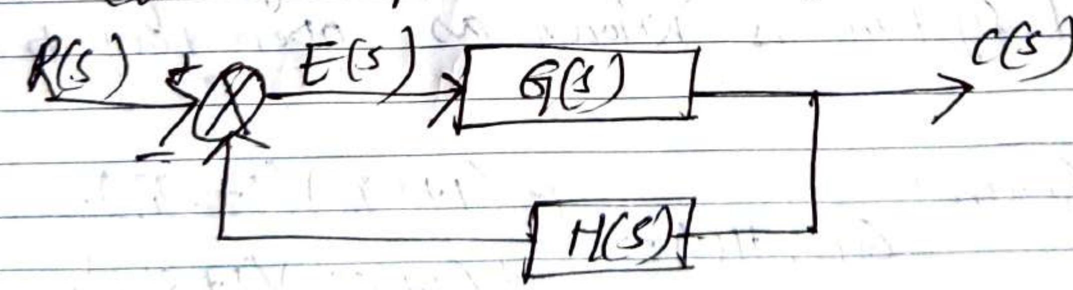
⇒ If system having no poles / zero poles at origin i.e $N=0$, the system is called type '0' syst.

⇒ If system having one pole at origin i.e $N=1$ the system is called type '1' syst.

⇒ If system having 2 poles at origin i.e $N=2$, the system is called type '2' system.

⇒ If the system having N poles at origin i.e $N=N$, the system is called type 'N' sys.

In a closed loop control system.



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s).H(s)} \quad \text{--- (1)}$$

$$\text{But } C(s) = E(s) \cdot G(s) \quad \text{--- (2)}$$

Comparing eqn ① & ② for C(s)

$$\Rightarrow E(s)\, G(s) = \frac{G(s) \cdot R(s)}{1 + G(s) \cdot H(s)}$$

$$\Rightarrow E(s) = \frac{R(s)}{1 + G(s) \cdot H(s)}$$

Applying final value Thm.

$$e_{ss} = \lim_{s \to 0} s E(s)$$

$$\boxed{e_{ss} = \lim_{s \to 0} \frac{s\, R(s)}{1 + G(s) \cdot H(s)}}$$

The actual output of a control system may be in any physical form, ie it is called as "position" or "displacement".

STATIC ERROR CO-EFFICIENTS:-

Steady state error is ~~eatt~~ also called as static error.

Static error is associated with static error co-efficient.
static error co-efficient is different for different input.

STATIC POSITIONAL ERROR CO-EFFICIENT (K_p)

Static positional error co-efficient (K_p) is associated with unit step input applied to a closed loop control system.

$$e_{ss} = \lim_{s \to 0} sR(s) \frac{1}{1+G(s) \cdot H(s)}$$

As input is $R(s) = \frac{1}{s}$

$$e_{ss} = \lim_{s \to 0} s \times \frac{1}{s} \frac{1}{1+G(s) \cdot H(s)}$$

$$e_{ss} = \frac{1}{1+\lim_{s \to 0} G(s) \cdot H(s)}$$

$$\boxed{e_{ss} = \frac{1}{1+K_p}}$$

Put $K_p = \lim_{s \to 0} G(s) \cdot H(s)$ = static positional error co-efficient.

STATIC VELOCITY ERROR CO-EFFICIENT (K_v)

Static velocity error co-efficient is associated with unit ramp input applied to a closed loop C.S.

$$e_{ss} = \lim_{s \to 0} s R(s) \frac{1}{1+G(s)\cdot H(s)}$$

As input $R(s) = \frac{1}{s^2}$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{1}{s^2} \frac{1}{1+G(s)\cdot H(s)}$$

$$= \lim_{s \to 0} \frac{1}{s+s\,G(s)\cdot H(s)}$$

$$e_{ss} = \frac{1}{\lim_{s \to 0} s + \lim_{s \to 0} s\,G(s)\cdot H(s)}$$

$$\boxed{e_{ss} = \frac{1}{K_v}}$$

put $K_v = \lim_{s \to 0} s\,G(s)\cdot H(s)$ = Static velocity error co-efficient.

STATIC ACCELERATION ERROR CO-EFFICIENT (K_a)

Static acceleration error co-efficient is associated with unit parabolic input applied to a closed loop control system.

$$e_{ss} = \lim_{s \to 0} sR(s) \frac{1}{1+G(s) \cdot H(s)}$$

As input $R(s) = \frac{1}{s^3}$

$$e_{ss} = \lim_{s \to 0} s \frac{1}{s^3} \frac{1}{1+G(s) \cdot H(s)}$$

$$e_{ss} = \frac{1}{\lim_{s \to 0} s^2 + \lim_{s \to 0} s^2 G(s) \cdot H(s)}$$

$$\boxed{e_{ss} = \frac{1}{K_a}}$$

Put $K_a = \lim_{s \to 0} s^2 G(s) \cdot H(s)$ = static acceleration error co-efficient.

TYPES OF TRANSFER FUNCTION & e_{ss}

Steady state error depends upon the types of the system and types of input.

FOR TYPE '0' SYSTEM.

With unit step input

For type '0' system.

$$G(s)\cdot H(s) = \frac{K(1+sT_a)(1+sT_b)\cdots}{(1+sT_1)(1+sT_2)\cdots}$$

$$K_p = \lim_{s\to 0} G(s)\cdot H(s) = K.$$

$$= \lim_{s\to 0} \frac{K(1+sT_a)(1+sT_b)\cdots}{(1+sT_1)(1+sT_2)\cdots} = K.$$

$$K_p = K.$$

Since $e_{ss} = \frac{1}{1+K_p}$

$$\boxed{e_{ss} = \frac{1}{1+K}.}$$

Wk	M	T	W	T	F	S	S
40		1	2	3	4	5	6
41	7	8	9	10	11	12	13
42	14	15	16	17	18	19	20
43	21	22	23	24	25	26	27
44	28	29	30	31			

b) with unit ramp input

For type (0) System.

$$G(s) \cdot H(s) = \frac{K(1+sT_a)(1+sT_b) \ldots}{(1+sT_1)(1+sT_2) \ldots}$$

$$K_v = \lim_{s \to 0} s\, G(s) \cdot H(s)$$

$$= \lim_{s \to 0} s \frac{K(1+sT_a)(1+sT_b) \ldots}{(1+sT_1)(1+sT_2) \ldots}$$

$$K_v = 0$$

$$e_{ss} = \frac{1}{K_v} = \infty, \quad \boxed{e_{ss} = \infty}$$

c) with unit parabolic input

For type (0) System.

$$G(s) \cdot H(s) = \frac{K(1+sT_a)(1+sT_b) \ldots}{(1+sT_1)(1+sT_2) \ldots}$$

$$K_a = \lim_{s \to 0} s^2 G(s) \cdot H(s)$$

$$= \lim_{s \to 0} s^2 \frac{K(1+sT_a)(1+sT_b) \ldots}{(1+sT_1)(1+sT_2) \ldots}$$

$$K_a = 0, \quad e_{ss} = \frac{1}{K_a} \quad \boxed{e_{ss} = \infty}$$

FOR TYPE '1' SYSTEM:

for type '1' system.

$$G(s)\cdot H(s) = \frac{K(1+sT_a)(1+sT_b)\ldots}{s(1+sT_1)(1+sT_2)\ldots}$$

a) With unit step input

$$e_{ss} = \lim_{s\to 0} \cancel{sR(s)}\cdot \frac{1}{1+}$$

$$K_p = \lim_{s\to 0} G(s)\cdot H(s)$$

$$= \lim_{s\to 0} \frac{K(1+sT_a)(1+sT_b)\ldots}{s(1+sT_1)(1+sT_2)}$$

$$K_p = \infty$$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+\infty} = 0.$$

$$\boxed{e_{ss} = 0}$$

b) With unit ramp input

$$K_v = \lim_{s\to 0} s\,G(s)\cdot H(s)$$

$$= \lim_{s\to 0} s\,\frac{K(1+sT_a)(1+sT_b)\ldots}{s(1+sT_1)(1+sT_2)}$$

$$K_v = K$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{K}$$

$$\boxed{e_{ss} = \frac{1}{K}}$$

November 2019							
Wk	M	T	W	T	F	S	S
44					1	2	3
45	4	5	6	7	8	9	10
46	11	12	13	14	15	16	17
47	18	19	20	21	22	23	24
48	25	26	27	28	29	30	

c) with unit parabolic input

$$K_a = \lim_{s \to 0} s^2 \, G(s) \cdot H(s)$$

$$= \lim_{s \to 0} s^2 \frac{K(1+sT_a)(1+sT_b)\ldots}{s(1+sT_1)(1+sT_2)\ldots}$$

$$K_a = 0$$

$$e_{ss} = \frac{1}{K_a} \quad e_{ss} = \frac{1}{0} \quad \boxed{e_{ss} = \infty}$$

FOR TYPE '2' SYSTEM.

for type '2' system open loop transfer fn

$$G(s) \cdot H(s) = \frac{K(1+sT_a)(1+sT_b)\ldots}{s^2(1+sT_1)(1+sT_2)\ldots}$$

a) with unit step input

$$K_p = \lim_{s \to 0} G(s) \cdot H(s)$$

$$K_p = \lim_{s \to 0} \frac{K(1+sT_a)(1+sT_b)\ldots}{s^2(1+sT_1)(1+sT_2)}$$

$$K_p = \infty$$

$$e_{ss} = \frac{1}{1+K_p} \quad e_{ss} = \frac{1}{\infty} \quad \boxed{e_{ss} = 0}$$

with unit ramp input:

$$K_V = \lim_{s \to 0} s\, G(s) \cdot H(s)$$

$$= \lim_{s \to 0} s \frac{K(1+sT_b)(1+sT_I)\ldots}{s^2(1+sT_1)(1+sT_2)\ldots}$$

$$K_V = \lim_{s \to 0} = \infty$$

$$e_{ss} = \frac{1}{K_V} \qquad e_{ss} = \frac{1}{\infty} \qquad \boxed{e_{ss} = 0}$$

with unit parabolic input

$$K_a = \lim_{s \to 0} s^2\, G(s) \cdot H(s)$$

$$= \lim_{s \to 0} s^2 \frac{K(1+sT_a)(1+sT_b)\ldots}{s^2(1+sT_1)(1+sT_2)\ldots}$$

$$K_a = K$$

$$e_{ss} = \frac{1}{K_a} \qquad \boxed{e_{ss} = \frac{1}{K}}$$

Frequency response Analysis

The magnitude and phase relationship betw^n the sinusoidal input and steady state output of a system is termed as frequency response.

POLAR PLOT

The sinusoidal transfer function $G(jw)$ is a complex function is given by.

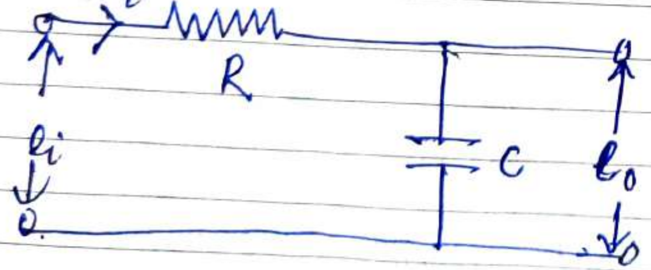
$$G(jw) = Re[G(jw)] + j\,Im[G(jw)]$$

$$G(jw) = |G(jw)| \underline{/G(jw)} = M\underline{/\phi}$$

$G(jw)$ may be represented as a phasor Magnitude M and phase angle ϕ

As w, the input frequency varied from 0 to ∞, the magnitude M and phase angle ϕ changes. hence the tip of the phasor $G(jw)$ traces a locus in the complex plane. The locus thus obtained is called polar plot

Consider a R-C Filter.



$$e_0 = i X_C = \frac{i}{wC}$$

$$E_0(s) = \frac{I(s)}{sC} \qquad s = jw.$$

$$e_i = iR + iX_C = i\left(R + \frac{1}{\omega C}\right)$$

$$E_i(s) = I(s)\left(R + \frac{1}{sC}\right)$$

$$G(s) = \frac{E_o(s)}{E_i(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{1 + RCs}$$

Where $T = RC$

$$G(s) = \frac{1}{1 + Ts}$$

$$G(j\omega) = \frac{1}{1 + j\omega T}$$

$$G(j\omega) = \frac{1}{\sqrt{1 + \omega^2 T^2}} \angle -\tan^{-1} \omega T$$

$$= M \angle \phi$$

Where $M = \frac{1}{\sqrt{1 + \omega^2 T^2}}$, $\phi = -\tan^{-1} \omega T$.

When $\omega = 0$, $M = 1$ and $\phi = 0$.

$\omega = \frac{1}{T}$ $M = \frac{1}{\sqrt{2}}$ and $\phi = -45°$.

$\omega \to \infty$ $M = 0$ and $\phi = -90°$

June						2012
S	M	T	W	T	F	S
		1	2	3	1	2
3	7	8	9	10	8	9
10	14	15	16	17	15	16
17	21	22	23	24	22	23
24	28	29	30	31	29	30

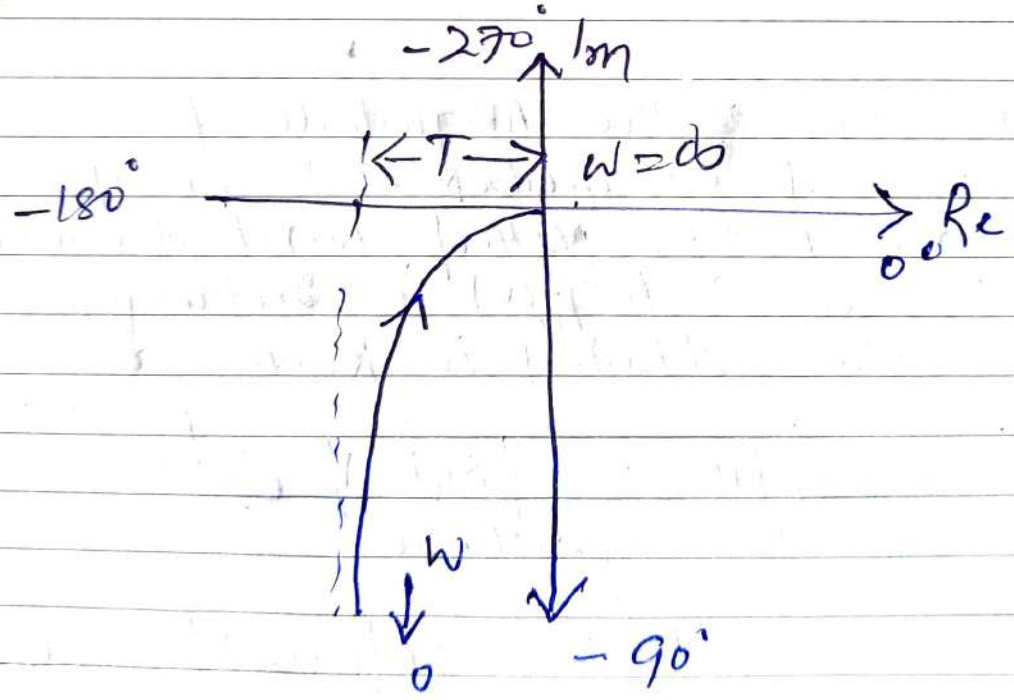
consider another transfer function.

$$G(j\omega) = \frac{1}{j\omega\,(1+j\omega T)}$$

$$G(j\omega) = \frac{-T}{(1+\omega^2T^2)} - j\,\frac{1}{\omega(1+\omega^2T^2)}$$

$$\lim_{\omega \to 0} G(j\omega) = -T - j\infty = \infty \angle -90^\circ$$

$$\lim_{\omega \to \infty} G(j\omega) = -0 - j0 = 0 \angle -180^\circ$$



BODE PLOT

One of the most useful representation of a transfer function is a logarithmic plot.

It consists of two graphs.

(1) The Logarithmic of $|G(j\omega)|$

(2) phase angle, both plotted,

Versus

frequency in logarithmic scale.

These plots are called Bode plots in honour of H. W. Bode

Or

The variation of the magnitude of sinusoidal transfer function expressed in decibel and corresponding phase angle in degree being plotted w.r.t frequency on a logarithmic scale (i.e $\log_{10} \omega$) in rectangular axes.

The plot thus obtained is known as Bode Plot.

$$G(j\omega) = |G(j\omega)| \, e^{j\phi} \quad \text{--- (1)}$$

Taking natural logarithmic of both sides

$$\ln G(j\omega) = \underbrace{\ln|G(j\omega)|}_{Re} + j\underbrace{\phi(\omega)}_{Im} \quad \text{--- (2)}$$

Real part is the natural logarithmic of magnitude and is measured in a basic unit called neper.

The imaginary part is the phase characteristic, Taking logarithmic of base 10, on both side of equ^n (1).

$$\log G(j\omega) = \log|G(j\omega)| + \log e^{j\phi(\omega)}$$

$$= \log|G(j\omega)| + j\phi(\omega)\log_{10} e$$

$$= \log|G(j\omega)| + j0.434\,\phi(\omega) \quad (3)$$

$20\log|G(j\omega)|$ and phase angle $\phi(\omega)$ versus $\log\omega$.

Unit of Magnitude $20\log|G(j\omega)|$ is decibel abbreviated as db.

The curve generally drawn on Semilog paper using log scale for frequency and linear scale for magnitude in db and phase angle in degrees.

Consider an example RC Filter.

$$G(j\omega) = \frac{1}{1+j\omega T} = \frac{1}{\sqrt{1+\omega^2T^2}} \angle -\tan^{-1}\omega T$$

The Log-magnitude is

$$20\log|G(j\omega)| = 20\log(1+\omega^2T^2)^{-\frac{1}{2}}$$

$$= -10\log(1+\omega^2T^2) \quad (4)$$

For low frequency $\omega \ll \frac{1}{T}$

$$20\log|G(j\omega)| = -10\log 1 = 0\,db \quad (5)$$

For High frequency $\omega \gg \frac{1}{T}$

$$20\log|G(j\omega)| = -20\log\omega T$$

$$= -20\log\omega - 20\log T \quad (6)$$

The logarithmic ~~scale~~ plot $20 \log |G(j\omega)| \sim \log \omega$ of eqn (5) is a horizontal axis.

The plot of eqn (6) is a straight line with a slope -20 db per unit change in $\log \omega$.

A unit change $\log \omega$ means.

$$\log\left(\frac{\omega_2}{\omega_1}\right) = 1 \qquad \omega_2 = 10\omega_1$$

The range of frequency is called decade.
Thus the slope -20 db/decade.

The range of frequency $\omega_2 = 2\omega_1$ is called octave.

$$-20 \log 2 = -6 \text{db}$$

Slope is called -6 db/octave.

The error ~~in~~ logmagnitude for $0 < \omega \leq \frac{1}{T}$

$$-10 \log(1+\omega^2 T^2) + 10 \log 1$$

Error at corner frequency. $\omega = \frac{1}{T}$ is

$$-10 \log(1+1) + 10 \log 1 = -3 \text{db}.$$

For $\frac{1}{T} \leq \omega < \infty$, the error in log magnitude

$$-10 \log(1+\omega^2 T^2) + 20 \log \omega T.$$

Error at corner frequ.

$\omega = \frac{1}{T}$

$-10\log(1+1) + 20\log 1 = -3\,dB.$

Bode plot (logarithmic plot) for Transfer function

$$G(s) = \frac{K[(1+sT_1)(1+sT_2)\cdots]\omega_n^2}{s^N(1+sT_a)(1+sT_b)\cdots(s^2+2\xi\omega_n s+\omega_n^2)}$$

$$G(j\omega) = \frac{K[(1+j\omega T_1)(1+j\omega T_2)\cdots]\omega_n^2}{(j\omega)^N(1+j\omega T_a)(1+j\omega T_b)\cdots(\omega_n^2-\omega^2+j2\xi\omega_n\omega)} \quad -(7)$$

The procedure for drawing the Bode plot for Transfer function

In decibel.

$$20\log_{10}|G(j\omega)| = 20\log K + 20\log|1+j\omega T_1| + \cdots - 20N\log_{10}|j\omega| - 20\log|1+j\omega T_a|$$
$$-20\log_{10}\left|\frac{(\omega_n^2-\omega^2)}{\omega_n^2} + j\frac{2\xi\omega_n\omega}{\omega_n^2}\right| \quad -(8)$$

For phase angle.

$$\angle G(j\omega) = \tan^{-1}\omega T_1 + \tan^{-1}\omega T_2 + \cdots - N\,\angle 90^\circ - \tan^{-1}\omega T_a$$
$$-\tan^{-1}\omega T_b \cdots - \tan^{-1}\left[\frac{2\xi\omega_n\omega}{\omega_n^2-\omega^2}\right] \quad -(9)$$

The Bode plot is a graph obtained from equn (8) & (9) consisting of two parts.

(i) $20\log_{10}|G(j\omega)| \sim \log_{10}\omega$

(ii) $\angle G(j\omega) \sim \log_{10}\omega$.

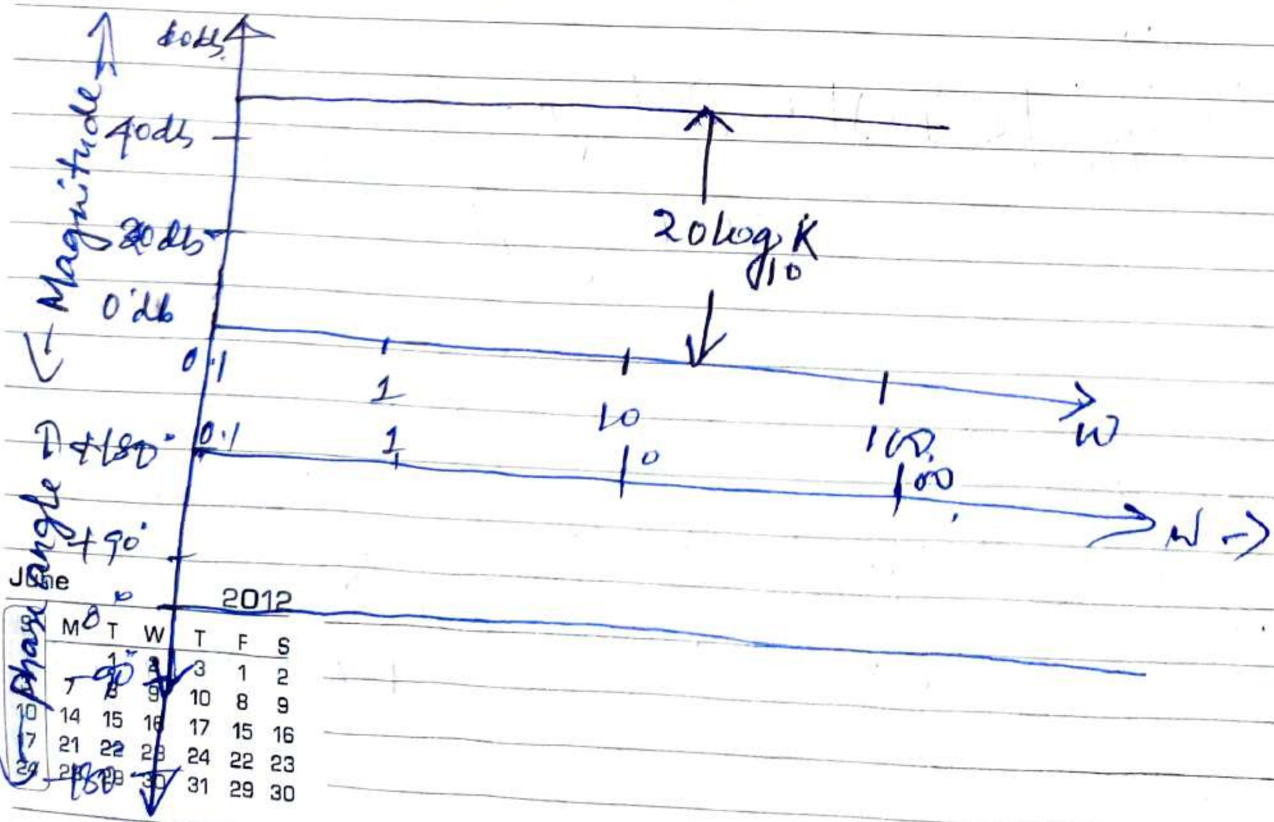
Graphs for the Gain Term k.

The magnitude in decibel for the term of k.

$$k(db) = 20\log_{10}(k) \quad \text{—— (10)}$$

equn (10) indicates that the magnitude is independent of $\log_{10}\omega$ and as k is considered positive real.

The phase angle is always zero what ever may be the value of ω. ~~fig~~.



17.04.20 L18-2004 - F16-2050, F17-2033.

Graphs for the Terms $\frac{1}{(j\omega)^N}$.

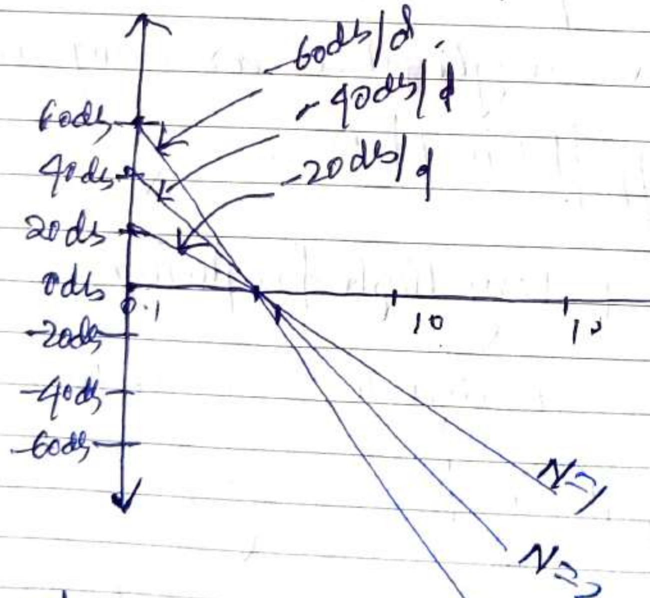
The magnitude of the term $\frac{1}{(j\omega)^N}$ in decibel is given by

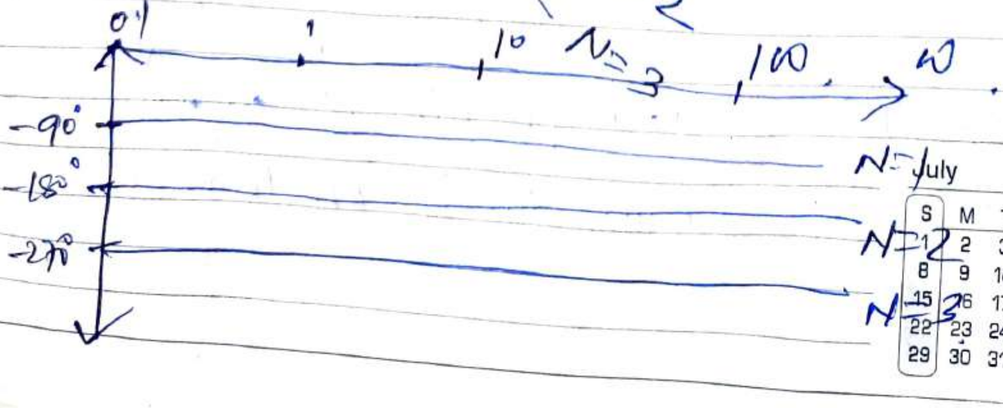
$$20\log_{10}\left|\frac{1}{(j\omega)^N}\right| = -20N\log_{10}\omega \quad \text{--- (11)}$$

The phase angle is given by $\angle\frac{1}{(j\omega)^N} = -90N^\circ$ --- (12)

From eqn (11) & eqn (12) the graphs are shown.

The graph for the magnitude versus $\log_{10}\omega$ is a straight line a slope of $-20N$ db/decade.





The term $\frac{1}{(j\omega)^N}$ has only imaginary term in the denominator the phase angle is $-90N°$.

Graphs for the Term $(1+j\omega T)$

* The magnitude in decibel for the term $(1+j\omega T)$ is given by

$$20\log_{10}|1+j\omega T| = 20\log_{10}\sqrt{1+\omega^2 T^2} \quad -(13)$$

Consider two cases

(i) $\omega << \frac{1}{T}$ (very low frequency)

ωT is negligible as compared to 1.

$$20\log_{10}|1+j\omega T| \simeq 20\log_{10} 1 = 0\,db. \quad -(14)$$

(ii) $\omega >> \frac{1}{T}$ (very high frequency)

1 is negligible as compared to ωT.

$$20\log_{10}|1+j\omega T| \simeq 20\log_{10}\sqrt{\omega^2 T^2} = 20\log\omega T.$$

$$= 20\log\omega + 20\log T \quad -(15)$$

Eqn (14) gives a graph which lies on 0 dbaxis.
For Case (ii) the graph has a slope 20db/decad.

These two graphs intersect on odb axis at a point.

$$0 = 20\log_{10}\omega + 20\log_{10}T.$$

$$20\log_{10}\omega = -20\log_{10}T.$$

$$20\log_{10}\omega = 20\log_{10}(T^{-1})$$

$$\omega = T^{-1}$$

$$\omega = \frac{1}{T}$$

Hence the two graphs intersect on odb axis. at $\omega = \frac{1}{T}$.

* The phase angle for the term $(1+j\omega T)$ is given by,

$$\phi = \tan^{-1}\left(\frac{\omega T}{1}\right)$$

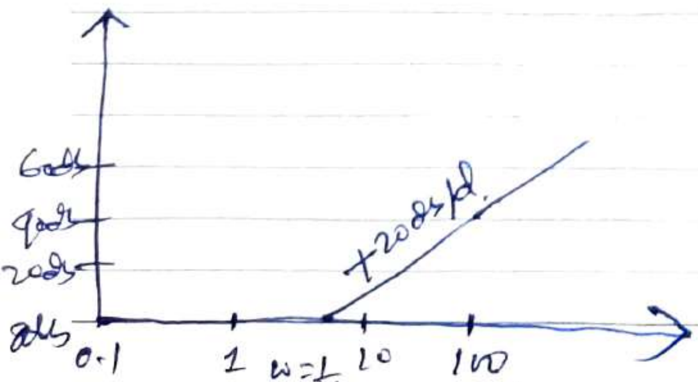
(i) At very low frequency ωT is very small

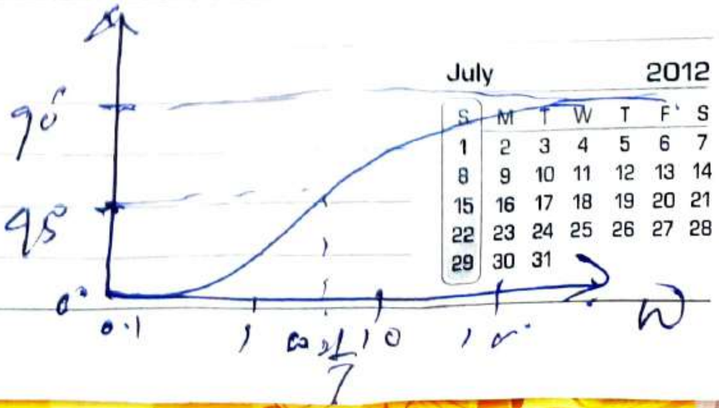
$$\phi = \tan^{-1}(0) \text{ or } \phi = 0^\circ$$

(ii) At $\omega = \frac{1}{T}$ $\quad \phi = \tan^{-1}(1) = 45^\circ.$

(iii) At high frequency.

$$\phi = \tan^{-1}(\infty) = 90^\circ.$$





Graphs for the term $\frac{1}{(1+j\omega T)}$

The magnitude of $\frac{1}{1+j\omega T}$

$$20\log\left|\frac{1}{1+j\omega T}\right| = 20\log\frac{1}{\sqrt{1+\omega^2T^2}}$$

$$= -20\log\sqrt{1+\omega^2T^2}$$

(i) $\omega << \frac{1}{T}$, $\omega T << 1$

$$M = -20\log 1 = 0 \text{ db.} \quad —(16)$$

(ii) $\omega >> \frac{1}{T}$ $\omega T >> 1$

$$M = -20\log \omega T$$

$$= -20\log\omega + 20\log\frac{1}{T} \quad —(17)$$

Graph of Equ^n (16) & (17) meets the 0 db axis.

$$20\log\omega = 20\log\frac{1}{T}$$

$$\omega = \frac{1}{T}$$

Hence the two graphs meets the 0 db axis at $\omega = \frac{1}{T}$.

phase angle for the term $\frac{1}{1+j\omega T}$.

$$\phi = -\tan^{-1}\left(\frac{\omega T}{1}\right)$$

For Low frequency $\phi = 0$.

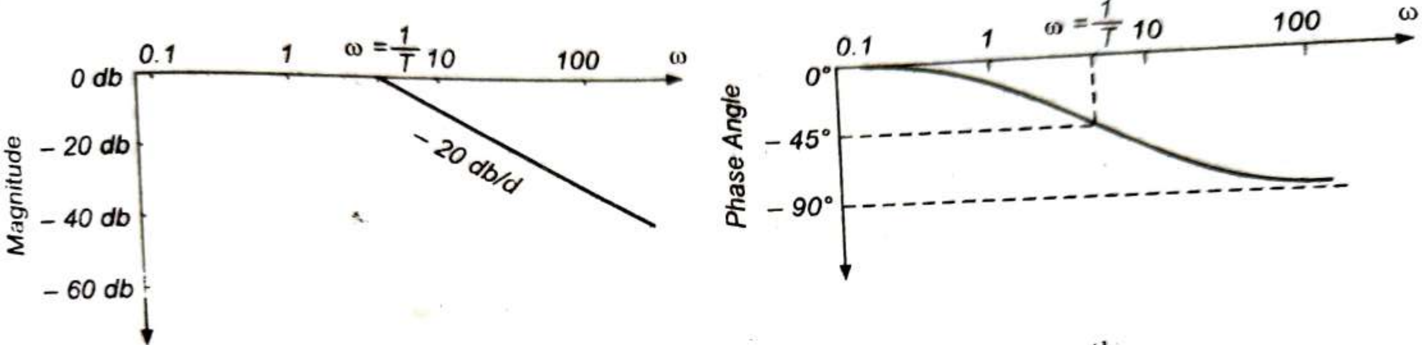
for frequ- $\omega = \frac{1}{T}$ $\phi = -45^\circ$.

For high frequency $\phi = -90^\circ$

fig: 7.184

fig. 7.18.5

~~fig~~ P-267.



(a) (b)

Fig. 7.18.4. Bode plot for the term $1/(1 + j\omega T)$.

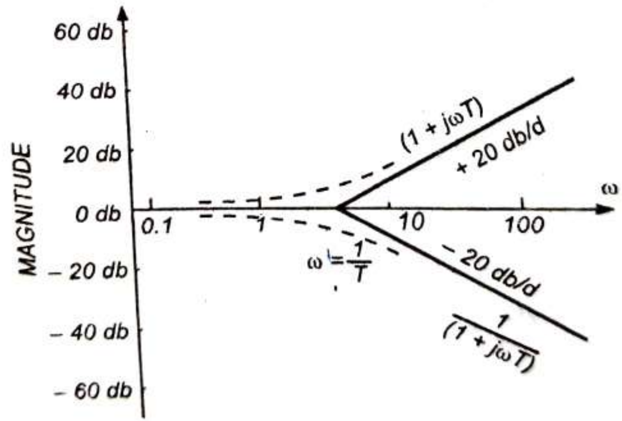


Fig. 7.18.5. Exact and asymptotic (approximate) bode plots for the terms $(1 + j\omega T)$ and $\dfrac{1}{(1 + j\omega T)}$

Fig. 7.18.5

INITIAL SLOPE OF BODE PLOT

The corner frequencies due to first order terms $(1+j\omega T_1)\,(1+j\omega T_2)\cdots\cdots \frac{1}{(1+j\omega T_a)}\,\frac{1}{(1+j\omega T_b)}\cdots$ etc.

are given by

$$\omega = \frac{1}{T_1}, \frac{1}{T_2} \cdots \frac{1}{T_a}, \frac{1}{T_b} \cdots \text{etc.}$$

For the frequencies lower than the lowest corner frequency the contribution towards gain of the transfer function is nil.

Transfer function for frequencies lower than the lowest corner frequencies can be expressed as.

$$G(j\omega) = \frac{K}{(j\omega)^N}.$$

Magnitude $20\log|G(j\omega)| = 20\log\left|\frac{K}{(j\omega)^N}\right|$

$$= 20\log K - 20N\log\omega \quad \text{--- (18)}$$

From this above eqn. the graph magnitude vs. $\log_{10}\omega$ has initial slope of $-20N$ dB/decade.

N = Types of the transfer function.

For Type '0' system i.e. N=0

$$20\log|G(j\omega)| = 20\log_{10}\left|\frac{K}{(j\omega)^0}\right| = 20\log_{10}K - 20\log_{10}1$$

$$= 20\log_{10}K.$$

Initial slope for type '0' system is 0.

fig.?

For Type '1' system i.e. N=1

For type '1' system initial part of Bode plot is

$$20\log_{10}|G(j\omega)| = 20\log_{10}\left|\frac{K}{(j\omega)^1}\right|$$

$$= 20\log_{10}K - 20\log_{10}\omega$$

Initial slope = -20 db/decade.

and the graph intersect the 0 db axis,

$$0 = 20\log_{10}K - 20\log_{10}\omega$$

$$20\log_{10}K = 20\log_{10}\omega$$

$$\omega = K$$

The graph intersect 0 db axis at $\omega = K$.

For Type '2' system i.e $N=2$

For type '2' system initial part of Bode plot.

$$20\log_{10}|G(j\omega)| = 20\log_{10}\left|\frac{K}{(j\omega)^2}\right|$$

$$= 20\log_{10}K - 20\log_{10}\omega^2$$

$$= 20\log_{10}K - 40\log_{10}\omega$$

Initial slope = -40 db/decade.
and the graph intersect the '0' db axis.

$$0 = 20\log_{10}K - 40\log_{10}\omega$$

$$20\log K = 20\log_{10}\omega^2$$

$$\omega^2 = K$$

$$\omega = \sqrt{K}$$

The graph intersect 0 db axis at $\omega = \sqrt{K}$

Similarly for type '3' system $N = 3$.

slope = -60 db/decade.

intersection point on 0 db axis $\omega = \sqrt[3]{K} = K^{1/3}$.

PROCEDURE FOR DRAWING BODE PLOT.

Ex. Draw Bode plot for the system whose open loop. T. f is

$$G(s) \cdot H(s) = \frac{4}{s\,(1+0.5s)\,(1+0.08s)}$$

$$G(j\omega)\,H(j\omega) = \frac{4}{j\omega\,(1+j0.5\omega)\,(1+j0.8\omega)}$$

1. The corner frequencies are

$\omega = \frac{1}{0.5} = 2$ rad/sec, and $\omega = \frac{1}{0.08} = 12.5$ rad/sec

2. Starting frequency is less than the lowest corner frequency.

As lowest corner frequency = 2 rad/sec

Starting frequency = 1 rad/sec.

3. As it is a Type '1' T.F.

Starting slope = -20 db/decade.

and intersection with '0' db axis

$\omega = 4$.

4. The denominator term $\frac{1}{(1+j0.5\omega)}$

corner frequency = 2 rad/sec.
slope = −20db/decade.

Before slope was = −20db/decade.
slope after ω = 2rad/sec is −40 db/decade
= −20db/decade + −20db/decad.
= − 40 db/ decade.

5. The denominator term $\frac{1}{(1+j\,0.8\omega)}$

Corner frequency ω = 12.5 rad/sec.

Slope due to this term = − 20db/ decade.

Before slope was = − 40 db/ decade.

slope after ω = 12.5 rad/sec

Slope = − 40db/decade + −20db/decade
= −60 db/decade.

This slope continues after ω = 12.5 rad/sec.

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6. Phase angle $\angle G(j\omega)H(j\omega)$ for frequencies betw^n 1 rad/sec to 100 rad/sec.

ω(rad/sec)	1	2	8	10	20	50
$\angle G(j\omega)H(j\omega)$	-121	-144°	-198	-207	-234	-252

fig:

Gain Margin:

The gain in db at phase cross over frequency frequencies greater than ω = 12.5 rad/sec.

5. $\angle G(j\omega)H(j\omega)$ for frequencies between $\omega = 1$ rad/sec to $\omega = 100$ rad/sec is calculated as below :

$$\angle G(j\omega)H(j\omega) = -90^\circ - \tan^{-1}(0.5\omega) - \tan^{-1}(0.08\,\omega)$$

ω (rad/sec)	1	2	8	10	20	50
$\angle G(j\omega)H(j\omega)^\circ$	– 121	– 144	– 198	– 207	– 234	– 252

The Bode plot $|G(j\omega)H(j\omega)|$ db and $\angle G(j\omega)H(j\omega)$ *versus* ω (log scale) is drawn and shown in Fig. 7.18.11.

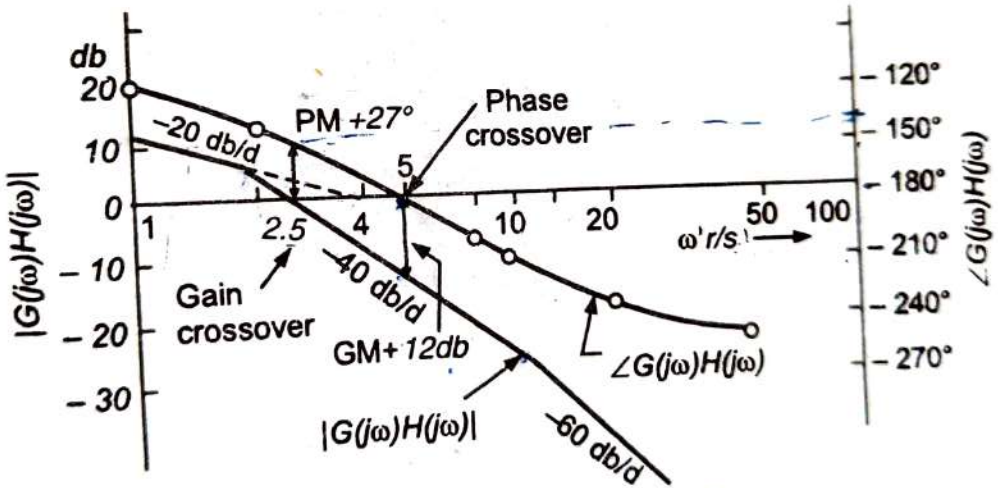

Fig. 7.18.11. Bode plot for $G(s)H(s) = \dfrac{4}{s(1+0.5s)(1+0.08s)}$

fig:

Imp **Gain Margin:**

The gain in db at phase cross over frequency is the gain margin. (G.M)

If Gain is $-ve$.
G.M is $+ve$.

Phase cross over frequency is 5 rad/sec.

and Gain $G(j\omega)\,H(j\omega) = -12\,db$

$G.M = +12\,db$.

Phase Margin (P.M)

The phase margin is

$$P.M = 180^\circ + \angle G(j\omega).H(j\omega)$$

The gain crossover frequeny is 2.5 rad/sec.

$$\angle G(j\omega) H(j\omega) = -153^\circ.$$

$$P.M = 180^\circ + (-153^\circ) = 27^\circ.$$

G.M & P.M both are +ve, Hence the closed loop system is Stable.

For Stable systems:

The gain cross over frequency < phase crossover frequeny.

For Unstable system:

The gain cross over frequency > phase cross over frequeny.

For Marginally Stable system

The gain cross over frequency = phase crossover frea

Gain Cross over frequency:-

The frequency at which the Gain plot crosses the '0' db axis.

phase Cross over frequency:-

The frequency at which the ~~Ga~~ phase plot Cross the '0' db axis.

1 NYQUIST PLOT

PRINCIPLE OF ARGUMENT

Principle of argument states that if there are P poles and Z zeros are enclosed by the 's' plane closed path, then the corresponding G(s).H(s) plane must encircle the origin P-Z times.

Number of encirclements

$$N = P - Z.$$

If the enclosed 's' plane close path contains only poles, then the direction of the encirclement in the G(s)·H(s) plane will be opposite to the direction of the closed path in the 's' plane.

If the enclosed 's' plane close path contains only zeros, then the direction of the encirclement in the G(s)·H(s) plane will be in the same direction as that of enclosed path in the 's' plane.

fig 9.3 fig 9.4

NYQUIST STABILITY CRITERION.

Let us now apply the principle of argument to the entire right half of 's' plane by selecting it as a closed path.

This selecting path is called the Nyquist Contour.

We know that the closed loop control system is stable if all the poles of the closed loop transfer function are in the left half on 's' plane.

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15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

poles of the closed loop transfer function are the nothing but the roots of the characteristic equ^n.
i.e $1+G(s)\cdot H(s)=0$.
As the order of the characteristic equ^n increases it is difficult to find the roots.

The poles of the characteristic equ^n $(1+G(s)\cdot H(s)=0)$ are same as that of the poles of the open loop transfer function $(G(s)\cdot H(s))$.

The zeros of the characteristic equ^n $(1+G(s)\cdot H(s)=0)$ are same as that of the zeros of the closed loop transfer function.

In order for the system to be stable there should be no zeros of $q(s)=1+G(s)\cdot H(s)$ on the right half s-plane.

$$Z=0$$
$$N=P$$

In special case of $P=0$ (ie the open loop stable system) the closed loop system is stable if

$$N=P=0.$$

Which means the net encirclements of the origin of the $q(s)$ plane by the Γ_q contour should be zero.

$$G(s)\cdot H(s)=[1+G(s)\cdot H(s)]-1$$

Γ_{GH} contour of $G(s)H(s)$. Corresponding to Nyquist contour in the s plane is the same as contour Γ_q of $1+G(s)\cdot H(s)$ drawn for the point $-1+j0$.

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		1	2	3	1	2
3	7	8	9	10	8	9
10	14	15	16	17	15	16
17	21	22	23	24	22	23
24	28	29	30	31	29	30

Encirclement of the origin by the contour Γ_q is equivalent to the encirclement of the point $(-1+j0)$ by the contour Γ_{GH}.

Fig 9.6. Fig 9.7

Along C_1
$s = j\omega$ with ω varying from $-j\infty$ to $+j\infty$.

and along C_2
$s = \underset{R\to\infty}{R e^{j\theta}}$ with θ varying from $+\pi/2$ to 0 to $-\pi/2$.

Statement of Nyquist Stability criterion.

If the contour Γ_{GH} of the open loop transfer function $G(s)H(s)$ corresponding to the Nyquist Contour in the s-plane encircles the point $(-1+j0)$ in the counter clockwise dir^n as many as times as the number of right half s-plane poles of $G(s)H(s)$, the closed loop system is stable.

Example 1.

RULES FOR DRAWING NYQUIST PLOTS

1) Locate the poles and zeros of the open loop transfer function G(s).H(s). in s-plane.

2) Draw the polar plot varying ω from 0 to ∞. if the poles or zero present at s=0.

3) Draw the mirror image of above polar plot for values of ω ranging from -∞ to 0⁻.

4) The number of infinite radius half circles will be equal to the number of poles or zeros at origin.

The infinite radius half circle will start at the point where the mirror image of the polar plot ends.

And the infinite radius half circle will end at the point where the polar plot starts.

After drawing the Nyquist plot, we can find the stability of the closed loop control system using the Nyquist stability criterion.

If the critical point (-1+j0) lies outside the encirclement, then the close loop control system is absolute stable.

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STABILITY ANALYSIS USING NYQUIST PLOT.

From the Nyquist plots, we can identify whether the control system is stable, marginally stable, unstable based on parameter.
(i) Gain cross over frequency (ω_{gc}) and phase cross over frequency (ω_{pc})
(ii) Gain Margin and phase Margin.

phase cross over frequency (ω_{pc})

The frequency at which the Nyquist plot intersects the negative real axis (phase angle is 180°) is known as phase cross over frequency (ω_{pc}).

Gain crossover frequency (ω_{gc})

The frequency at which the Nyquist plot is having the magnitude of one is known as ~~gain~~ gain cross over frequency (ω_{gc}).

For stable system $\omega_{pc} > \omega_{gc}$.

For Marginally stable system $\omega_{pc} = \omega_{gc}$.

For unstable system $\omega_{pc} < \omega_{gc}$.

Gain Margin: (GM)

The gain margin GM is equal to the reciprocal of the Magnitude of the Nyquist plot at the phase crossover frequency.

$$G.M = \frac{1}{M_{pc}}$$

Where M_{pc} is the magnitude at phase cross over frequency in normal scale.

Phase Margin: (PM)

The phase margin (PM) is equal to the sum of 180° and the phase angle at the gain cross over frequency.

$$PM = 180^\circ + \phi_{gc}$$

Where ϕ_{gc} is the phase angle at gain cross over frequency.

For stable system GM > 1, PM is +ve.

For marginally stable system GM = 1 PM = 0 degree

For unstable system GM < 1 PM is -ve.

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22	23	24	25	26	27	28

NYQUIST STABILITY CRITERION APPLIED TO INVERSE POLAR PLOT.

Occasionally, it is found more convenient to work with the inverse function $\frac{1}{G(j\omega)H(j\omega)}$ rather than the direct function $G(j\omega)H(j\omega)$.

Nyquist Stability Criterion can be applied to inverse polar plot from direct polar plot after minor modification.

Let us consider a open-loop transfer function.

$$G(s)H(s) = K\frac{(s+Z_1)(s+Z_2)\cdots(s+Z_m)}{(s+P_1)(s+P_2)\cdots(s+P_n)} ; m \le n. \quad -(1)$$

For stable system, no roots of the characteristic eqn should lie in the right half of s-plane.

$$q(s) = 1+G(s)H(s) = \frac{(s+Z_1')(s+Z_2')\cdots(s+Z_n')}{(s+P_1)(s+P_2)\cdots(s+P_n)} \quad -(2)$$

Dividing eqn (2) by eqn (1) we get

$$q'(s) = \frac{1}{G(s)H(s)}+1 = \frac{(s+Z_1')(s+Z_2')\cdots(s+Z_n')}{(s+Z_1)(s+Z_2)\cdots(s+Z_m)} \quad -(3)$$

From eqn (2) & (3), it is found that

(i) Zeros of $q(s)$ & $q'(s)$ are same.

(ii) poles of $q(s)$ and $G(s)\cdot H(s)$ are same.

(iii) poles of $q'(s)$ and $\frac{1}{G(s)\cdot H(s)}$ are same.

and also same with Zeros of $G(s)\cdot H(s)$.

If $\frac{1}{G(s)H(s)}$ has P right half s-plane poles

and characteristic eqn has Z right half s-plane zeros.

The Locus $\frac{1}{G(s)H(s)}$ encircles the point $(-1+j0)$

N-times in counter clockwise dirn.

$$N = P - Z$$

For stability $Z = 0$,

so, $N = P$.

If the Nyquist plot $\frac{1}{G(s) \cdot H(s)}$, corresponding

to the Nyquist contour in the s-plane.

encircles $(-1+j0)$ in counter clockwise as many as

the right half s-plane poles of $\frac{1}{G(s) \cdot H(s)}$.

Then the close loop system is stable.

special case of no poles on right half s-plane

of $\frac{1}{G(s)H(s)}$

$N = 0$, for stable system.

Exa 9.9

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RELATIVE STABILITY FROM NYQUIST PLOT.

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CONSTANT - M-CIRCLES (MAGNITUDE).

The open-loop transfer function $G(s)$ of a unity feedback control system is a complex quantity.

$$G(s) = x + jy \quad , \quad H(s) = 1$$

$$M = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s).H(s)} \qquad s = j\omega.$$

$$\text{Magnitude} = M = \frac{x+jy}{1 + x + jy} \quad \text{——— (1)}$$

Taking modulus

$$|M| = \frac{\sqrt{x^2+y^2}}{\sqrt{(1+x)^2 + y^2}} \quad \text{——— (2)}$$

Squaring eqn (2) and simplifying

$$M^2 = \frac{x^2+y^2}{(1+x)^2 + y^2}$$

$$\Rightarrow M^2\left[(1+x)^2 + y^2\right] = x^2 + y^2$$

$$\Rightarrow M^2\left(1 + x^2 + 2x + y^2\right) = x^2 + y^2$$

$$\Rightarrow M^2 + M^2x^2 + 2M^2x + M^2y^2 = x^2 + y^2$$

$$\Rightarrow (1-M^2)x^2 - 2M^2x + (1-M^2)y^2 = M^2.$$

$$\Rightarrow x^2 - \left(\frac{2M^2}{1-M^2}\right)x + y^2 = \frac{M^2}{1-M^2} \quad (3)$$

Making perfect square adding $\left(\frac{M^2}{1-M^2}\right)^2$ to both side to equⁿ (3)

$$\Rightarrow x^2 - \frac{2M^2}{1-M^2}x + \left(\frac{M^2}{1-M^2}\right)^2 + y^2 = \frac{M^2}{1-M^2} + \left(\frac{M^2}{1-M^2}\right)^2$$

$$\Rightarrow \left(x - \frac{M^2}{1-M^2}\right)^2 + y^2 = \frac{M^2(1-M^2)+M^4}{(1-M^2)^2}$$

$$\Rightarrow \left(x - \frac{M^2}{1-M^2}\right)^2 + y^2 = \left(\frac{M}{1-M^2}\right)^2 \quad (4)$$

For different values of M, equ^n (4) represents a family of circles with centre at

$$\left(x = \frac{M^2}{1-M^2},\ y = 0\right)$$

and radius $\frac{M}{1-M^2}$

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For a particular circle the values of M (Magnitude of closed loop transfer function) is const, therefore these circles are called const. M-circles.

Fig 7.11.1.

CONSTANT N-CIRCLE (PHASE ANGLES)

From eqn (1) the phase angle of the closed loop transfer function of a unity feedback control system is given by.

$$\phi = \angle \frac{C(s)}{R(s)} = \angle \frac{x+jy}{1+x+jy} \quad \text{—— (5)}.$$

The phase angle ϕ

$$\phi = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{y}{1+x}\right)$$

Taking tan on both sides.

$$\tan\phi = \tan\left(\tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{y}{1+x}\right)\right).$$

$$= \frac{\tan\tan^{-1}\left(\frac{y}{x}\right) - \tan\tan^{-1}\frac{y}{1+x}}{1 + \tan\tan^{-1}\frac{y}{x} \cdot \tan\tan^{-1}\left(\frac{y}{1+x}\right)}$$

$$= \frac{\frac{y}{x} - \frac{y}{1+x}}{1 + \frac{y}{x} \times \frac{y}{1+x}}.$$

$$\tan\phi = \frac{\dfrac{y(1+x) - yx}{(x(1+x))}}{\dfrac{x(1+x) + y^2}{(x(1+x))}}$$

$$\tan\phi = \frac{y}{x^2+x+y^2}$$

put $\tan\phi = N$

$$N = \frac{y}{x^2+x+y^2}$$

$$\Rightarrow \quad x^2+x+y^2 = \frac{y}{N}$$

$$\Rightarrow \quad x^2+x+y^2 - \frac{y}{N} = 0.$$

Making perfect square

$$x^2 + 2\cdot x\cdot\frac{1}{2} + \frac{1}{4} - \frac{1}{4} + y^2 - 2\cdot\frac{y}{2N} + \frac{1}{4N^2} - \frac{1}{4N^2} = \frac{1}{4} + \frac{1}{4N^2}.$$

$$= \left(x+\frac{1}{2}\right)^2 + \left(y - \frac{1}{2N}\right)^2 = \left(\frac{1}{4} + \frac{1}{4N^2}\right) \quad (6)$$

for different values of N equⁿ (6) represent a family of circle with centre at $x = -\frac{1}{2}$, $y = \frac{1}{2N}$.

$$r = \sqrt{\frac{1}{4} + \frac{1}{4N^2}}$$